ALGEBRAIC FORMULÆ:

SHOWING THE METHOD OF

DEDUCING THE MOST IMPORTANT RULES

OF

Arithmetic and Mensuration:

WITH EXAMPLES

ILLUSTRATING THEIR USE AND APPLICATION.

BY JOHN SANGSTER,

TEACHER IN THE NORMAL AND MODEL SCHOOLS, TORONTO.

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ALGEBRAIC FORMULÆ.

I. RATIOS AND PROPORTION.

Let a, b, c, and d be any four quantities such that $a : b :: c : d$; then $a : b = \frac{a}{b}$; $c : d = \frac{c}{d}$; and $\because a : b = c : d$; $\frac{a}{b} = \frac{c}{d}$; $ad = bc$ $\therefore d = \frac{bc}{a}$ (1).

II. INTEREST

Let $P =$ Principal, $I =$ Interest, $A =$ Amount, $t =$ Time in years, and $r =$ Interest of *one* pound for *one* year. $I = Prt$ (2); $\therefore P = \frac{I}{rt}$ (3); $r = \frac{I}{Pt}$ (4); and $t = \frac{I}{Pr}$ (5); $A = P + I = P + Prt = P(1+rt)$ (6); $\therefore P = \frac{A}{1+rt}$ (7); $r = \frac{A-P}{Pt}$ (8); and $t = \frac{A-P}{Pr}$ (9). From (9) the time in which any sum as S will amount to n times S is represented by $t = \frac{nS-S}{Sr} = \frac{n-1}{r}$ (10); $\therefore n = tr + 1$ (11); and $r = \frac{n-1}{t}$ (12).

III. DISCOUNT.

D = Discount; other symbols as before.

$D = A - P =$ from (6) and (7) $\frac{A(1+rt)}{1+rt} - \frac{A}{1+rt} = \frac{Art}{1+rt}$ (13);

$\therefore$ Present Worth or $P = \frac{A}{1+rt}$ (14).

IV. COMPOUND INTEREST.

Since £1 at the end of 1st year amounts to $1 + r$.

$1 : 1 + r :: P : P(1+r)$ = Amount of P at the end of the 1st year.

$1 : 1 + r :: P(1+r) : P(1+r)^2$ = " ....2nd "

$1 : 1 + r :: P(1+r)^2 : P(1+r)^3$ = " ....3rd "

And so on, therefore at the end of the tth year $A = P(1+r)^t$ (15);

$\therefore P = \frac{A}{(1+r)^t}$ (16); $r = \sqrt[t]{\left(\frac{A}{P}\right)} - 1$ (17); $t = \frac{\log. A - \log. P}{\log. (1+r)}$ (18). From (18) the time in which any sum, as S, will amount to n times S at Compound Interest is represented by $t = \frac{\log. n.}{\log. (1+r)}$ (19).

V. ARITHMETICAL PROGRESSION.

Let a = first term, l = last term, d = common difference, n = number of terms, and S = sum of the series.

$S = a + (a+d) + (a+2d) + (a+3d) + \ldots\ldots (l-d) + l$, Reversing the series.

$S = l + (l-d) + (l-2d) + (l-3d) + \ldots (a+d) + a$ Adding.

$2S = (a+l) + (a+l) + (a+l) + (a+l) \ldots (a+l)$ to n terms $= n(a+l)$.

$\therefore S = \frac{n}{2}(a+l)$ (20); $a = \frac{2S}{n} - l$ (21); $l = \frac{2S}{n} - a$ (22);

$n = \frac{2S}{a+l}$ (23). Also $l = a + (n-1)\,d$ (24).

$\therefore\ a = l - (n-1)\,d$ (25); $d = \frac{l-a}{n-1}$ (26); $n = 1 + \frac{l-a}{d}$ (27). Substitute (24) in (20); and,

$S = \frac{n}{2}\left\{ 2\,a + (n-1)\,d \right\}$ (28); $\therefore\ a = \frac{S}{n} - \frac{(n-1)\,d}{2}$ (29); $d = \frac{2\,(S - an)}{n\,(n-1)}$ (30); and $n = \frac{d - 2a \pm \sqrt{(2\,a - d)^2 + 8\,d\,S}}{2\,d}$ (31). Substitute (25) in 28 and, $S = \left\{ 2\,l - (n-1)\,d \right\} \frac{n}{2}$ (32);

$\therefore\ d = \frac{2\,(\mathrm{nl} - S)}{n\,(n-1)}$ (33); $l = \frac{S}{n} + \frac{(n-1)\,d}{2}$ (34); $n = \frac{d + 2\,l \pm \sqrt{(2\,l + d)^2 - 8\,d\,S}}{2\,d}$ (35). Substitute (27) in (20) and $S = \frac{a+l}{2} + \frac{l^2 - a^2}{2\,d}$ (36); $\therefore\ d = \frac{l^2 - a^2}{2\,\mathrm{S} - \mathrm{a} - \mathrm{l}}$ (37); $a = \frac{d \pm \sqrt{(2\,l + d)^2 - 8\,d\,S}}{2}$ (38); $l = \frac{\sqrt{(2\,a - d)^2 + 8\,d\,S} - d}{2}$ (39).

