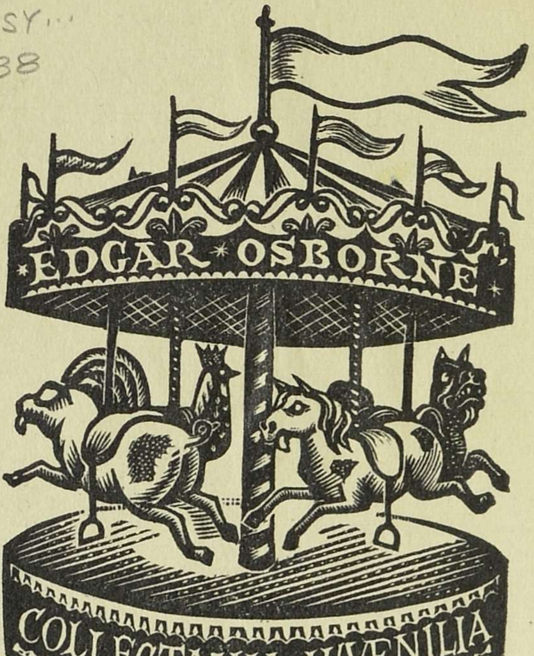
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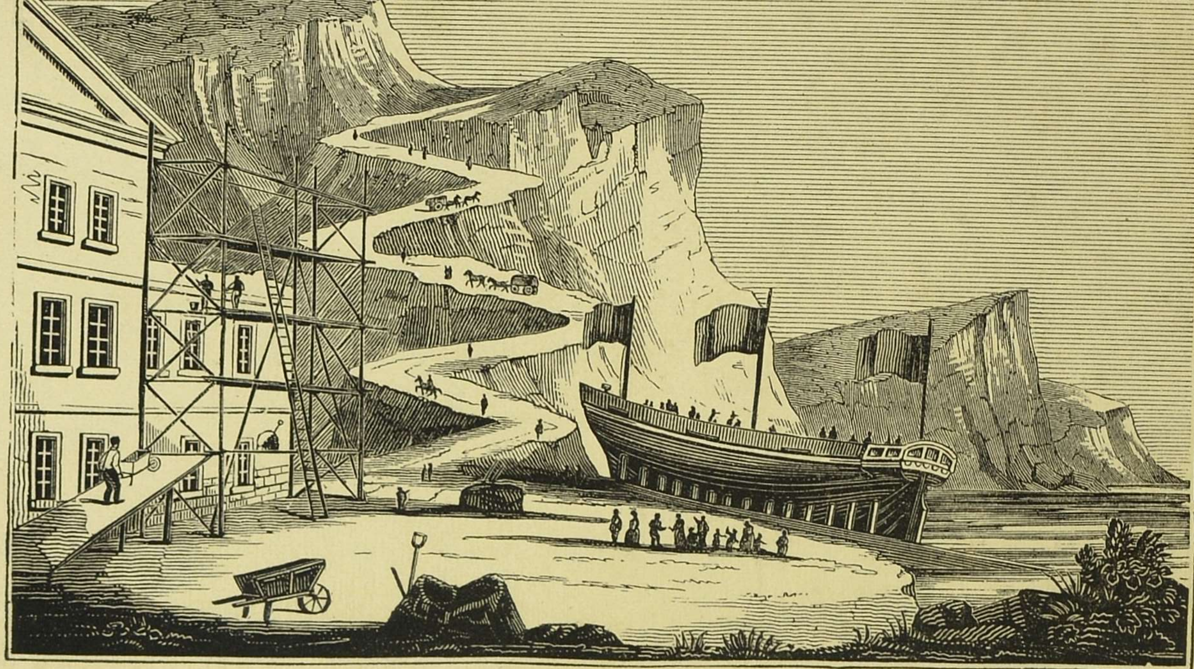

The Inclined Plane.

# EASY LESSONS

IN

# MECHANICS;

WITH

FAMILIAR ILLUSTRATIONS

SHOWING

THE PRACTICAL APPLICATION OF THE VARIOUS
MECHANICAL PRINCIPLES.

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*PUBLISHED UNDER THE DIRECTION OF
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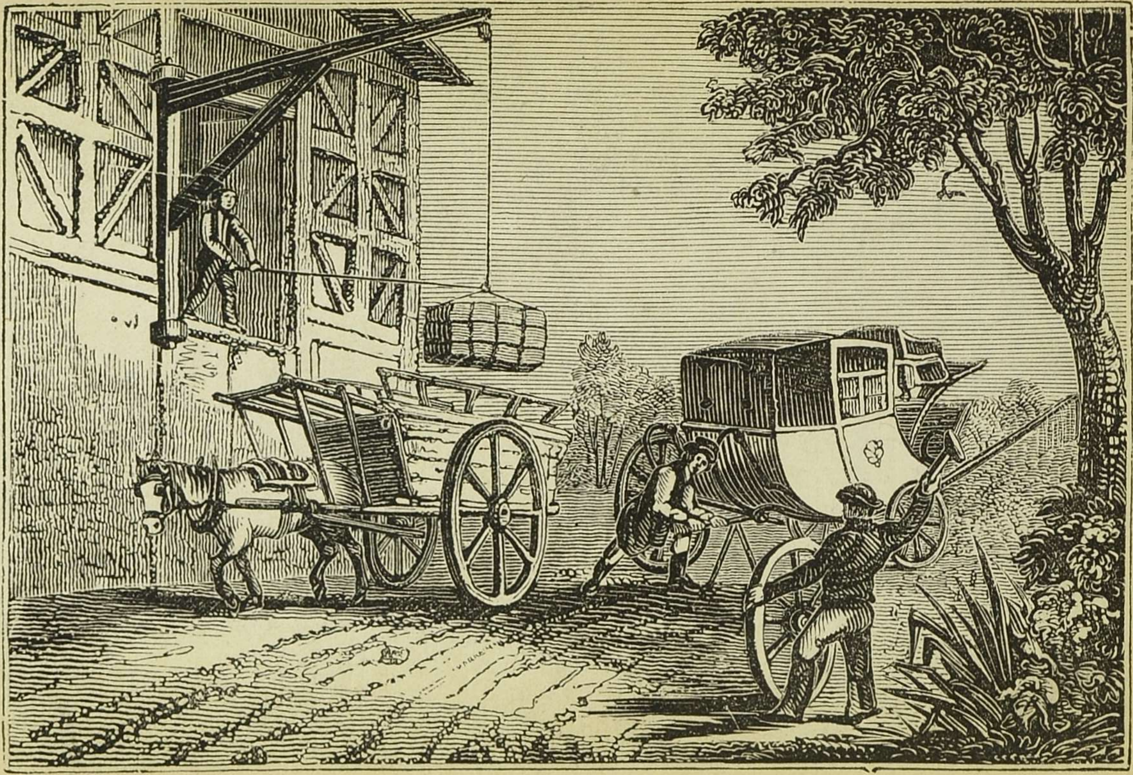
# PREFACE.

