EUCLID

PINNOCK'S CATECHISM OF Geometry AND the First Principles of Trigonometry.

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A

CATECHISM

OF

GEOMETRY;

CONTAINING

A VARIETY OF DIAGRAMS,

FAMILIARLY EXPLAINED;

TO WHICH ARE ADDED THE FRIST PRINCIPLES OF

TRIGONOMETRY.

FOR THE USE OF YOUNG PEOPLE.

EIGHTH EDITION.

London:
PRINTED FOR G. AND W. B. WHITTAKER,
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ADVERTISEMENT.

THE study of Geometry being the basis of all mathematical learning, and as no art or science can be practised without its assistance, the author hopes, that this little Work will be found as acceptable to the rising generation, as his other Catechisms; his object being to render the study of Geometry easy and familiar to young people.

PRINTED BY W. CLOWES, NORTHUMBERLAND-COURT, STRAND.

A

CATECHISM

OF

GEOMETRY.

CHAPTER I.

Introduction.

Question. WHAT is Geometry?

Answer. Geometry is that science which treats of lines, surfaces, and solids; and of extension and magnitude in general.

Q. Into how many parts is geometry divided?

A. Three; namely,

Lineametry, which refers to lines;

Planimetry, which refers to surfaces, and

Stereometry, which refers to solids.

Q. What difference is there between a line, a surface, and a solid?

A. A line is a space only in length; a surface consists of length, and width; a solid extends to length, width, and thickness.

Q. What country gave birth to this very useful science?

A. Egypt; and the most early cultivators of this science were *Thales*, *Pythagoras*, *Plato*, and *Euclid*; but the most celebrated of these is Euclid, whose "*Elements of Geometry*," have been held in the highest estimation by all nations, in all ages.

Q. To whom is geometry useful?

A. Geometry is of the greatest use to

The *Astronomer*, in measuring the distances, motions, and magnitudes of the heavenly bodies;

The *Geographer*, in determining the figure and magnitude of the earth, and in delineating the divisions of its empires, kingdoms, and states;

The *Architect*, in the structure of edifices, so as to give them strength and elegance;

Military Engineers, in the construction of forts, &c.;

Navigators, in conducting a fleet on the ocean; and is also of great use in music, drawing, perspective, optics, and all kinds of mechanics.

CHAP. II.

Definition of a Point, &c.

Q. What is a *definition?*

A. A definition is a short explanation of any name or term.

Q. What is a *point* ?*

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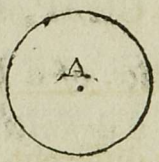
* The learner will observe, that a *point* in geometry or mathematics, is ideal, indivisible, and invisible, and that the dot made by a pen or pencil, usually called a point, is not in reality a mathematical point, but one that is real, divisible, and visible, and is made a datum from which geometrical figures may be more readily worked.

A. A point is that which has no parts or magnitude, and is usually made by the prick of a pin, point of a compass, pen, or pencil, as the point marked under the letter A.

A.

Q. What is a central point?

A. A central point is that from which a circle or circumference is made as at A.



Q. What is a secant point?

A. A secant point, or the point of intersection, is that when two or more lines cross one another, as the point at E.

CHAP. III.

Definition of a Line.

Q. What is a *line?*

A. A *line* is length without breadth; as the line A B*.

A_____B.

Q. How many different kinds of lines are there?

A. Three, a *right*, a *curve* and a *mixed line.*

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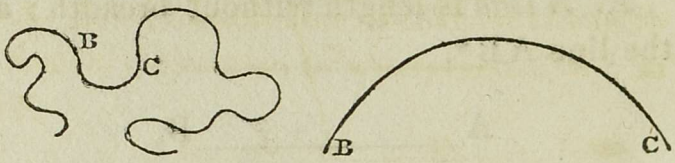
* What has been said of the point is also applicable to the definition of a line, for what is drawn upon paper with a pen or pencil, is not in fact a mathematical line, but the representation of a line; and, however fine this representation may be made, it will still have breadth and thickness. You will therefore notice, that lines in geometry are all imaginary.

Q. What is a *right line?*

A. A right or straight line is that which lies between its points, as A B above-mentioned.

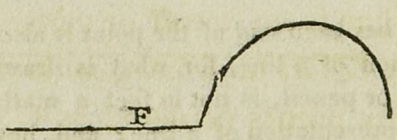
Q. What is a *curve line?*

A. A *curve line* is that which turns out of its way by one or more deviations; as B C.



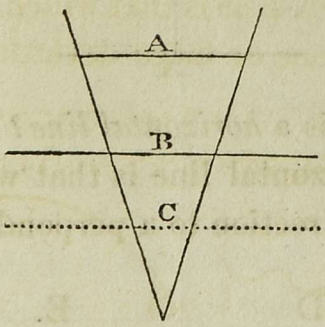
Q. What is a *mixed line?*

A. A mixed line is that which is both straight and a curve: as F.



Q. What is a *finite line?*

A. A *finite line* is that which is supposed to contain a necessary length; as A.



Q. What is an *infinite line?*

A. An *infinite line* is that which is undetermined having no precise length; as B.

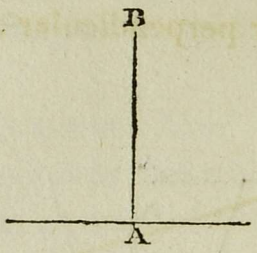
Q. What is an *occult* or *white line?*

A. An *occult* or *white line* is that which is made by points or dots; as C.

Q. What is a *perpendicular line?*

A. A perpendicular line is that which is

let fall or erected upon another, making the angles on each side equal; as A B.



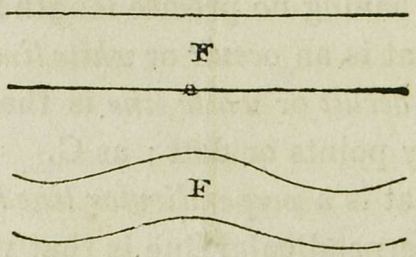
Q. What is a *horizontal line?*

A. A horizontal line is that which is in a contrary direction to a perpendicular; as D E.

D E.

Q. What are *parallel lines?*

A. Parallel lines are such as follow one another at an equal distance, and if infinitely continued, would never meet; as F.