VI. GEOMETRICAL PROGRESSION.

Let r = common ratio, other symbols as before.

$S = a + ar + ar_2 + ar_3 + ar_4 + \ldots\ldots ar^{n-2} + ar^{n-1}$ (A). Multiplying by r.

$Sr = ar + ar^2 + ar^3 + ar^4 + \ldots\ldots ar^{n-2} + ar^{n-1} + ar^{n}$ (B). Subtracting (A) from (B).

$Sr - S = ar^{n} - a$, or, $S(r-1) = a\,(r^{n} - 1)\ \therefore\ S = \frac{a(r^n - 1)}{r-1}$ (40); $a = S\left(\frac{r-1}{r^n - 1}\right)$ (41); Also from (A) $l = ar^{n-1}$ (42); $\therefore\ a = \frac{l}{r^{n-1}}$ (43); $n = 1 + \frac{\log.\ l - \log.\ a}{\log.\ r}$ (44); and

$r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}}$ (45); multiplying (42) by r and substituting it in (40.)

$S = \frac{rl - a}{r - 1}$ (46); $a = rl - S\,(r - 1)$ (47); $l = S - \frac{S - a}{r}$ (48); $r = \frac{S - a}{S - l}$ (49). When n is infinite, and r a *proper* fraction, ar^n in (40) becomes $= o$. Hence for an *Infinite Series* $S = \frac{a}{1 - r}$ (50); $a = S\,(1 - r)$ (51); and $r = \frac{S - a}{S}$ (52).

VII. ANNUITIES AT SIMPLE INTEREST.

Let $A =$ a single payment of the Annuity, $M =$ Amount, $t =$ number of payments, and $r =$ Interest of *one* pound for *one period*. Then when the annuity is forborne any number of payments, the last payment being received at the time it falls due, $= A$; last but one $= A + Ar$, last but two $= A + 2Ar$, last but three $= A + 3Ar$, 1st $= A + (t - 1)\,Ar$; hence $M = A + (A + Ar) + (A + 2Ar) + (A + 3Ar) + \ldots\ldots\ldots (A + (t - 1)\,Ar)$. Whence from (28), $M = At\left(1 + \frac{(t - 1)\,r}{2}\right)$ (53), $A = \frac{2\,M}{t\,(2 + r\,(t - 1))}$ (54); $r = \frac{2\,(M - At)}{At\,(t - 1)}$ (55); and $t = \frac{\sqrt{8\,r\,\frac{M}{A} + (2 - r)^2} - (2 - r)}{2\,r}$ (56); Let $v =$ present value of an annuity to continue any number of payments; from (6) $v\,(1 + rt) = M = At\left(1 + \frac{(t - 1)\,r}{2}\right)$; hence $v = \frac{2 + (t - 1)\,r}{2}\left(\frac{At}{1 + rt}\right)$ (57); $r = \frac{2\,(At - v)}{(2\,v - (t - 1)\,A)\,t}$ (58); $A = \frac{2\,v\,(tr + 1)}{(2 + (t - 1)r)}$ (59).

VIII. ANNUITIES AT COMPOUND INTEREST.

Symbols same as before. Then last payment being received as before $= A$, last but one $= A(1+r)$ last but two $= A(1+r)^2$, last but three $= A(1+r)^3$; and so on, hence 1st payment $= A(1+r)^{t-1}$ $\therefore M = A + A(1+r) + A(1+r)^2 + A(1+r)^3 + \ldots\ldots\ldots A(1+r)^{t-1}$; a geometrical progression, whence by (40) $M = \frac{A((1+r)^t - 1)}{r}$ (60); $A = \frac{Mr}{(1+r)^t - 1}$ (61); and $t = \frac{\log.(Mr + A) - \log. A}{\log.(1+r)}$ (62). From (15) $v(1+r) = M = \frac{A((1+r)^t - 1)}{r} \therefore v = \frac{A((1+r)^t - 1)}{r(1+r)^t}$ (63); $A = \frac{vr(1+r)^t}{(1+r)^t - 1}$ (64); and $t = \frac{\log. A - \log.(A - vr)}{\log.(1+r)}$ (65). To find the present value of an annuity which is to commence after s years and continue for t years; from (63) v for $s + t$ years $= \frac{A}{r}\left(\frac{(1+r)^{s+t} - 1}{(1+r)^{s+t}}\right)$; and for t years only $v = \frac{A}{r}\left(\frac{(1+r)^t - 1}{(1+r)^t}\right)$ $\therefore$ for t years to commence after s years, $v = \frac{A}{r}\left(\frac{1}{(1+r)^s} - \frac{1}{(1+r)^{s+t}}\right)$ (66).