The object of this little work is to give a familiar and connected account of the first principles of Mechanics. Since no accurate knowledge can be gained without a clear perception of the meaning of the terms employed, great care has been taken to define all technical words as they occur. Very plain illustrations and experiments have been referred to, throughout the work; and it is hoped that, although the expressions and processes of Mathematics have been necessarily excluded, the reasoning by which the several parts are connected, will be found to be sound and convincing.

As every part depends closely upon that which goes before, it is very important that the young student should thoroughly comprehend one lesson, before he proceeds to another.

# CONTENTS.



Different Kinds of Force.

# MECHANICS.

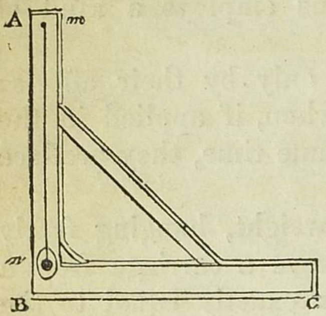
## LESSON I.

### ON THE OBJECTS OF MECHANICS, AND ON FORCES IN GENERAL.

MECHANICS is the science which treats of the laws of equilibrium, or of the motion of bodies.

If we observe what takes place around us, we perceive that certain effects are always produced by certain causes. For instance, if weights are successively put into the scale of a balance, they begin to lift another weight in the opposite scale, as soon as the weights exceed a certain amount. Suppose this amount to be ten pounds. Then we feel persuaded that the same effect would always be produced under the same circumstances, however frequently the experiment was repeated; that a weight of nine pounds would not be heavy enough, and a weight of eleven pounds too heavy. By extending the same kind of reasoning, we arrive at the conclusion, that mechanical effects are not produced at random, but are connected with their causes by the regular operation of certain fixed laws. It is the object of mechanics to discover and trace the consequences of these laws.

begin to move. Thus, if a bullet be let fall from the top of a perpendicular tower, it falls in a straight line to the bottom of the tower. The *direction*, therefore, of the *force of gravity* is, sensibly, a *vertical* line. This property gives one of the easiest methods of finding whether any building is strictly




upright. If A B C is a frame, the sides of which are accurately at right angles to each other, and a plumb-line, *m n*, hangs freely upon the bar A B, the part A B will be vertical, and the part B C horizontal, when the string *m n* exactly falls upon the notch at *n*.

The direction of a force by which motion is stopped, or prevented, is *opposite* to the direction of a force which would cause the same motion.

Thus, if a bullet falls from a height upon the ground, and is there stopped, the pressure of the ground against the bullet, which stops its motion downwards, is in a direction perpendicularly *upwards*.

If a weight rests on a table, the direction of the pressure of the weight is in like manner, downwards, while the direction of the pressure of the table against the weight is upwards.

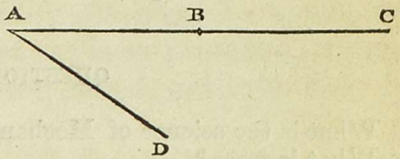
The *magnitude* of a force is measured by the effect which it would produce under given circumstances, compared with the effect produced, under the same circumstances, by some other force taken as a standard.

Thus, suppose a magnet is just able to support a weight of one ounce; and that, after it has gained strength by constant use, it is able to support two ounces. The *magnitude* of the sustaining force which it exerts is exactly double of what it was at first.

As the *effects* produced by forces are different, any of those effects may be taken as the measure of the magnitude of force, care being taken to distinguish the different circumstances.

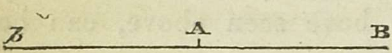
For the present we shall have to consider only pressures, which, as we have seen above, can be always measured by weights.

It is often very convenient to represent forces by lines, drawn in the directions in which the forces act, the lengths of the lines representing also the magnitude of the forces. Thus any line, as A B, may be taken to represent any force, acting in the direction A B. If we wish to represent a force twice as great, acting in the same direction, we must take a line A C, to represent it, which is in the direction A B, and twice as long: and so on for any other force in the same direction.




In like manner, a line A D, inclined to A B, will represent another force acting at the same point A, in the direction in which A D is drawn, and bearing the same proportion to the first force, that A D bears to A B; and the same principle of representation may be extended to any number of forces.

It must be well observed, that in so describing forces, we take the *direction* of the forces to be that indicated by the relative position of the letters; thus "a force A B," implies a force acting upon A, proportional to the line A B, and tending to cause motion *from* A *towards* B. Whereas "a force B A," implies an equal force, tending to cause motion *from* B *towards* A.




It is often most convenient to take a line, as A *b*, equal to A B, and measured from A in the direction opposite to A B, in order to represent a force acting upon A, and tending to cause motion in the direction opposite to A B.

## QUESTIONS.

What is the science of Mechanics?
What is force?
What are animal force, gravity, pressure, and impact?
When are two forces equal?
How can force producing pressure be measured?
What is the direction of a force?
What is the magnitude of a force?
Show that the direction and magnitude of forces can be represented by lines.

## LESSON II.

### ON THE PROPERTIES OF MATTER.

---

Force may be employed either to keep bodies *at rest*, or else to *put* them *in motion*, or act upon them when they *are* in motion.