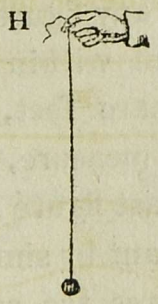


Q. What is an *oblique line?*

A. An *oblique line* is that which is neither horizontal nor perpendicular; as O P.

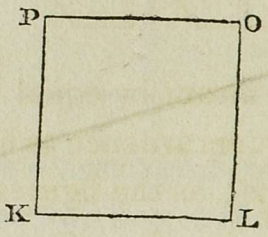
Q. What is a *plumb line?*

A. A plumb line is that which hangs down without inclining to the right or to the left, as H.



Q. What is the *base line* of a figure?

A. The base line of a figure is that upon which it is supposed to rest; as K L.



Q. What is the altitude of a figure?

A. The altitude of a figure is its perpendicular height from the base to the highest part; as K P, L O.

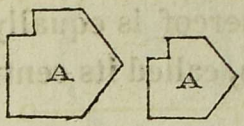
Q. What is meant by the *area* of a figure?

A. The area of any figure is the quantity of space contained within its boundaries, expressed in square feet, yards, or any other superficial measure, as the admeasurement of the last figure; K L O P.

Q. What is meant by similar figures?

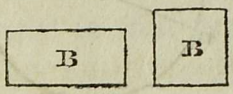
A. Similar figures are such as have the

same angles, and whose sides are in the same proportion; as the figures A, A.



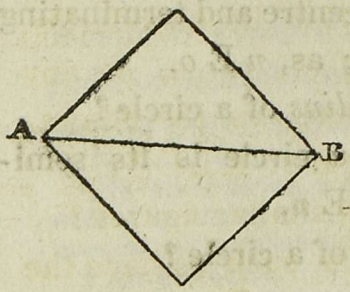
Q. What is meant by equal figures?

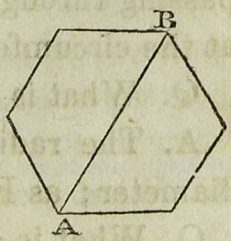
A. Equal figures are such as have the same area or contents; as the figures B. B.



Q. What is a *diagonal line?*

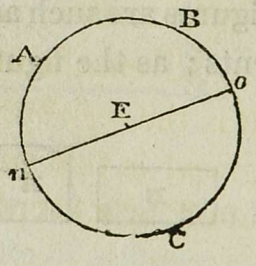
A. A *diagonal line* is that which crosses a figure from one angle or corner to another; as A B.





Q. What is a *circle*?

A. A circle is a plane figure bounded by a curve line called the circumference, as ABC, every part whereof is equally distant from a point within, called its centre; as E.



Q. What is the *diameter* of a circle?

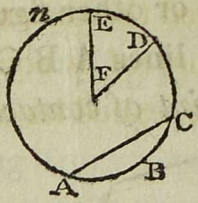
A. The diameter of a circle is a right line passing through the centre and terminating at the circumference; as, *n* E *o*.

Q. What is the *radius* of a circle?

A. The radius of a circle is its semi-diameter; as E *o*, or E *n*.

Q. What is an *arc* of a circle?

A. An *arc* of a circle is any part of the circumference; as A B C.



Q. What is a *chord* or *subtense*?

A. A *chord* or *subtense* is a right line that joins the two extremities of an arc; as, A C.

Q. What is the *segment* of a circle?

A. A *segment* is that portion of a circle which is cut off by a chord*; as A B C. and A *n* C.

Each segment made by a diameter is called a semi-circle†.

Q. What is meant by the *sector* of a circle?

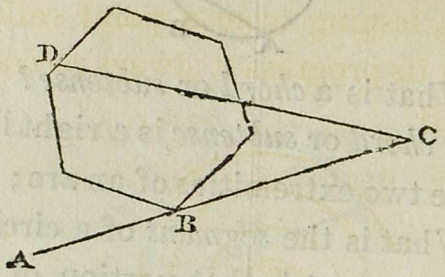
A. The sector is that portion of a circle which is comprehended between any two radii or semi-diameters and the intercepted part of the circumference; as D E F (above).

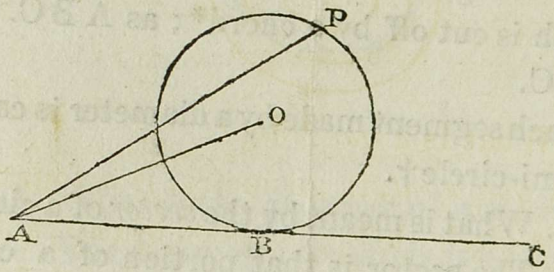
* In other terms, a chord is a right line, joining two ends of an arc, as A C; which divides the circle into two unequal parts, called *segments*.

† Or half circle.

Q. What is a *tangent?*

A. A tangent* in geometry is a line that touches a circle or other figure without cutting it, as the lines A B C. The point B is called the *point of contact.*





Q. What is a *secant?*

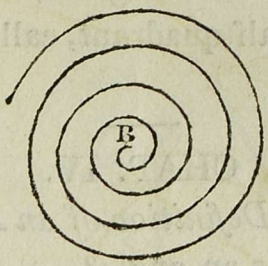
A. A *secant* in *geometry* is a line which cuts another, whether right or curved; as, C D and A P; but in *trigonometry* it is

* From *tango*, to touch.

a right line drawn from the centre of a circle to meet the extremity of any tangent to the same circle; as A O.

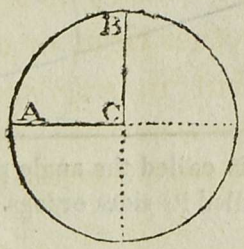
Q. What is a *spiral line?*

A. A *spiral line* is a curve line of the circular kind, issuing from a point called its centre; in its progress it always recedes, as the figure B.



Q. What is a *quadrant?*

A. A *quadrant*, in geometry, is a fourth part of a circle, being bounded by two radii perpendicular to each other, and a quarter of the circumference, or 90 degrees; as A B C.



Q. Into how many parts is the circumference of a circle divided?

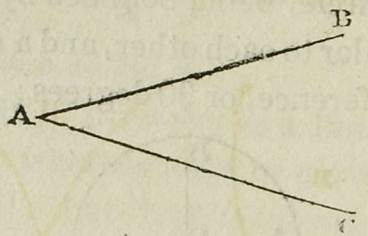
A. The circumference of every circle in geometry is supposed to be divided into 360 parts, called degrees; and each degree is subdivided into 60 minutes, and each minute into 60 seconds, and so on; hence a semicircle contains 180 degrees, a quadrant 90 degrees, an half quadrant, called an octant, 45 degrees.

CHAP. IV.

Of the Definition of an Angle.

Q. WHAT is an *angle?*

A. An *angle* is the inclination and opening of two lines meeting in a point; as, ABC*.



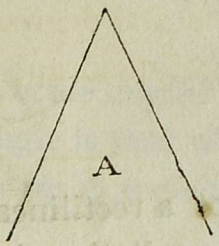
* The point A is called the angle point, and the lines AB and AC are called its sides or legs.