When an annuity lasts for ever, as in the case of landed property, $\frac{1}{(1+r)^t}$ in (63) $= \frac{1}{\infty} = 0$; hence for a perpetual annuity $v = \frac{A}{r}$ (67); $A = vr$ (68); $r = \frac{A}{v}$ (69). The present value of a freehold estate to a person to whom it will revert after s years is found from (66) and is represented by $v = \frac{A}{r(1+r)^s}$ (70).

IX. AREAS OF SURFACES.

Let $A =$ area, $s =$ side, $d =$ diagonal, then

Square.—$A = s^2$ (71); $s = \sqrt{A}$ (72); $A = \frac{d^2}{2}$ (73); $d = \sqrt{2A}$ (74).

Rectangle.—Let $b =$ base, and $p =$ perpendicular, $A = bp$ (75); $b = \frac{A}{p}$ (76); $p = \frac{A}{b}$ (77); also $A = b\sqrt{(d+b)(d-b)}$ (78). *Parallelogram.*—$A = bp$ (79).

Triangle.—$A = \frac{bp}{2}$ (80); $b = \frac{2A}{p}$ (81); $p = \frac{2A}{p}$ (82).

Let $a, b, c,$ be the three sides of any triangle and let $s = \frac{a+b+c}{2}$, $A = \sqrt{s(s-a)(s-b)(s-c)}$ (83); when the triangle is equalateral $A = \frac{b^2\sqrt{3}}{4}$ (84).

Quadrulateral in a circle or whose *opposite* angles $=$ two right angles.—$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$ (85), where $s =$ half the sum of the four sides.

Regular Polygon.—Let $s =$ side, $n =$ number of sides, $p =$ apothem or perpendicular from the centre. $A = \frac{nsp}{2}$ (86).

Circle.—Let $c =$ circumferance, $r =$ radius, $d =$ diameter, $\pi = 3{\cdot}1416$. $c = 2r\pi$ (87); $r = \frac{c}{2\pi}$ (88); $A = \pi r^2$ (89); $A = \frac{cr}{2}$ (90); $A = mc^2$ (91); where $m = {\cdot}0796$.

Sector.—$A = \frac{rl}{2}$ (92); where $l =$ length of circular arc $A = \frac{\pi n r^2}{360^\circ}$ (93); where $n =$ number of degrees in the arc.

Circular Annulus.—$A = m(c+c')(c-c')$ (94); where c' $=$ circumference of inner circle; and $A = \frac{\pi(d+d')(d-d')}{4}$ (65); where $d' =$ the diameter of the inner circle.

Ellipse.—Let $C=$ circumference, $t=$ transverse axis, $c=$ conjugate axis, $a=$ abscissa, $o=$ ordinate, $d=$ distance of abscissa from centre. $C=\frac{\pi\,(t+c)}{2}$ (96); $A=\frac{\pi tc}{4}$ (97); $o=\frac{c}{t}\sqrt{(t-a)\,a}$ (98); $a=\frac{t}{2}\pm d$; and $d=\frac{t}{c}\sqrt{\left\{(\frac{c^2}{2}+o)\,(\frac{c^2}{2}-o)\right\}}$ (99); $t=\frac{ca}{o^2}\left\{\frac{c}{2}+\sqrt{(\frac{c^2}{4}-o^2)}\right\}$ (100); $c=\left\{\frac{ot}{\sqrt{(t-a)a}}\right\}$ (101).

Parabola.—Let $p=$ parameter, a, and $a'=$ abscissas, o and o' $=$ ordinates, $b=$ base, $=$ double ordinate, $l=$ length of parabolic curve, $p=\frac{o^2}{a}$ (102); $o'=o\sqrt{\left(\frac{a'}{a}\right)}$ (103); $a'=a\left(\frac{o'}{o}\right)^2$ (104); $A=\frac{2ab}{3}$ (105); and $l=2\sqrt{\left(o^2+\frac{4a^2}{3}\right)}$ (106).