That part of Mechanics which treats of keeping bodies *at rest* is called Statics, from a Latin word (stare), which implies standing still: and that part which treats of motion has received the name of Dynamics, from a Greek word (δυνάμις, dynamis,) which signifies power or force.

The bodies of which the science of Mechanics treats are *material* bodies, that is, they are composed of matter, or are the objects of our senses, and possess certain properties.

Matter possesses *extension;* it has a certain magnitude of length, breadth, and thickness.

Matter possesses *solidity;* it occupies space, so that if one body, as a stone, is in the hand, it prevents the hand from being closed, until the body is removed.

Matter is *moveable.* All the matter with which we are acquainted can be moved by the application of a sufficient force; whence we conclude that this is a general property of all matter.

Matter is *divisible.* If we take a piece of iron, we can divide it into two parts: the nature of those two parts is the same as that of the whole piece; each of them, therefore, is as divisible as the whole piece. Thus each division leaves the matter as capable of division as it was at first; and we cannot imagine any limit to such a division.

The division of matter may be carried to a great extent, by grinding or pounding certain substances, so as to reduce them to a very fine powder. By dissolving a body in a fluid, the division into parts may be carried still farther: and we have no reason to think we have reached the limit of smallness, when we have arrived as far as our senses extend.

Matter possesses *gravity*. We cannot lift a body without exerting a force: and if we leave a body to itself, it falls to the earth. This tendency to move in the direction of a line drawn from the body towards the centre of the earth, is called *gravitation*. It does not depend upon the form of the body, or upon the arrangement of the particles of which it is composed. If we take a stone of a pound weight, and bruise it in a mortar, provided none of the parts are lost, they are found to weigh still exactly a pound.

Matter is also *inactive:* it cannot move itself, nor be moved without the exertion of a sufficient force. This property of matter is often called its *inertia*, a Latin word which implies inactivity.

Since the gravity of a body is not altered by any change in its form, the *degree* of its gravity, or *its weight*, at any given place of the earth's surface, may be taken as the measure of its *quantity of matter.* If a portion of matter of a given *weight* is taken as an unit, for instance if it is called one *pound*, another portion of matter of the *same* weight is also called one pound: ten such portions together ten pounds, and so on.

The *density*, or *specific gravity* of a body is measured by the weight of a given bulk.

For instance, a cubic, or solid foot of water weighs one thousand ounces avoirdupois. A cubic foot of lead is found to weigh 11,325 ounces, and a cubic foot of iron, 7645 ounces. The *densities*, therefore, of these bodies are in the proportion of 1000, 11,325, and 7645 respectively: and those numbers are called the *specific gravities* of the bodies.

## QUESTIONS.

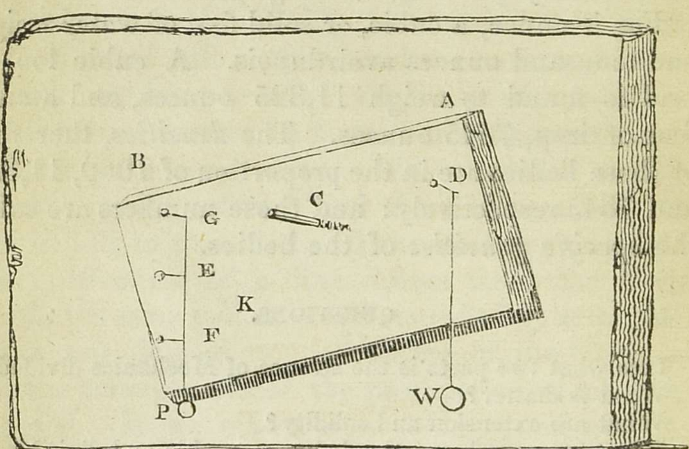
Into what two parts is the science of Mechanics divided?
What is matter?
What are extension and solidity?
What is meant by matter being moveable and divisible?
What is gravity?
What is meant by the inactivity of matter?
How is density measured?
What is specific gravity?

## LESSON III.

### ON FORCES APPLIED TO A POINT.

#### Proposition 1.

The effect of any force is the same, at whatever point in the direction of the force it is applied.



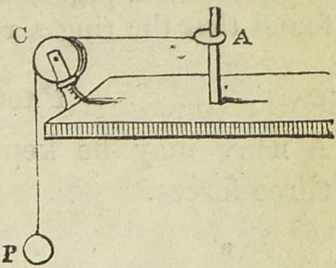

Suppose a piece of wood, A B, has a hole bored through it, and is hung freely upon a peg C; then let two weights, P and W, be hung upon two small nails, driven into the wood at D and E, and let the whole be left to balance itself, the two strings, D W and E P, hanging perpendicularly along the surface of the wood A B.

Then if the string E P be taken from the nail E, and hung upon a nail, driven at any point, as G or F, in the same *vertical line* G E P, the whole will be still found to balance itself.

This experiment shows that the effect of the force P is the *same*, at whatever point in the *direction* of the force it is applied.

But if the weight P be hung upon any other point, as K, not in the line G E F, it is then found that the wood will no longer balance itself upon the peg C.

If a weight P, is hung to a string, P C A, which passes over a wheel, or fixed pulley, C, and is fastened to a peg A, the force which presses against A is exactly equal to the weight P, since the effect of the pulley is only to change the *direction* of the force.




## Proposition 2.

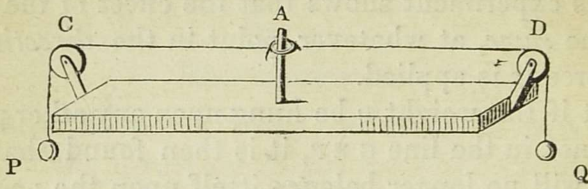
If a body be acted upon by two equal and opposite forces, it will remain at rest.

Let two pulleys C and D be screwed into a piece of wood, and a weight P be hung over C, the string being fixed to a small ring, which is hung over a peg at A, placed between C and D.

Then the force acting upon A is equal to the weight of P, and acts in the direction A C.

Now let a weight Q, equal to P, be hung over the pulley D, and attached to the ring A.