Q. How are angles distinguished?

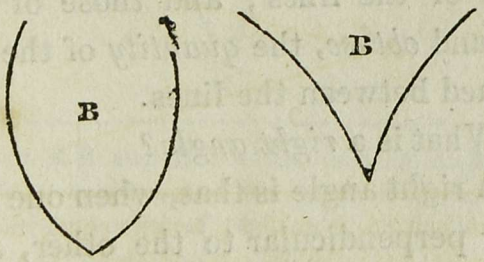
A. Angles are distinguished by the several names of *rectilineal*, *curvilineal*, and *mixtilineal* or *compound*.

Q. Describe these several angles.

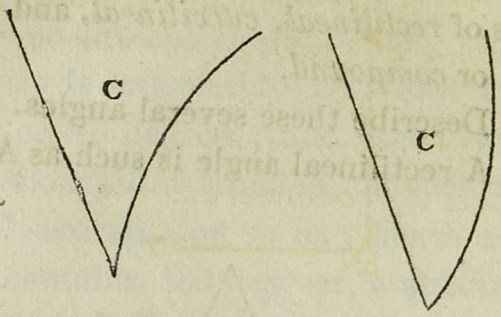
A. A rectilineal angle is such as A.



A curvilineal angle is such as B.



A mixed or compound angle as C.



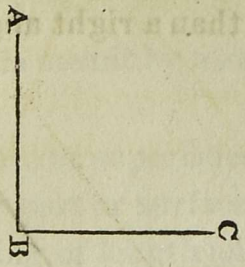
Q. Does not a rectilineal angle receive several other appellations?

A. Yes; according as it has a greater or less aperture or opening; as, *right*, *acute*, *obtuse*; thus, the terms of *rectilineal*, *curvilineal*, and *mixed* or *compound* express the *quality* of the lines; and those of *right*, *acute*, and *obtuse*, the *quantity* of the space contained between the lines.

Q. What is a *right angle*?

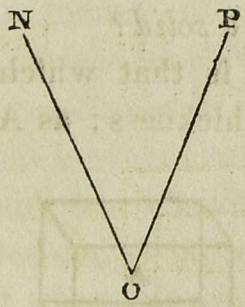
A. A right angle is that, when one of the lines is perpendicular to the other, and is

equal to the one-fourth of a circle; as ABC*.



Q. What is an *acute angle?*

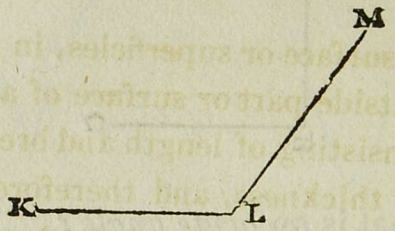
A. An acute angle is that when its aperture is less than a right angle; as NOP.



* The lines A B and B C which form the angle, are called the legs or sides, and the point B where they meet, is called the vertex of the angle, or the angular point.

Q. What is an *obtuse angle?*

A. An obtuse angle is that where its aperture is greater than a right angle; as KLM

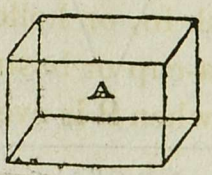


Q. What is a *figure?*

A. A figure is a bounded space, and is either a surface * or a solid.

Q. What is a *solid?*

A. A solid is that which has length, breadth, and thickness; as A.



* For the explanation of a surface, see next chapter.

CHAP. V.

Definition of a Surface or Superficies.

Q. What is meant by a *surface* or *superficies?*

A. A surface or superficies, in geometry, is the outside part or surface of any magnitude, consisting of length and breadth only, without thickness, and therefore forms no part of the substance or contents of the body.

Q. How is a *superficies* divided?

A. A superficies is divided into three kinds, namely, *plane*, *concave*, and *convex*.

Q. Describe them?

A. If the surface be elevated or raised, like that of a tea-cup or basin when turned upside down, it is said to be *convex** ; but if depressed, sunk in, or hollow, like the interior of a tea-cup or basin, it is called a concave; but when it is even or flat like the

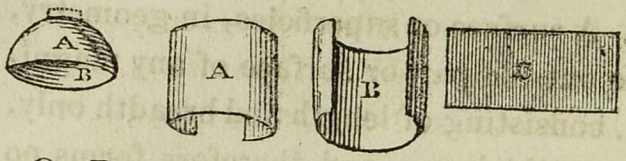
* Here the learner will observe, that the bottom of the tea-cup (which is a plane) is supposed to be excluded.

top of a table, or a marble slab, it is a *plane superficies**; thus

A is a *convex superficies.*

B is a *concave superficies.*

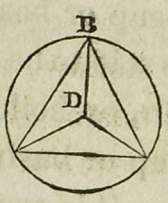
C is a *plane superficies.*



Q. By what name are plane figures, having more than four sides, generally called?

A. *Polygons*, but plane figures also receive particular names, according to the number of their sides; thus,

A figure of three sides is called a *trigon* or *triangle*; as,



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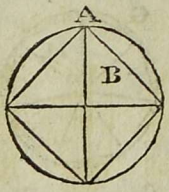
* The extremities of a superficies are lines. A superficies may be bounded by straight lines, curved lines, or both these.

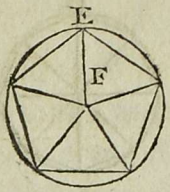
Q. What is a figure of four sides called?

A. A figure of four sides is called a *tetragon* or *quadrangle;* as the figure B. inscribed in the circle A.



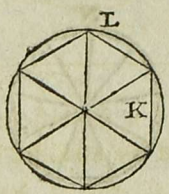
Q. What is a figure of five sides called?

A. A figure of five sides is called a *pentagon;* as the figure F inscribed in the circle E.



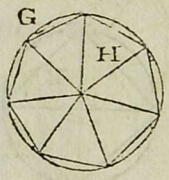
Q. What is a figure of six sides called?

A. A figure of six sides is called an *hexagon;* as the figure K inscribed in the circle L.



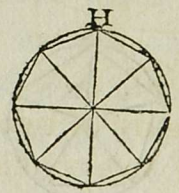
Q. What is a figure of seven sides called?

A. A figure of seven sides is called an *heptagon;* as the figure H inscribed in the circle G.



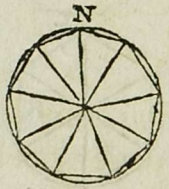
Q. What is a figure of eight sides called?