Hyperbola.—Symbols same as in ellipse.

$o=\frac{c}{t}\sqrt{(t+a)\,a}$ (107); $a=\frac{t}{2}\pm d$; and $d=\frac{t}{c}\sqrt{\left(\frac{c^2}{4}+o^2\right)}$ (108); $c=\frac{ot}{\sqrt{(t+a)\,a}}$ (109); $t=\frac{ca}{o^2}\left\{\frac{c}{2}\pm\sqrt{(\frac{c^2}{4}+o^2)}\right\}$ (110); and $A=4ca\left\{\frac{3\sqrt{7a(7t+5a)}+4\sqrt{ta}}{75t}\right\}$ (111.)

Regular solids.—Let $s=$ surface, $v=$ volume, $e=$ one edge

Tetraedron.—$V=\frac{e^3\sqrt{2}}{12}$ (112); and $s=e^2\sqrt{3}$ (113). *Hexaedron or cube.*—$V=e^3$ (114); $s=6\,e^2$ (115). *Octaedron.*—$V=\frac{e^3\sqrt{2}}{3}$ (116); $s=2e^2\sqrt{3}$ (117).

Dodecaedron.—$V=5e^3\times 1.53262$ (118); $s=15e^2\times 1.376385$ (119).

Icosaedron.—$V=\frac{5e^3}{6}\times 2.61803$ (120); and $s=5e^2\sqrt{3}$

Cylinder or prism.—$S = ph + 2\ A$ (122); where $p =$ perimeter of base, $h =$ perpendicular height, and $A =$ area of base.

Pyramid or cone.—$S = \frac{pl}{2} + A$ (123); where $l =$ length of slant side.

Frustum of Pyramid or Cone. $S = \frac{P + P'}{2} l + A + A'$ (124); where P' and A' represent the perimeter and area of smaller base.

Sphere. $S = 4\pi r^2$ (125); $S = cd$ (126).

Spherical Segment. $S = 2\pi rh$ (127); where $h =$ height of segment.

X. VOLUME OF SOLIDS.

$V =$ volume, $h =$ height, then,

Prism or Cylinder. $V = Ah$ (128); $A = \frac{V}{h}$ (129).

Pyramid or Cone. $V = \frac{Ah}{3}$ (130); $A = \frac{3V}{h}$ (131).

Frustum of Pyramid or Cone. $V = \frac{h}{3}(A + A' + \sqrt{A A'})$ (132), where A and A' represent the areas of the ends of the Frustum.

Sphere. $V = \frac{4\pi r^3}{3}$ (133); $r = \sqrt[3]{\left(\frac{3v}{4\pi}\right)}$ (134).

Spherical Segment. $V = \frac{\pi}{6}(3d - 2h)h^2$ (135); $V = \frac{\pi}{6}(3r^2 + h^2)h$ (136). where $r =$ radius of base of segment, and $d =$ diameter of sphere.

Spherical Zone. $V = (r^2 + r'^2 + \frac{1}{3}h^2)\frac{\pi h}{2}$ (137); where r and $r' =$ the radii of the ends; for the middle zone,

$V = (d^2 - \frac{1}{3} h^2) \frac{\pi h}{4}$ (138); or $V = (d'^2 + \frac{2}{3} h^2) \frac{\pi h}{4}$ (139); where d and d' = the diameters of the sphere and of the segment.

Spheroid. $V = \frac{KPE^2}{6}$ (140), where P = Polar and E = Equatorial axis.*

EXAMPLES.

NOTE.—The numbers inclosed within parentheses, *besides* serving to number the questions, indicate the Formula to be used.

(1) Find the fourth proportional—
to 9, 36 and 17. Ans. d=68:
Also to 11, 165 and 14. Ans. d=210.

(2) What is the Interest of £537 12s. 6d. for 5 years at 4 per cent? Ans. I=£107 10s. 6d.

(3) What principal will produce £117 1s. 10¾d. in 7 years at 6 per cent? Ans. P=£278 16s.

(4) At what rate per cent will £672 16s. 4d. produce £518 1s. 4½d. in 7 years? Ans. r=11 per cent.

(5) In what time will £2000 produce £280 at 3½ per cent? Ans. t=4 years.

(6) What is the amount of £219 16s. for 3 years at 6 per cent? Ans. A=£259 7s. 3½d.

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* A spheroid is generated by the revolution of an ellipse about one of its axes; the fixed axis is called the Polar, the revolving one the Equatorial. In an oblate spheroid the transverse axis revolves: in a prolate spheroid the conjugate.