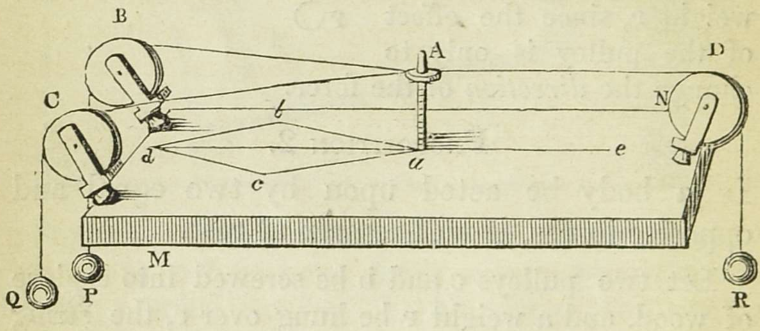
Then the point A is acted upon by two *equal* and *opposite* forces, in the directions A C and A D.




And if the peg at A be now removed, it will be found that the ring A remains at rest.

## PROPOSITION 3.

A BODY may be kept at rest by the action of three forces.




Let there be two pulleys B C, and any two weights P and Q, hung over them by two strings, which are attached to a ring hung over a peg at A, in such a manner that B A, A C, may be horizontal.

From the point *a* on the horizontal board M N, draw two lines *a b*, *a c*, parallel to the directions of the forces P P, Q Q, respectively, and proportional to them. For instance, if P is three ounces, and Q five ounces, let *a b* be three *inches*, and *a c* five *inches*. Then from *b* and *c* draw *b d* parallel to *a c*, and *c d* parallel to *a b*, and join *a d*.

Then let another pulley D be placed in the direction *d a e*, and a third weight R be suspended, which is represented in magnitude by *a d*, and attached to the ring A, and it will be found that the ring will *exactly be kept at rest* by the action of the three forces P, Q, and R, if the peg be removed.

Now, by Proposition 2, the force R *would* be kept at rest by an *equal* and *opposite force* acting at A, in a direction parallel to *a d*.

Hence the *two forces* P, Q, acting on a point at A, and represented in magnitude and direction by the *two sides of a parallelogram*, are equivalent to a *single force*, represented by the *diagonal* of the same parallelogram*.

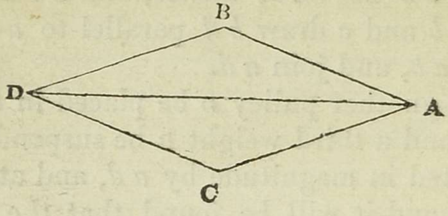
Such a single force is said to be the *resultant* of two such forces.

## Proposition 4.

Any force acting on A, represented in quantity and direction by A D, is equivalent to any other *two* forces, represented by A B, A C, respectively,

* A parallelogram is a figure of which the opposite sides are *equal* and *parallel* straight lines; and a *diagonal* is the straight line drawn from any angle to the opposite angle. Thus, in p. 14, A B D C is a parallelogram, and A D a diagonal.

provided A D is the diagonal of a parallelogram, of which A B, A C, are the sides.



In such a case the single force A D is said to be *resolved* into the two forces A B, A C.

A simple machine, such as that described in the plate of Proposition 3, which can be constructed for a few pence, affords the means of showing easily the composition and resolution of force, by placing different weights at pleasure at P, Q, and R.

## Proposition 5.

If a point be acted upon by three forces, which are represented in *quantity* and *direction* by the three sides of a triangle, taken in order, it will be kept at rest.

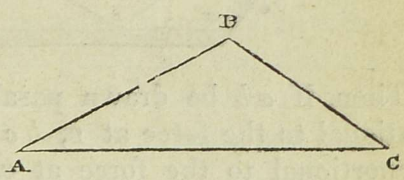
In the figure of Proposition 4, suppose a point A, acted upon by two forces, represented in quantity and direction by the two lines A B, A C. Then the effect will be the same as if a *single force*, A D, acted upon A. And if another single force, D A, equal and opposite to A D, acts also upon A, A will be kept at rest, by Proposition 2.

Now, by the nature of a parallelogram, the line B D is exactly equal to A C, and is parallel to it.

And the three lines A B, B D, D A, are the three sides of a triangle, A B D, taken in order.

Hence, if a body, A, be acted upon by three forces, which are represented by the three sides of a triangle taken in order, it will be kept at rest.

Hence also it follows, that two forces acting upon a point, A, and represented in quantity and direction by two sides, A B, B C, of a triangle, taken in order, are equivalent to *a single force*, represented by A C, the remaining side of the triangle, A B C.



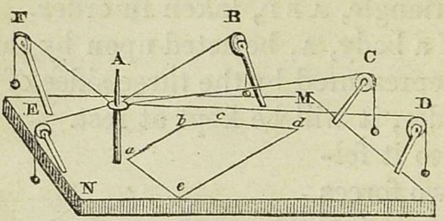

## Proposition 6.

If several forces act at once upon a point, and are such that their directions and magnitudes are represented by the sides of a plane rectilineal figure*, taken in order, the point will be kept at rest.

Suppose five weights hung over as many pulleys,

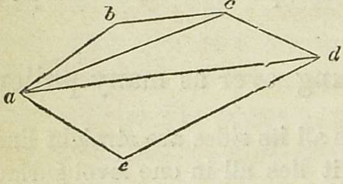
* A figure is *rectilineal* when all its sides are straight lines: and it is a *plane* figure, when it lies all in one level surface. A square is a rectilineal figure, and also a plane figure. A circle is a plane figure, but not a rectilineal figure.

and all attached to a ring, A, kept in its place by a pin; all the strings, A B, A C, &c., being parallel to the horizontal plane M N.




Then, if *a b* be drawn parallel to A B, and proportional to the force at B, *b c* parallel to A C, and proportional to the force at C, *c d* parallel to A D, and proportional to the force at D; *d e* parallel to A E, and proportional to the force at E; and *e a* parallel to A F, and proportional to the force at F: and if the lines so drawn complete a rectilineal figure, *a b c d e a*, the point A will be found to be kept at rest by the five forces, so as to remain fixed when the pin is withdrawn.