A. A figure of eight sides is called an *octagon;* as the figure inscribed in the circle H.



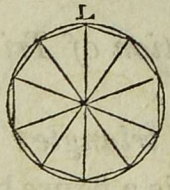
Q. What is a figure of nine sides called?

A. A figure of nine sides is called a *nonagon;* as the figure inscribed in the circle N.



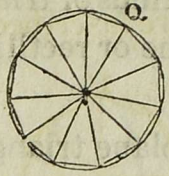
Q. What is a figure of ten sides called?

A. A figure of ten sides is called a *decagon;* as the figure inscribed in the circle L.



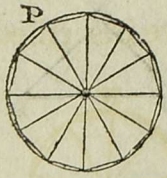
Q. What is a figure of eleven sides called?

A. A figure of eleven sides is called a *cendecagon;* as the figure described in the circle Q.



Q. What is a figure of twelve sides called?

A. A figure of twelve sides is called a *dodecagon;* as the figure described in the circle P.



N. B. When Polygons have more than 12 sides, they are usually called polygons of 13 sides, of 14 sides, and so on.

CHAP. VI.

Definition of Triangles.

Q. What is a *triangle?*

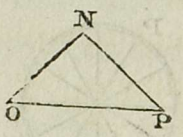
A. A triangle is a figure bounded by three sides, and consequently contains three angles; whence it derives its name.

Q. How many kinds of triangles are there?

A. Three; plane or rectilineal, spherical, and curvilineal.

Q. What is a plane triangle?

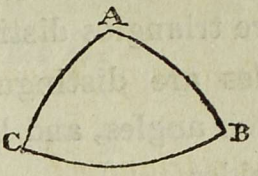
A. A plane triangle is that which is bounded by three right or straight lines; as N O P.



Q. What is a *spherical triangle?*

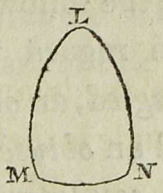
A. A spherical triangle is that which is

bounded by three arcs of great circles of this sphere*; as A B C.



Q. What is a *curvilineal triangle?*

A. A curvilineal triangle is that which is bounded by any three curve lines; as, L M N.



Q. Are not each of these three distinct classes of triangles subdivided into other distinguishing denominations?

A. Yes, according to the relation of their sides and angles.

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* A sphere or globe, in geometry, is a solid contained under one uniform surface, every point of which is equally distant from a point within, called its centre.

CHAP. VII.

Of Plane Triangles.

Q. How are triangles distinguished?

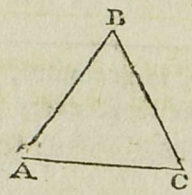
A. Triangles are distinguished by the quality of their angles, and by the disposition of their sides.

Q. Under what denomination are plane triangles usually known?

A. By the several names of *equilateral*, *isosceles*, and *scalene*, which have reference to their *sides*, but the following receive their denominations in regard to their *angles*; namely, a *right-angled*, an *oblique-angled*, an *acute-angled*, and an *obtuse-angled* triangle.

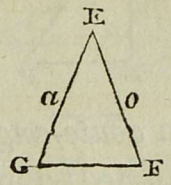
Q. What is an *equilateral* triangle?

A. An equilateral triangle is that which has its three sides equal, and consequently its angles equal, whence it is sometimes called an *equi-angular triangle*; as, A B C.



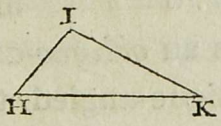
Q. What is an *isosceles* triangle?

A. An isosceles triangle is that which has only two of its sides equal; as the sides *a o* in the triangle E F G.



Q. What is a *scalene* triangle?

A. A scalene triangle is that whose sides are all unequal; as, H I K *.



Q. What is a *right-angled* triangle?

A. A right-angled triangle is that which

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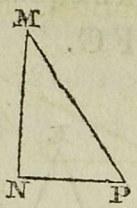
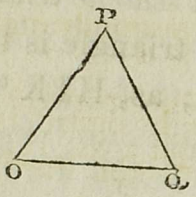
* This triangle is termed a *scalene triangle* with reference to its sides, and *oblique* with regard to its *angles*.

has one of its angles right; as, the angle N in the triangle N M P*.



Q. What is an *acute-angled* triangle?

A. An acute-angled triangle is that which has all its angles acute; as, O P Q.



Q. What is an *oblique-angled* triangle?

A. An oblique-angled triangle is that which has not a right angle, and is either *acute* or *obtuse*†.

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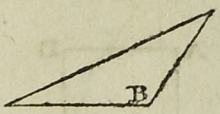
* A right angle contains 90 degrees; as the angle N. The side opposite the right angle is called the hypothenuse; as, P M; the upright line N M is the perpendicular; and the line N P is nominated its *base*. The *base* and *perpendicular* are sometimes called legs.

† See the scalene triangle.

Q. What is an *obtuse* triangle?

A, An obtuse triangle is that which has one of its angles obtuse; as, the angle B.



CHAP. VIII.

Of Quadrangles or Quadrilateral Figures.

Q. What is a quadrangle or quadrilateral figure?

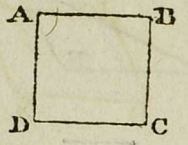
A. A quadrangle figure is that which is made by four straight lines, and is usually called a *parallelogram*, a *trapezium*, or *trapezoid.*

Q. Do not parallelograms receive particular denominations?

A. Yes, parallelograms are severally named according to the equality or inequality of their sides and angles; and are called a *square*, a *rectangle*, a *rhombus* or a *rhomboid.*

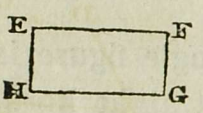
Q. What is a *square*?

A. A square is a figure whose four sides are equal, and its angles right; as, A B C D.



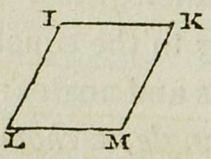
Q. What is a *rectangle*?

A. A rectangle* has all its angles right or equal, but its sides unequal; as, E F G H.



Q. What is a *rhombus*:

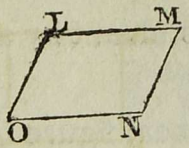
A. A rhombus is a figure whose sides are all equal to one another, but not its four angles; as, I K L M.



* By some improperly called a long square

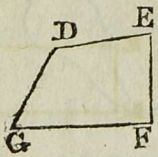
Q. What is a *rhomboid?*

A. A rhomboid is that which has its opposite angles and sides equal, without being equiangular or equilateral*; as, L M N O.



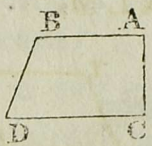
Q. What is a *trapezium?*

A. A trapezium is that figure which is contained under four right lines, but neither of the opposite sides are parallel; as, D E F G.