(7) What principal will amount to £387 16s. $3\frac{1}{2}$d. in $5\frac{3}{4}$ years at $3\frac{3}{4}$ per cent? Ans. P=£319 0s. 6d.

(8) At what rate per cent will £537 12s. 6d. amount to £645 3s. in five years? Ans. r=4 years.

(9) In what time will £926 12s. amount to £1130 9s. $0\frac{1}{2}$d. at 4 per cent? Ans. t=$5\frac{1}{2}$ years.

(10) In what time will £130 double itself at 7 per cent? Ans. t=$14\frac{2}{7}$ years.

In what time will any sum of money quadruple itself at 6 per cent? Ans. t=50 years.

(11) To how many times itself will any sum of money amount in 25 years at 8 per cent? Ans. n=3.

(12) At what rate per cent will any sum of money amount to 6 times itself in 20 years? Ans. r=$2\frac{1}{2}$.

(13) What is the discount on a bill of £550 10s. payable in nine months at 5 per cent? Ans. D=£19 17s. $11\frac{1}{4}$d.

(14) What is the present worth of a bill for £100 payable in 2 years at 5 per cent. Ans. P=£90 18s. $2\frac{2}{11}$d.

(15) What is the amount and compound interest of £142 for 8 years at 3 per cent half-yearly? Ans. A=£227 17s. $4\frac{1}{2}$d. and I=£85 17s. $4\frac{1}{2}$d.

(16) What sum will amount to £432 in 8 years at 3 per cent Compound Interest. Ans. P=£341 0s. 4d.

(17) At what rate per cent will £500 amount to £629 17s. $1\frac{11}{25}$d. in 3 years allowing Compound Interest? Ans. r=8 per cent.

(18) In what time will £1500 arise from £1219 12s. 9d. at 3 per cent allowing Compound Interest? Ans. t=7 years.

(19) In what time willl any sum of money double itself at 5 per cent Compound Interest. Ans. t=$14\frac{1}{4}$ years.

(20) Find the sum of the series of which 6 is the first term, 796 the last term, and 80 the number of terms. Ans. S=32080.

(21) Find the first term of the series whose sum=444, last term =70 and number of terms=12. Ans. a=4.

(22) A person travels 300 miles in 10 days, his first day's journey was one mile, what was his last? Ans. $l=59$ miles.

(23) A boy won 33700 marbles, his winnings were in arithmetical progression—on the first day he won 3 and on the last 671; how many days did he play? Ans. $n=100$ days.

(24) and (28) A body falling through space, falls through 16 feet the first second, through 48 feet the next second, 80 feet the next and so on, what will be its entire descent during 21 seconds and through how many feet will it fall the last second? Ans. $l=656$ feet, and $S=7056$ feet.

(25) A person purchasing an estate agrees to pay for it in 29 payments, each payment being £25 more than the preceding one; what must the first payment be that the last may =£897? Ans. $n=$£197.

(26) Given the number of terms$=7$; last term$=27$; first term $=3$; to find common difference. Ans. $d=4$.

(27) A drover bought a flock of sheep, paying 8s. for the first, 8s. 6d. for the second, 9s. for the third, and £2 4s. for the last, how many sheep were there in the flock? Ans. $n=73$ sheep.

(28) Find the sums of the following series:

$1+5+9+13+$&c. to18 terms. Ans. $S=630$.
$2+2\frac{1}{3}+2\frac{2}{3}+3+$&c. to 16 terms. Ans. $S=56$.
$\frac{5}{7}+1+1\frac{2}{7}+$&c. to 15 terms. Ans. $S=40\frac{5}{7}$.
and of $-9-7-5-3-$&c. to 20 terms. Ans. $S=200$.

(29) A person worked 47 weeks upon condition that his wages should be increased 2s. every week, at the end of the time he received £110 9s. as the amount of his wages, what was the wages of the first week? Ans. $a=1$ shilling.

(30) Find the common difference of the series of which the first term$=2$, sum$=8675$, and number of terms$=50$. Ans. $d=7$.

(31) Given the sum of the series$=34850$, first term$=2$, and common difference$=7$; to find the number of terms. Ans. $n=100$.

(32) The last term of a series $=157$, common difference $=3$, and number of terms $=51$; find the sum of the series. Ans. $S=4182$.

(33) Given the last term of a series $=97$, number of terms $=11$, sum of the series $=2489$, to find the common difference. Ans. $d=20\frac{2}{5}$.

(34) A person spends $\frac{1}{2}$d. more on the 2nd January than on the first; $\frac{1}{2}$d. more on the 3rd than on the 2nd, and so on; at the end of the year he finds that he has spent £155 2s. 6d., what was his outlay on the 31st December. Ans. $l=16$s. 1d.