The reason of this is plain, from the last proposition.




Two forces, *a b*, *b c*, acting upon a point, are equivalent to a single force, *a c*, acting upon the same point.

Suppose such a force substituted for them.

Then the three forces, *a b*, *b c*, *c d*, produce the same effect as the two forces, *a c*, *c d;* or as a single force, *a d*.

In like manner, the *four* forces, *a b*, *b c*, *c d*, *d e*, produce the same effect as the *two* forces, *a d*, *d e*, which are equivalent to a single force, *a e*. And when an *equal and opposite* force, *e a*, is applied, the point is kept at rest.

Hence, if a point is acted upon by any number of forces, acting in one plane, we can find a single force equivalent to them all, by drawing lines successively proportional to the forces, and in their respective directions.

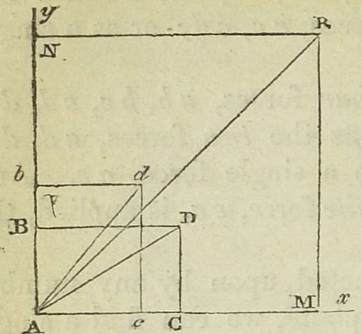
If those lines complete a rectilineal figure, the forces, by their united action, will keep a point at rest.

If they do not complete such a figure, they are equivalent to a force represented in quantity and direction by a line drawn from the first point so as to complete the rectilineal figure.

Thus, in the figure of Proposition 6, the four forces at B, C, D, E, and proportional to *a b*, *b c*, *c d*, *d e*, are equivalent to a single force acting in the direction F A, and proportional to the line *a e*.

## Proposition 7.

Any number of forces, acting in one plane upon a point, may be resolved into two forces, at right angles to each other, and their resultant found.




If A D represents the direction and magnitude of any force, acting on A, and A $x$, A $y$ are at right angles to each other, and D C, D B parallel to A $y$, A $x$ respectively, we may substitute for the single force A D, the two forces A C, A B, in the direction of A $x$, A $y$, which are sometimes called co-ordinates.

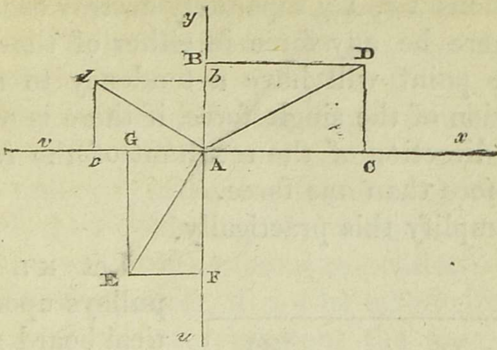
In like manner, if A $d$ represents the direction and magnitude of any other force acting upon A, in the same plane in which A $x$, A $y$ lie, and $d$ $c$, $b$ $d$, are respectively parallel to A $y$, A $x$, we may substitute for the force A $d$, the two forces A $c$, A $b$, in the directions A $x$, A $y$, respectively.

Hence, instead of the *two* forces, A D, A $d$, we may substitute the forces, A C, A $c$, (or one force, A M, equal to A C and A $c$ together,) in the direction A $x$, and the forces, A B, A $b$, (or one force, A N, equal to A B, A $b$ together,) in the direction A $y$.

And, by completing the parallelogram, A N R M, we get a single force, A R, which is the resultant of the two forces, A M, A N, and equivalent to the first two forces, A D, A $d$.

The same principle may be applied to any number of forces.

Suppose there are three such forces, A D, A *d*, A E.




Then, when these forces are resolved in the directions of A *x*, A *y*, the forces in the line *v* A *x* will be A C, *tending to move the point* A *in the direction* A *x*, and two forces A *c*, A G, *tending to move the point* A *in the opposite direction* A *v*.

And if the force A C be equal to the two forces A *c*, A G, the point A will have no tendency to move in the line *v* A *x*.

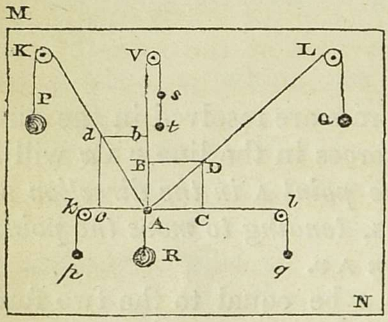
In like manner, the forces acting in the line *y* A *u* are A B, A *b*, tending to cause motion in the direction A *y*; and A F, tending to cause motion in the opposite direction A *u*.

And if A B, A *b* together are exactly equal to A F, the point A will have no tendency to move in the line *y* A *u*.

And therefore the three forces A D, A *d*, A E, will keep the point A at rest.

Any forces, acting in one plane upon a point, will *not keep it at* rest, unless the resolved parts in the directions A $x$, A $y$ separately destroy each other. For, if there be any force in either of those directions, the point will have a tendency to move in the direction of the single force, if there is only one, or in the direction of the resultant of the forces, if there is more than one force.

To exemplify this practically.




Let K L be two pulleys upon a vertical board M N; and let two weights, P, Q, be suspended upon a cord passing over the pulleys, and a third weight, R, be hung by a loose ring, A, upon the cord, so as to balance.

In order that the results may be expressed in whole numbers, suppose P to be 20 ounces, Q, 15 ounces, and R, 25 ounces.

When the whole is balanced, let two other pulleys, $k$, $l$, be placed so that their upper surfaces are in the horizontal line $k$ A C; and another pulley, V, so that one of its horizontal sides may be in the vertical line R A V.

Then taking, on any given scale, A D equal to 15, to represent the force of Q in the direction A D, and drawing D C, D B, vertically and horizontally, A C will be found equal to 12, and A B to 9. Hence the force 15, in the direction A D, is equivalent to two forces, one 12, in the direction A C, the other 9, in the direction A B.

This may be shown by hanging a weight, *q*, of 12 ounces, over the pulley *l*, and a weight *s*, of 9 ounces, over the pulley V, attaching the ends of the strings at A, and then removing the weight Q, when the equilibrium will still be found to subsist.