Q. What is a *trapezoid?*

A. A trapezoid is that quadrangle which has only two sides parallel; as, A B C D.



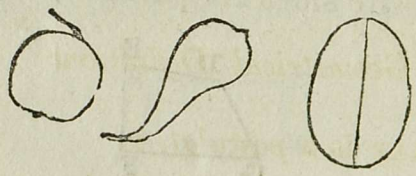
* Or, in other terms, a rhomboid is an *oblique-angled* parallelogram.

CHAP. IX.

Of Curves or Curvilineal Figures, &c.

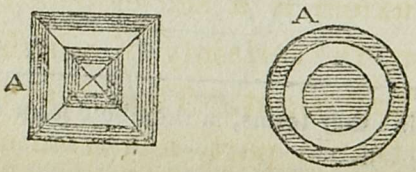
Q. What is meant by a *curve*, or a *curvilineal* figure?

A. A curvilineal figure is that whose surface or extremity is bounded by curves; as that of an egg, apple, or pear.



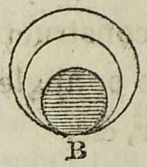
Q. What is meant by a concentric figure?

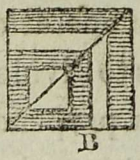
A. A concentric figure is that which has the same centre; as, A.



Q. What is an *excentric* figure

A. An excentric figure is that which is described upon different centres; as, B.





CHAP. X.

Geometrical Definitions

Q. WHAT is a *postulate*?

A. A postulate is a *demand*, *petition*, or *supposition*, and differs from an axiom only in the manner in which it is put, namely, as a *request* instead of an *assertion*.

Q. What is an *axiom*?

A. An axiom is a self-evident truth or proposition, the certainty of which is perceived at first sight. Thus, that a whole is greater than a part, and from nothing,

nothing can arise, are self-evident truths or axioms*.

Q. What is a *problem?*

A. A problem is a question or proposition wherein an operation or construction is required; as to divide a line or angle, to erect or let fall a perpendicular, &c.

Q. What is a *theorem?*

A. A theorem is a proposition in which some truth is proposed to be demonstrated†.

Q. What is a *lemma?*

A. A lemma denotes a previous proposition, laid down in order to clear the way for some following demonstration, and is prefixed either to theorems in order to render their demonstration less perplexed and intricate, or to problems to make their resolution more easy, clear, and short.

Q. What is a *corollary?*

---

* *Axiom*, when applied to other sciences, is usually called a *maxim*.

† PROCLUS defines a theorem and problem thus; a *problem* is something proposed to be done; a *theorem* is something which is proposed to be demonstrated.

A. A corollary is a consequent truth, gained from some preceding truth or demonstration; thus, that if the three sides of a triangle are equal, *the three angles are equal also.*

Q. What is a *scholium?*

A. A scholium is a note, annotation, or remark, occasionally made on some passage, proposition, or the like, with the view to make it more easy and clear.

CHAP. XI.

Of Postulates, Petitions, or Demands.

Q. WHAT is it to make, in geometry, a postulate or demand?

A. 1. To grant that a point may be posited*, or imagined to be posited, wherever we please.

2. That a line can be drawn, or imagined

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* From *positus* (Latin) *placed.*

to be drawn, from any one given point to another.

3. That a straight line may be produced, or imagined to be produced, either way or both ways, as far as we please.

4. That a straight line may be drawn parallel or inclined to any given straight line.

5. That from any given point, in a given straight line, may be drawn a perpendicular to the given straight line.

6. That a circle may be described from any point as a centre, with any radius, or at any distance from that centre.

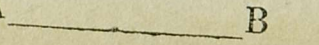
Q. How do you *posit* or *place* a dot?

A. I posit or place a dot by the point of a pin, pen, or pencil; as, at A.

A.

Q. How do you draw a line?

A. By applying a ruler to any two given points; as, A B.



Then draw the line demanded,..... A B
by carrying the pen or pencil along
the ruler, and close to it from the
point..... A
to the point..... B

Q. How do you produce a line either way, or both ways at pleasure ?

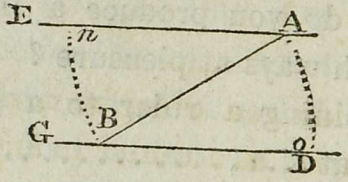
A. By joining a ruler to a given point ; as, at..... A
from which I draw a line at plea-
sure to..... B
then by turning my paper and
ruler I carry my pencil from
A to C.

C A B

Q. How do you draw a line parallel, or incline it, to another ?

A. To draw it parallel, I first draw
at pleasure the oblique line..... A B
then upon the point,..... A
I describe the arc..... B *n*
Upon the point,..... B
I make the arc,..... A D

I then make the arc..... B *n*
equal..... A D
and draw the lines required..... A E
and ..... G D
through the points ..... A *n* B *o*

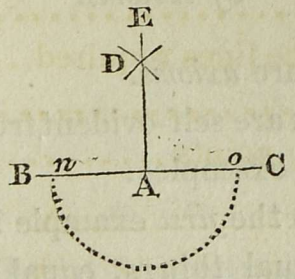


To *incline* one line to another, as from the given line A B, B D, I draw the line at pleasure.

Q. How do you draw a perpendicular from a given point in a given straight line?

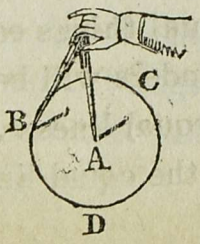
A. from the given point..... A
in the line..... B C
I describe the semicircle..... N O
upon the points..... N O
I make the section..... D
then from the point ..... A

I draw the line demanded ..... A E
through the section ..... D



Q. How do you describe a circle from any given point as a centre to any distance, at pleasure?

A. I first set out one of the points of the compass upon the given point.. A
then open the other to the given point ..... B
turn the compasses about the point ..... A
and by trailing the point..... B
I draw the circle demanded ... B C D



CHAP. XII.

Of Axioms.

Q. WHAT are *axioms.*

A. Axioms are self-evident truths ; as in the following examples.

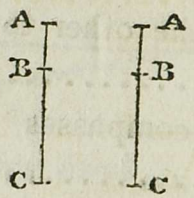
Q. What is the *first* example ?

A. If to equal things, equal things be added, the whole will be equal ; as,

The lines A B, A B are equal

The lines B C, B C added are equal

A. The whole, B C, B C are also equal.



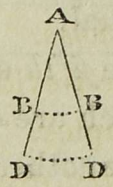
Q. What is the *second* example ?