(35) Find the number of terms in the series of which 18000 is the sum, 10 the common difference, and 595 the last term. Ans. $n=60$.

(36) Find the sum of the odd numbers from 1 to 99 inclusive. Ans. $S=2500$: Also of the even numbers from 2 to 100 inclusive. Ans. $S=2550$.

(37) Given the sum of a series $=2625$, first term $=5$, and last term $=245$, to find the common difference. Ans. $d=12$.

(38) Find the first term of the series of which the sum $=2288$, last term $=95$, and common difference $=2$. Ans. $a=9$.

(39) What is the last term of the series of which the first term $=-5$, the sum $=196$, and the common difference $=11$. Ans. $l=61$.

(40) Required the sum of the series 1, 2, 4, 8, 16, &c. to 10 terms. Ans. $S=1023$. Required the sum of the series 2, 6, 18, 34, &c. to 8 terms. Ans. $S=6560$.

(41) What is the first term of the geometrical series of which the sum $=682$, the number of terms $=5$, and the common ratio $=4$? Ans. $a=2$.

(42) Required the last term of a series of which the first term $=3$, the number of terms $=10$, and the common ratio $=8$. Ans. $l=402653184$.

(43) What is the first term of a series when the last term $=1024000$, the common ratio $=4$, and number of terms $=6$? Ans. $a=1000$.

(44) Given the first term $=2$, the last term $=512$, and the common ratio $=4$, what is the number of terms? Ans. $n=5$.

(45) What is the common ratio of a series of which the first term $=64$, last term $=\frac{1}{65536}$, and number of terms $=12$? Ans. $r=\frac{1}{4}$.

(46) The extremes of a series are 12 and 175692 and the common ratio is 11; what is the sum? Ans. $S=193260$.

(47) Find the first term of a series of which the common ratio $=3$, the last term $=4374$, and the sum $=6560$. Ans. $a=2$.

(48) Given the sum of the series $=4095$, common ratio $=2$, first term $=1$, to find the last term. Ans. $l=2048$.

(49) Given the sum of the series $=1023$, the last term $=512$, and the first term $=1$, to find the common ratio. Ans. $r=2$.

(50) Find the sum of an Infinite series of which the first term $=\frac{7}{10}$ and the ratio $=\frac{1}{10}$. Ans. $S=\frac{7}{9}$.
Find the value of $\cdot\dot{4}6\dot{3}$ ad infinitum. Ans. $S=\frac{463}{999}$.

(51) Given the sum of an Infinite series $=2$, and the common ratio $=\frac{1}{2}$ to find the first term. Ans. $a=1$.

(52) Find the common ratio of an Infinite series of which the first term is 17 and the sum 18. Ans. $r=\frac{1}{18}$.

(53) What is the amount of an annuity of £436 forborne 12 years at $3\frac{1}{2}$ per cent. simple interest? Ans. M=£6239 3s. $2\frac{1}{2}$d.

(54) What annuity will amount to £385; if forborne 5 years at 5 per cent. simple interest? Ans. A=£70.

(55) At what rate per cent. will an annuity of £356 amount to £3972 19s. $2\frac{1}{2}$d.; if forborne 9 years, allowing simple interest? Ans. $r=6$ per cent.

(56) In what time will an annuity of £37 amount to £508 15s. at 5 per cent. simple interest? Ans. $t=11$ years.

(57) How much ready money should be paid for a pension of £30 to continue for 5 years at $4\frac{1}{2}$ per cent. simple interest? Ans. v=£133 9s. $4\frac{1}{2}$d.

(58) If £133 9s. $4\frac{1}{2}$d. ready money be paid for an annuity of £30 to continue 5 years, what is the rate per cent. simple interest? Ans r=$4\frac{1}{2}$ per cent.

(59) A person sells 6 years of his pension, which is payable quarterly with simple interest at 5 per cent. per annum, for £263 18s. 10d.; what was the annual value of his pension? A=£50.

(60) What will an annuity of £33 17s. 9d. amount to, in 14 years at 4 per cent. compound interest? Ans. M=£619 17s. 4d.

(61) What annuity, accumulating at $3\frac{3}{4}$ per cent. compound interest, will amount to £600, in 40 years? Ans. A=£6 13s. 11d.

(62) In how many years will an annuity of £8 per annum, amount to £187 6s. $3\frac{3}{4}$d. at 3 per cent. compound interest? Ans. t=18 years.

(63) What is the present value of an annuity of £154 for 19 years, at 5 per cent. compound interest? Ans. v—£1861 2s. $7\frac{1}{4}$d.