In the same manner, if A *d* be taken equal to 20, on the same scale, to represent the force of P, in the direction A *d*, and *d c*, *d b*, be drawn vertically and horizontally, the force 20, in the direction A *d*, will be found to be equivalent to two forces, one 12, in the direction A *c*, and the other 16, in the direction A *b*.

And this also may be shown, by hanging a weight, *p*, of 12 ounces, over the pulley *k*, and a weight, *t*, of 16 ounces, over the pulley P, attaching the ends of the strings to A; and then removing the weight P.

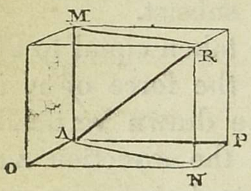
The point A will be found to be sustained in the same position as at first.

Here, then, the oblique forces have been resolved into horizontal and vertical forces, the points of which have been shown to be separately equivalent to those oblique forces: and we have now two equal forces, each of 12 ounces, acting in the horizontal directions A *l*, and A *k*: and two other equal forces, each of 25 ounces, acting in the vertical directions

A R, A V. And these resolved forces separately destroy each other, and keep the point A at rest.

## Proposition 8.

A force can always be resolved into three forces, each of which is at right angles to the plane in which the other two lie.




Suppose that a single force, represented in quantity and direction by the line A R, acts upon a point A.

And let A P, A O, A M, be drawn in the directions of the *edges* of a solid figure, M P N R O, which is such, that the angles M A P, M A O, O A P, are all right angles, and A R is in the direction of the *diagonal*, or straight line drawn across from A to the opposite angle R.

Then, if A N, M R, be joined, M R N A is a parallelogram, of which A R is the diagonal.

Hence the single force A R is equivalent to the *two* forces A M, A N, by Proposition 4. Again, A P N O is a parallelogram.

Hence the single force A N is equivalent to the *two* forces A P, A O. Let A P, A O, be substituted for A N: and the single force A R is equivalent to the *three* forces A M, A P, A O, each of which, as A M, is at right angles to the plane in which the other two, as A P, A O, lie.

The same principle may be extended to any number of forces acting upon a point, and is of the greatest importance in all problems depending upon forces acting in different planes. But the subject is somewhat too complicated to be farther entered upon here.

### QUESTIONS.

How is it shown that the effect of any .orce is the same, at whatever point in the direction of the force it is applied?

If a body is acted on by two equal and opposite forces, what effect is produced?

Can you show that a body may be kept at rest by three forces?

What must be the proportion of three forces which can keep a body at rest?

When is a force said to be the resultant of two other forces?

When is a single force said to be resolved?

If several forces act upon a point, in one plane, and are represented by the sides of a plane rectilineal figure, show that the point will be kept at rest.

Show that any number of forces, acting in one plane, may be resolved into two forces, at right angles to each other.

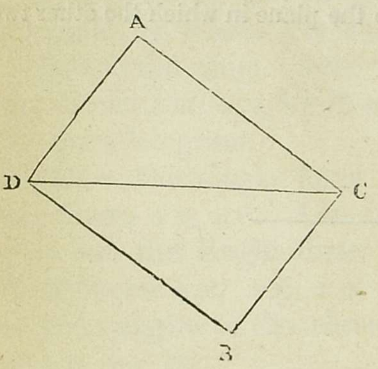
Show that a single force can always be resolved into *three* forces, each at right angles to the plane in which the other two lie.

# LESSON IV.

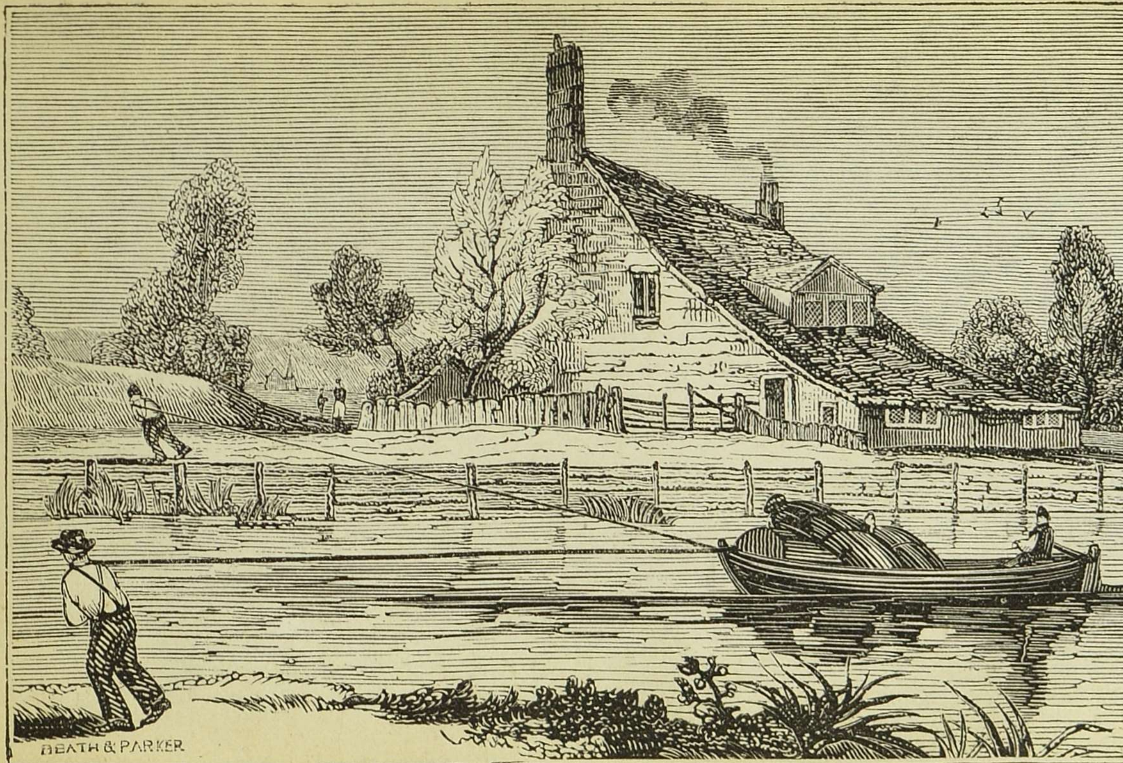
## EXAMPLES OF THE COMPOSITION AND RESOLUTION OF FORCES.