A. If from equal things equal things be taken, the remainders will be equal ; as,

If from the equal lines .... A D, A D

I take away the equal lines, A B, A B

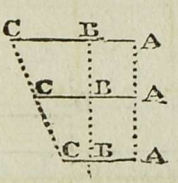
The remaining parts will be
also equal..... BD, BD



Q. What is the *third* example?

A. If to unequal things, equal things be added, the whole will be unequal; namely,

If the unequal lines..... BC, BC
be added to the equal lines AB, AB
the whole..... AC, AC
will be unequal.



Q. What is the *fourth* example?

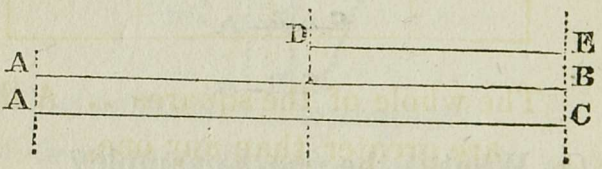
A. If from unequal things, equal things be taken, the remainder will be unequal; namely *(see last figure.)*

If from unequal lines ..... B C, B C
you take away the equals ... A B, A B
The remainders ..... B C, B C
will be unequal.

Q. What is the *fifth* example?

A. Things which are double of the same are equal to one another; namely,

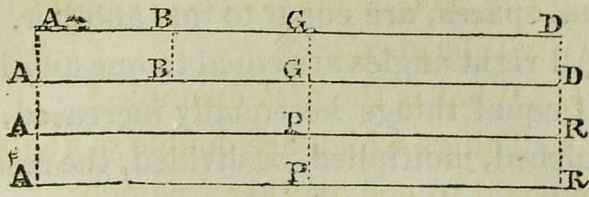
Supposing the lines ..... A B, A C
to be each double to ..... D E
the line ..... A B
is twice the measure of .... D E
and so also is ..... A C
It is therefore plain that A B and A C are also equal to each other.



Q. What is the *sixth* example?

A. The halves of equal things are equal,

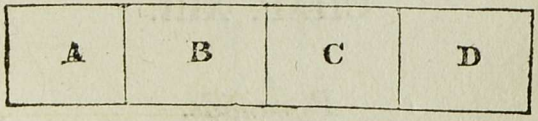
and the doubles of equal things are equal ; namely,



The halves of.....A B G, A B G
are equal the lines ..... A P, A P
and the double of ..... A P, A P
are equal .... A R, A R

Q. What is the *seventh* example ?

A. The whole is greater than a part, and equal to all its parts ; namely,



The whole of the squares .. A B C D
are greater than any one,
two, or three of its parts,
but the parts added are
equal to the whole .... A B C D

Q. What is the *eighth* example?

A. Things that mutually agree or fill equal spaces, are equal to one another.

All right angles are equal to one another.

If equal things be equally increased, diminished, multiplied, or divided, the results will always be equal.

If unequal things be equally increased, diminished, multiplied, or divided, the results will be greater or less than one another, according as the unequals from which they are derived, are greater or less than one another.

CHAP. XIII.

Pyramids.

Q. What is a *pyramid?*

A. A pyramid* is a solid, standing on a

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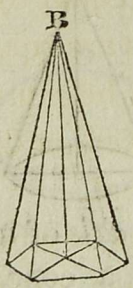
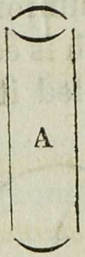
* The pyramids of Egypt, the burial-place of their kings, are much celebrated in history for their great height and magnitude.

square or polygonal basis, and terminating at the top in a point usually called its vertex; as B.



Q. What is a *cylinder?*

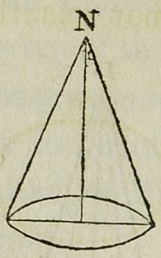
A. A cylinder is a round solid, having its two ends circular, equal and parallel; as A.



Q. What is a *cone?*

A. A cone is a solid, having a circle for its base, and its side a convex surface, whose

top terminates in a point, called its vertex; as N*.

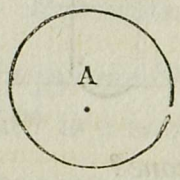


Q. What is the *axis* of a cone?

A. The axis of a cone is a line drawn from the vertex to the centre of its base; as the line falling from the point N in the above figure.

Q. What is meant by a *sphere?*

A. A sphere, in geometry, is a solid contained under one uniform round surface, every point of which is equally distant from a point within, called its centre†; as the figure A.

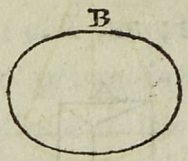


* A sugar-loaf is a *right* cone.

† Sphere, in *astronomy*, signifies the orb or expanse of the heavens, in which all the heavenly bodies ap-

Q. What is a *spheroid?*

A. A spheroid is a solid resembling a sphere, though not exactly round, but oblong; as, B.



Q. What means a *frustum?*

A. A frustum, in geometry, signifies a piece cut off from a solid, by a plane passing through it, and parallel to its base, as the frustum of a cone, a pyramid, &c.

CHAP. XIV.

Of Prisms.

Q. What is a *prism?*

A. A prism, in geometry, is a solid whose

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pear fixed; this sphere is the subject of trigonometry. *Sphere*, in geography, is the surface of the earth, whose great circles contain 60 geographical, or 69½ English miles in a degree: all other circles vary in measurement according to their situation.

ends are any plain figure which are parallel, equal, and similar; and its sides connecting those ends are parallelograms; hence every section parallel to the base, is equal and similar to the base; as, A.

Q. How are prisms denominated?

A. Prisms are denominated according to the figure of their bases, as a *triangular* prism, a *square* prism, a *pentagonal* prism, an *hexagonal* prism, and so on.

Q. What is the *axis* of a prism?

A. The axis of a prism is the line conceived to be drawn lengthways through the middle of it, connecting the centre of one end with that of the other end.

Q. Are not prisms again divided into right and oblique.

A. Yes, they are subdivided into *right* and *oblique*.

Q. What is a *right* prism?

A. A right prism is that whose sides and axis are perpendicular to its ends, like an upright tower.

Q. What is an *oblique* prism?

A. An oblique prism is when the axis and sides are oblique to the ends; so that, when set upon one end, it inclines on one hand more than on the other.

TRIGONOMETRY.

CHAPTER XV.

Definitions, &c.

Q. What is trigonometry?

A. Trigonometry is that part of geometry which teaches how to measure the sides and angles of triangles.

Q. How is trigonometry divided?

A. Trigonometry is divided into two parts; namely, *plane* and *spherical.*

Q. What is *plane trigonometry?*

A. Plane trigonometry treats of the application of numbers, to determine the relations of the sides and angles of a plane triangle to one another.