(64) What annuity to continue 12 years at £4 5s. per cent. compound interest may be purchased for £231 5s. $2\frac{1}{2}$d. ready money? Ans. A=£25.

(65) For how many years may an annuity of £22 be purchased for £308 12s. 10d., allowing compound interest at 4 per cent? Ans. t=21 years.

(66) Required the present value of a deferred annuity of £80 to be entered upon at the expiration of 12 years, and then to be continued for 7 years, at 4 per cent. compound interest? Ans. v=£299 18s. 1d.

(67) What is the present value of an estate whose rental is \$1500 allowing 5 per cent. compound interest? Ans. v=30000 dollars, or 20 years' purchase.

(68) What is the annual rental of a freehold estate purchased for £3000 when the rate of interest is at 4 per cent? Ans. A=£120.

(69) If a perpetuity of £563 can be purchased for £11260 ready money, what is the rate of interest allowed? Ans. r=5 per cent.

(70) A freehold estate producing £75 per annum, is mortgaged for the period of 14 years; what is its present value reckoning compound interest, at 4 per cent? Ans. v=£1082 15s. 4d.

MENSURATION.

(71) How many acres are there in a square field whose side is 809 links? Ans. A=6 acres, 2 roods, $7\frac{1}{6}$ poles.

(72) What is the side of a square whose area is 3025 yards? Ans. s=55 yards.

(73) What is the area of a square room whose diagonal is 31 feet? Ans. A=$480\frac{1}{2}$ feet.

(74) Required the diagonal of a square table whose area is 16 square feet? Ans. d=5.65685 feet.

(75) Find the number of square inches in a sheet of paper whose length is 11 inches and breadth $8\frac{1}{2}$ inches. Ans. A=$93\frac{1}{2}$. inches.

(76) If the area of a rectangle, whose perpendicular is 11 yards, be 2112 yards, how long is its base? Ans. b=192 yards.

(77) The area of a rectangular pond is 43750 square yards, one side is 350 yards, what is the length of the other? Ans. P=125 yards.

(78) Find the area of a rectangle whose base is 21 and diagonal 35 yards. Ans. A=588 yards.

(79) Find the area of a parallelogram whose base=90, and perpendicular height=$12\frac{1}{2}$ feet. Ans. A=1125 feet.

(80) Required the area of a triangle whose base=81 feet, and altitude 46 feet. Ans. A=1863 feet.

(81) What is the base of a triangle whose area=2560, and perpendicular height 40 feet? Ans. b=128 feet.

(82) Required the altitude of a triangle whose area=117.5625 square yards, and base=49 feet 6 inches. Ans. P=42 feet, 9 inches.

(83) Find the area of a triangular field, whose sides are 1200, 1800 and 2400 links. Ans. A=10 acres, 1 rood, 33 poles.

(84) Find the area of an equilateral flower bed whose side=25 yards. Ans. A=270.625 yards.

(85) The four sides of a trapezium inscribed in a circle are 75, 40, 60 and 55 links, what is its area? Ans. A=3146.427 links

(86) Find the area of a park in the form of a regular octagon, whose side=12 chains, and apothem 14.485 chains. Ans. A=69 acres, 2 roods, 4.6 poles.

(87) What is the circumference of a circle whose diameter is 44 feet? Ans. c=138.23 feet.

(88) Required the diameter of a circle whose circumference is 78.54 yards. Ans. d=2r=25 yards.

(89) What is the area of a circle whose diameter=80 feet? Ans. A=5026.56 feet.

(90) Required the area of a circular garden whose diameter is 200 yards and circumference 628.32 yards. Ans. A=31416 square yards.

(91) Find the area of a circle whose circumference=200 feet. Ans. A=3184 square feet.

(92) The radius of a circle is 50 feet, what will be area of the sector whose circular arc is 30 feet in length? Ans. A=750 feet.

(93) Find the area of a sector, the circular arc of which contains 40°, the diameter being 60 feet. Ans. A=314.16 feet.

(94) Find the area of a circular annulus, the circumferences of the containing circles being 90 and 60. Ans. A=358.2.

(95) The diameters of two concentric circles are 50 and 30 feet, what is the area of the included annulus? Ans. A=1256.64 feet.

(96) Required the circumference of an ellipse whose diameters are 600 and 400. Ans. C=1570.8.

(97) What is the area of an ellipse whose diameters are 5 and 10? Ans. A=39.27.

(98) The axes are 30 and 10, and one abscissa 24; find the ordinate. Ans. o=4.

(99) The axes are 70 and 50, and an ordinate 20; what are the abscissas? Ans. a=56 and 14.