It is very important to get a clear notion of the *composition* and *resolution* of forces. We will proceed to show the application of the principle in a few plain examples.

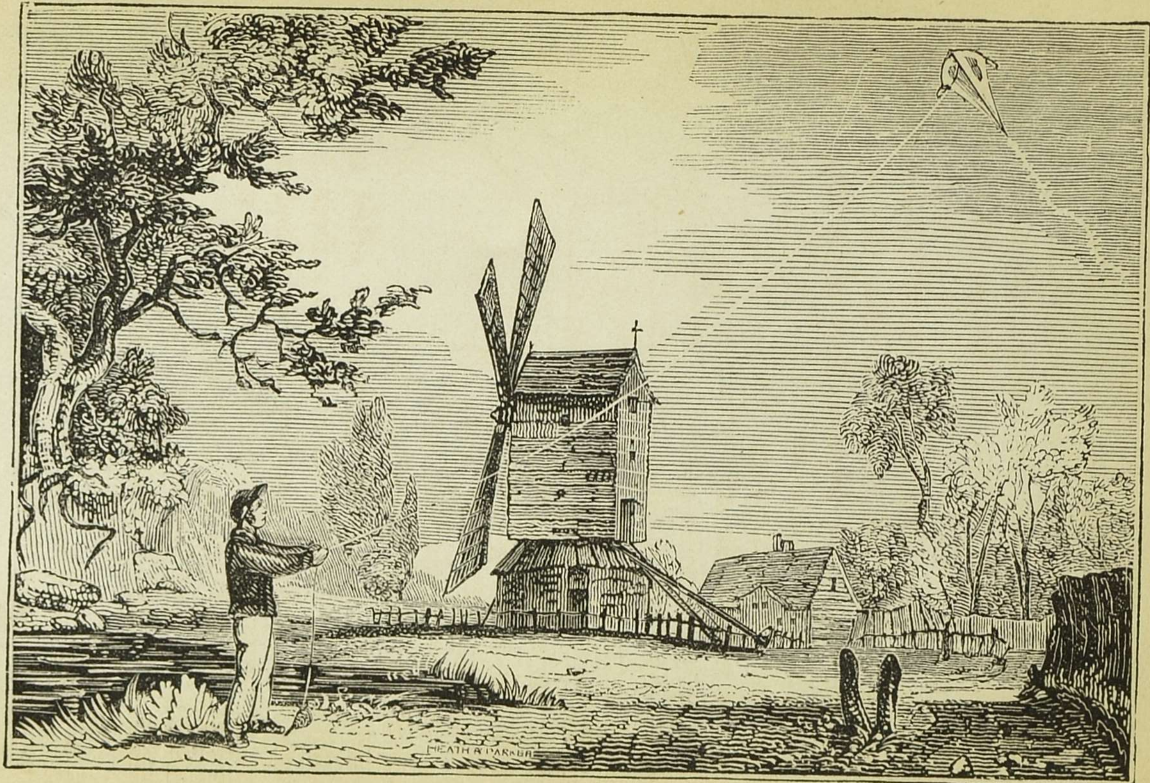
Example 1.—Suppose a man, A, is able to pull with a force of 400 lbs. in towing a boat, C, against the stream: and a boy, B, on the opposite side of the river, can pull with a force of 300 lbs.; and that we wish to know the force which the *stream* exerts upon the boat, assuming that the man and boy are *just able* to keep the boat from running down the stream; and that the two ropes, C A, C B, are at right angles to each other.




To find what the force of the stream is, if we lay down by a scale the line C A to represent 400, and C B at right angles to it, to represent 300, and complete the parallelogram, we shall find the diagonal C D, to be 500, on the same




Example of the Resolution of Force

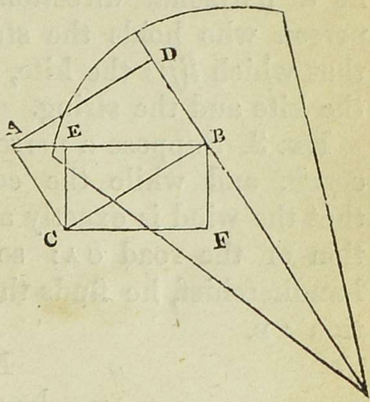



Example of the Resolution of Force.

scale: which shows that the force of the stream, in that case, is equal to the pressure of 500 lbs.

Ex. 2. A common kite, sustained in the air, gives a good example of the manner in which force is resolved.

The *tail* keeps the lower part of the kite always inclined *from* the wind, so that the wind acts obliquely upon the lower surface of the kite.




Suppose the kite quite flat, and the whole force of the wind upon it* to be applied at some point B, to which also the string is attached.

Let a horizontal line, A B, be taken to represent the *pushing* force of the wind. Then if A D is drawn perpendicular to the surface of the kite, and A D B C is a parallelogram, the effect upon the point B will be the same as if the force A B were removed, and two *pushing* forces applied instead of it, one represented by D B, the other by C B.

The force D B is applied *along the surface* of the kite, and has no effect *upon* the surface, so that we need consider only the force C B applied

* It will be shown that the supposition may be made, in the chapter on the Centre of Gravity.

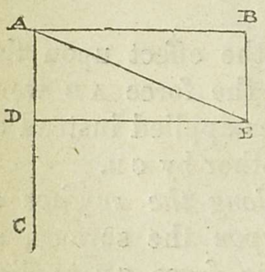
at B in a direction perpendicular to the surface of the kite.

Now again, let C E, B F be drawn perpendicular to A B, and C F parallel to B E.

Then the effect of the *pushing* force, C B, will be the same as if there were *two* pushing forces applied instead of it, represented by E B and F B.