Q. What is *spherical trigonometry?*

A. Spherical trigonometry treats of the application of numbers in like manner to sphe-

rical triangles. For this purpose, it is not only requisite that the peripheries of circles, but also certain right lines in and about a circle are supposed to be divided into some assigned number of equal parts. These lines are denominated sines, tangents, secants, &c.

Q. In what manner are the sides and angles of plane triangles estimated?

A. The sides of plane triangles may be estimated in feet, yards, &c., or by any other definite measures; but the angles are measured by the arcs of a circle, contained between the two legs, having the angular point for its centre.

Q. How are circles supposed to be divided?

A. Every circle is supposed to be divided into 360 equal points *, called degrees; each degree into 60 equal parts called minutes:

* The learner will observe, that the division of the circle is perfectly arbitrary, and that any other number might have been employed instead of 360, and according by the French writers have now adopted a different division. They divide the circle into 400 equal parts, or the quadrant into 100, which they call degrees, and

each minute into sixty equal parts, called seconds.

Q. How are degrees, minutes, and seconds marked?

A. Degrees, minutes, &c., are marked at the top of the figures by which the arc is denoted; thus, 24°, 40′, 19″, signify twenty-four degrees, forty minutes, and nineteen seconds.

CHAP. XVI.

Of Properties or Principles in Trigonometry.

Q. What is the first principle in trigonometry?

A. That every triangle consists of six parts; namely, three sides and three angles.

Q. What is the second principle?

A. That the sum of the three angles of

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these degrees are subdivided into 10ths, 100ths, and 1000ths, &c.; and hence the measure of angle with them is expressed the same as any other integral and decimal quantity, which mode of division is so decidedly superior to that which is so commonly used, that it is much to be regretted that this plan is not generally adopted.

every plane triangle is equal to two right angles or 180°*.

Q. What is the third principle?

A. That the greater side is opposite to the greater angle, and the less side to the less angle.

Q. What is the fourth principle?

A. That the sum of any two sides is greater than the third; and the difference of any two sides is less than the third.

Q. What is the fifth principle?

A. That a triangle is equilateral, isoceles, or scalene, according as its three angles are all equal, or only two of them equal, or all three unequal.

Q. What is the sixth principle?

A. That the angles opposite to the two least sides are acute; and if there is an obtuse angle, it is opposite to the greatest side.

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* Hence, if one of the angles be known, the sum of the other two may be found by subtracting the given angle from 180°; also if two of the angles be known, their sum taken from 180° will give the third angle.

CHAP. XVII.

Of the several Lines made use of Trigonometry.

Q. WHAT are the lines made use of in trigonometry?

A. The lines that are particularly made use of in trigonometry are

The *sine* or *right sine* of an arc,

The *versed sine* of an arc,

The *tangent* of an arc, and

The *secant* of an arc, with their complements, usually called *cosine*, *coversed sine*, *cotangent*, *cosecant*.

Q. What is meant by the *complement of an arc or angle?*

A. The complement of an arc or angle is what it wants of 90° or of a quadrant.

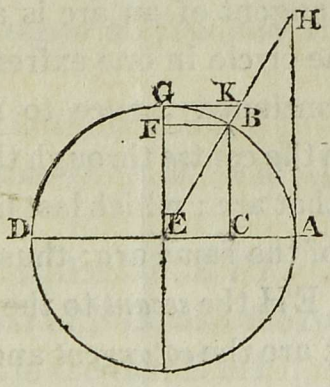
Q. What is meant by the *supplement of an arc or angle?*

A. The supplement of an arc or angle is what it wants of a semicircle or 180°.

Thus, if an angle measures 30° its complement is 150°, and its supplement 150°.

Q. What is the *sine* of an arc?

A. The *sine* of an arc, is a line drawn from one extremity of an arc, perpendicular to the diameter which passes through the other extremity; thus, B C is the sine of the arc A B, or of its supplement B D.



Q. What is meant by the *versed sine* of an arc?

A. The *versed sine* of an arc is that part of the diameter, which is intercepted between the *arc* and its *sine;* thus, A C is the *versed sine* of the arc A B, and C D the *versed sine* of the arc B D.

Q. What is the *cosine* of an arc?

A. The *cosine* of an arc is the part of the

diameter intercepted between the *centre* and the *sine*, and is equal to the *sine* of the *complement* of that arc: thus, E C is the *cosine* of the arc A B, and is equal to B F, the *sine* of its complement B G.

Q. What is the *tangent* of an arc?

A. The tangent of an arc is a right line touching the circle in one extremity of that arc, and continued thence to meet a line drawn from the centre through the other extremity of that arc; which last line is called the secant of the same arc; thus, A H is the *tangent* and E H the *secant* to the arc A B.

Q. What are the *cotangent* and *cosecant* of an arc?

A. The *cotangent* and *cosecant* of an arc, are the *tangent* and *secant* of the *complement* of that arc; thus, G K and E K are the *cotangent* and *cosecant* of the arc A B.

Questions for Examination—promiscuously arranged.

Q. WHAT is the line A H?

A. *Tangent* to the arc A B and *cotangent* to B G.

Q. What is the line A C?

A. The *versed sine* of A B, and the *coversed sine* of B G.

Q. What is the line G K?

A. The *tangent* of B G, and the *cotangent* of A B.

Q. What is the line B C?

A. The *sine* of A B, and the *cosine* of B G.

Q. What is the line B F?

A. The *sine* of B G, and the *cosine* of A B

Q. What is the line E H?

A. The *secant* of A B, and the *cosecant* of B G.

Q. What is the line E K?

A. The *secant* of B D, and the *cosecant* of A B.

Q. What is the line D K?

A. The *versed sine* of B G, and the *coversed sine* of A B.

APPENDIX.

Terms explained.

Apex, the vertex, top, or summit of any thing.

Apparent, a term used in mathematics and astronomy, to signify things as they appear to us, in contradistinction from real or true; and in this respect the apparent state of things is often very different from their real state, as is the case of distance, magnitude, &c.

Catenary, (in the higher geometry), a mechanical curve, which a chain or rope forms itself into by its own weight, when hung freely between two points of suspension, whether those points be in the same horizontal plane or not.

Complement of an *arc* or *angle*, is what it wants of 90 degrees; thus, the complement

of 60° is 30°, and the complement of 30° is 60°.

CONTENT, a term commonly used to denote the surface or solidity of bodies; so that to find the content of a body, is to find the measure or quantity of its surface or solidity.