(100) The conjugate axis is 10, the smaller abscissa 6, and the ordinate 4; required the transverse axis. Ans. t=30.

(101) The transverse axis is 280, an ordinate 80, and one abscissa 56; what is the conjugate axis? Ans. c=200.

(102) If an ordinate of a parabola is 20, and its abscissa 36, what is the parameter? Ans. p=$11.\dot{1}$.

(103) Two abscissas are 9 and 16, and the ordinate of the former is 6, find that of the latter. Ans. o′=8.

(104) Given the two ordinates 6 and 8, and the abscissa of the former=9; to find that of the latter. Ans. a′=16.

(105) Find the area of a parabola whose base or double ordinate is 15, and height or abscissa 22. Ans. A=220.

(106) Required the length of the parabolic curve whose abscissa is 6, and ordinate 12. Ans. l=27.71.

(107) The transverse axis of an hyperbola is 15, the conjugate axis 9, the smaller abscissa 5; required the ordinate. Ans. o=6.

(108) The transverse and conjugate axes are 60 and 45, and one ordinate is 30; what are the abscissas? Ans. a=67½ and 7½.

(109) The transverse axis is 60, an ordinate 24, the smaller abscissa 20; what is the conjugate axis? Ans. c=36.

(110) The conjugate axis is 45, the less abscissa 30, and the ordinate 30; required the transverse axis. Ans. t=90.

(111) What is the area of an hyperbola whose transverse and conjugate axes=15 and 9, and the less abscissa=5? Ans. A=37.919.

(112) and (113) Find the solidity and surface of a tetrahedron whose edge=8. Ans. v=60.3: and S=110.85.

(114) and (115) Find the volume and surface of a cube or hexahedron whose edge is 11. Ans. v=1331: and S=726.

(116) and (117) Find the volume and surface of an octahedron whose edge is 10. Ans. v=471.4: and S=346.4.

(118) and (119) Find the volume and surface of a dodecahedron whose edge is 4. Ans. v=490.44: and S=330.33.

(120) and (121) Find the volume and surface of an icosahedron whose edge is 6. Ans. v=471.245: and S=311.76.

(122) What is the surface of a right cylinder whose length (h) =20 and circumferance (p)=6? Ans. S=125.73.

(123) What is the surface of a regular pentagonal pyramid, each side of its base being $1\frac{2}{3}$ feet, and its slant side 10 feet? Ans. S=46.4456.

(124) Find the surface of a frustum of a right cone, its length being 31, and the circumferences of its two ends 62.832, and 37.6992. Ans. S=1985.49.

(125) What is the surface of a sphere whose diameter=800 inches? Ans. S=2010624 inches.

(126) Find the surface of a globe whose diameter=12 and circumference 37.6992. Ans. S=452.39.

(127) Find the surface of a spherical segment whose height is 2, the diameter of the sphere being 10. Ans. S=62.832.

(128) What is the volume of a prism whose length is 18 feet, its base being a regular hexagon each side of which is 16 inches? Ans. v=83.138 feet.

(129) If the volume of a triangular prism is 7.656, and its length $10\frac{1}{2}$; what is the area of the base? Ans. A=·729.

(130) What is the solidity of a cone whose altitude is 12 feet, the diameter of its base being 10 feet? Ans. v=314.16.

(131) Find the area of the base of a cone whose volume is 282.74 and altitude 30. Ans. A=28.274.

(132) Required the volume of a frustrum of a square pyramid, the side of the greater base being 16, of the lesser 10; and its length 18. Ans. v=37152.

(133) What is the solidity of a sphere whose diameter is 30? Ans. v=14137.2.

(134) What is the diameter of a sphere whose volume is equal to 65449.85 feet. Ans. d=2r=50 feet.

(135) What is the solidity of a segment of a sphere, the height of the segment being 2, the diameter of the sphere 10? Ans. v=54.4544.

(136) What is the volume of a spherical segment, whose height is 10, and the diameter of its base 20? Ans. v=2094.4.

(137) Find the volume of a spherical zone, the diameters of its ends being 10 and 12, and its height 2. Ans. v=195.9159.

(138) Required the solidity of the middle zone of a sphere, its height being 32 feet, and the diameter of the sphere 40. Ans. v=31633.8.

(139) Find the volume of the middle zone of a sphere, its height being 8, and end diameters 6. Ans. v=494.278.

(140) Find the solidity of an oblate spheriod whose axes are 20 and 12. Ans. v=2513.28.

What is the volume of a prolate spheriod, its polar axis being 7, and equitorial axis 5? Ans. v=91.63.