CUBE, in solid geometry, is a regular solid body, consisting of six equal square sides or faces.

CUSP, in geometry, is used for the point or corner, formed by two parts of a curve meeting.

CYCLOGRAPH, an instrument used for describing arches of circles. This instrument is used in those cases which require a greater radius than can be obtained by a pair of compasses.

DATA, in mathematics, a term used for such quantities as are given or known, in order to find others that are unknown.

DATUM, the singular number of *data*.

DECUPLE, tenfold.

DIAGRAM, is a scheme for the explanation

or demonstration of the properties of any figure, whether triangle, square, circle, &c.

DIMENSION, in *geometry*, is either *length*, *breadth*, or *thickness*; hence, a line has only *one* dimension,—*length*; a surface, *two*,—*length* and *breadth*; and a body or solid, *three*,—*length*, *breadth*, and *thickness*.

EQUIANGULAR, in geometry, is applied to figures whose *angles* are all equal.

EQUILATERAL, is applied to those figures whose sides are all equal.

GIBBOUS, denotes convexity or protuberance; hence the moon is said to be gibbous in the second and third quarters, being during this period more than half enlightened, and consequently convex or gibbous on both sides.

INCLINATION, denotes the mutual approach of two bodies, lines, or planes, towards each other, so that the lines of their direction make at the point of contact any angle of greater or less magnitude.

LEAGUE, a measure of three miles.

LONGIMETRY, the measuring of length or distances, both accessible and inaccessible.

MACHINE, any instrument employed to produce motion, so as to save time or force.

MEASURE, in geometry, denotes any certain quantity assumed as one, or unity, with which other homogeneous* or similar quantities are compared.

Measure of an angle, is the number of degrees, minutes, &c., contained in the arc of a circle comprised between the two lines forming that angle, its angular point being the centre.

Measure of a line, is its length compared with some determinate line; as a foot, a yard, &c.

Measure of a surface, is its contents in square units, as a foot, yard, &c.

Measure of a solid or *cube*, is its contents in square inches, feet, yards, &c.

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* Homogeneous (from two Greek words, signifying *like* and *kind*) is a term applied to various subjects, to denote that they consist of similar parts, or of parts of the same nature and kind, in contradistinction to heterogeneous, where the parts are of different natures, &c.

MECHANICS, that branch of practical mathematics which considers motion and moving powers, their nature and laws, with their effects on machines*.

Mechanism, the construction or employment of machinery.

OBLATE, flattened or shortened.

OCCULT LINE, in geometry, an obscure

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* The knowledge of mechanics is one of those things, says Mr. M'Laurin, that serve to distinguish civilized nations from barbarians. It is by this science that the utmost improvement is made of every power and force in nature; and the motions of the elements, water, air, and fire, are made subservient to the various purposes of life; for, however weak the force of man appears to be, when unassisted by this art; yet, with its aid, there is hardly any thing above his reach.

This science is divided, by Sir Isaac Newton, into practical and rational mechanics, the former of which relates to the mechanical powers; namely, the lever, the balance, axle and wheel, pulley, wedge, screw, and inclined plane; and the latter or rational mechanics, to the theory or laws of motion; shewing when the forces or powers are given, how to determine the motion that will result from them; and conversely, when the circumstances of the motion are given, how to trace the forces or powers from which they arise.

The mechanical powers are simple engines that enable men to raise weights, to move heavy bodies, and overcome resistances, which they could not do with their natural strength alone. Their importance to society is incalculable. Every machine whatever is composed of one or more of them; sometimes of several combined together.

line, which is drawn as a necessary part of the construction of a figure or problem, but which is not intended to appear after the plan is finished.

Octant, the eighth part of a circle.

Odometer, is an instrument for measuring the distance travelled over by a post chaise or other carriage; it is attached to the wheel, and shews, by means of an index and dial plate, the distance gone over.

Orbit, the path of a planet, comet, or other celestial body.

Perambulator, an instrument for measuring distances. It is sometimes called a pedometer, or surveying-wheel*.

Perimeter, the bounds or limits of any figure or body. The perimeters of surfaces are lines; those of bodies are surfaces †.

Periphery, the circumference of a circle,

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* The circumference of this wheel is ninety-nine inches, or half pole; and its proper office is in surveying roads and large distances, for which it is extremely convenient and expeditious, but it is not always quite accurate, in consequence of the unevenness of the road.

† *Perimeter* is usually applied to right-angled or right-lined figures, and *periphery*, to circles or curvilineal ones.

ellipsis, or any other regular curvilineal figure.

PLANISPHERE, a projection of the sphere or globe, and its various circles on a plane, as upon paper, &c., usually called maps.

POLYGRAM, a figure consisting of many lines.

QUADRANGLE, in geometry, a figure consisting of four angles, and is sometimes called quadrilateral in relation to its four sides.

QUADRANT, a quarter or the fourth part of a circle. It also means an instrument used for measuring altitudes, &c.

QUADRATURE, in geometry, denotes the squaring, or reducing a figure to a square. Thus, the finding of a square which shall contain just as much surface or area as a circle, an ellipsis, a triangle, &c., is the quadrature, or measure of a circle, ellipsis, &c.

QUADRILATERAL, a figure of four sides, and therefore of four angles, under which general term is included the *rectangle, square, rhombus, rhomboid, parallelogram, trapezium,* and *trapezoid.*

QUADRUPLE, four-fold.

QUINTUPLE, five-fold.

RADIUS, the semidiameter of a circle, or a right line drawn from a centre to the circumference.

RADII, the plural of radius.

RECTIFICATION, in geometry, is the finding a right line equal in length to a curve.

REGULAR FIGURE, in geometry, is that which has all its sides and all its angles equal; if these are not so, the figure is said to be irregular.

SCHEME, a draught or representation of any geometrical or astronomical subject, and is otherwise called a diagram.

SEMIDIAMETER, the same as radius.

SEXTUPLE, six-fold.

SOLIDITY, is that property of matter by which it excludes all other bodies from the place it possesses, whose content is estimated by the number of solid or cubic inches, feet, yards, &c., which it contains.

TERM, in geometry, is the extreme of any magnitude, or that which bounds and limits

its extent. So the terms of a line, are points; of a superficies, lines; of a solid, superficies.

THEODOLITE, an instrument used by surveyors for measuring angles, heights, and distances, of remote objects.

TRANSFORMATION*, in geometry, is nearly the same as reduction, being used to denote the change of one figure or body into another of the same area, or solidity, but of a different form, as a triangle into a square, &c.

TRILATERAL, *three-sided*, a term applied to any figure of three sides.

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* Sometimes called *transmutation*.

THE END.

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