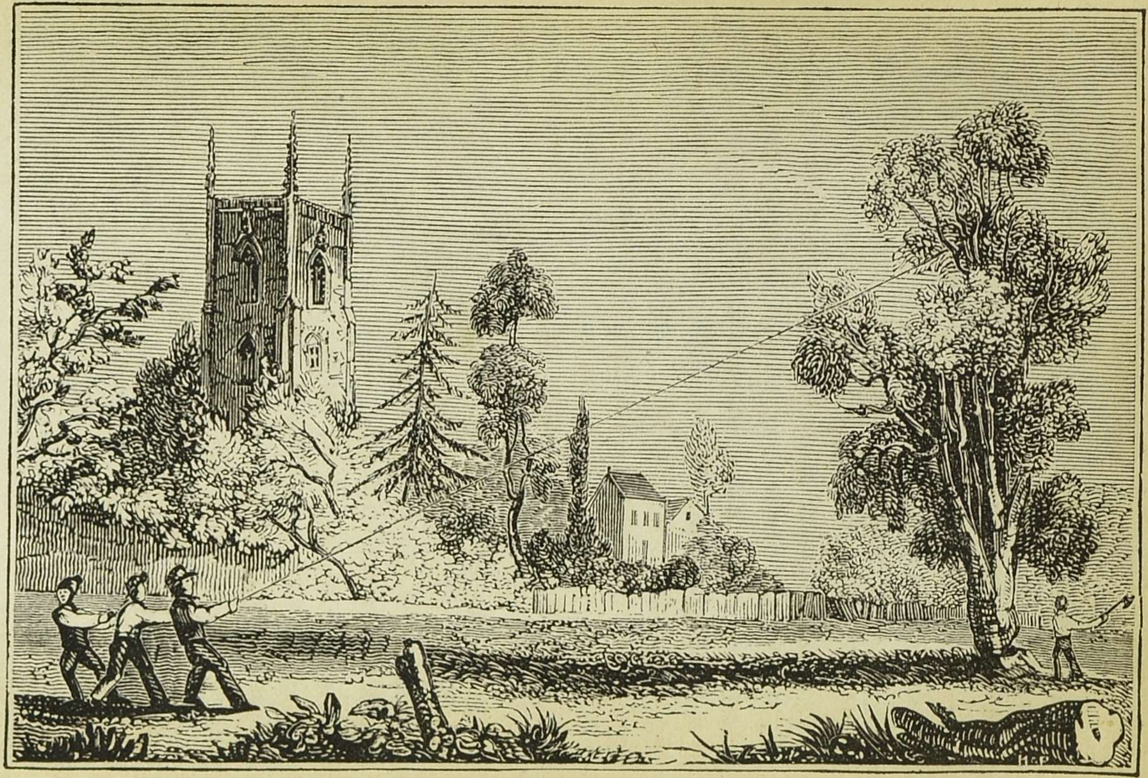
The force E B is that which causes the kite to *pull* in a horizonal direction against the hand of the person who holds the string; and the force F B is that which *lifts* the kite, and sustains the *weight* of the kite and the string.

Ex. 3. Suppose a person is travelling by a stage-coach, and while the coach is stopped, observes that the wind is exactly at right angles to the direction of the road C A: so that, if he holds up his handkerchief, he finds that it is blown in the direction A B.




Now suppose the coach to be set in motion in the direction C A. Then the effect upon the handkerchief will be the same as if a second current of air had begun to blow in the *opposite* direction A C, moving with the same velocity as the coach itself moves in the direction C A.

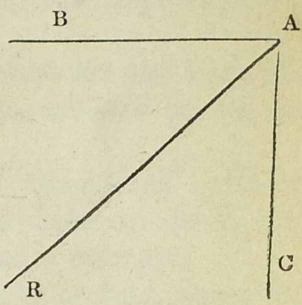
If then we take A B to represent the force of the wind in the direction A B, and A D to represent, on



Example of the Resolution of Force.

the same scale, the force occasioned by the resistance of the air to the motion of the coach, and complete the parallelogram A B E D, drawing the diagonal A E, the line A E will be the direction in which the handkerchief will now be blown, and the line A E will measure, upon the same scale, the force with which it is urged in that direction.

If the original direction of the wind, A B, is observed, and also the direction, A E, in which the wind *appears* to blow, when the coach is in motion, the proportion may be found between the force of the wind, A B, and the force A D, occasioned by the resistance of the air to the motion of the carriage.




Ex. 4. In pulling down a tree, the force A R, applied at the point where the rope is attached, may be resolved into two; one, A C, in the direction of a line passing from that point to the root of the tree; the other, A B, at right angles to that direction: and if there be no other force acting upon the tree, it will begin to fall, as soon as the effect of the last-mentioned force, acting perpendicularly at A, is sufficient to overcome the toughness of the fibres at the root of the stem.

## QUESTIONS.

If two men, on opposite sides of a river, draw a boat against the stream, how do you find the force of the stream when the boat is just kept at rest, the two strings being at right angles to each other?

Show how a kite is sustained in the air.

By what means can the force of the wind be compared with the resistance of the air opposed to the motion of a carriage?

Show how the force of the rope employed in pulling down a tree, may be resolved.

## LESSON V.

### ON THE MECHANICAL POWERS.

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It is very seldom convenient to apply a force directly to produce a mechanical effect. Any intermediate instrument which is employed for the purpose is called a *machine;* and the simplest parts of which all machines are composed are called the *mechanical powers.* It must be borne in mind, however, as will be shown by-and-by, that no increase of *power*, properly so called, is gained by the employment of any machine.

The simplest mechanical powers are the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw.*

A *lever* is a bar, usually considered without weight, or so arranged as to balance itself, and resting upon a fixed point called a *fulcrum.*

The lever is said to be *straight* or *bent*, according as the *arms*, or parts on each side of the fulcrum, are in the same straight line or not.

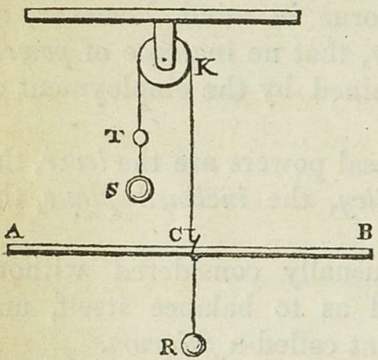
The lever is the most simple and most common of all the mechanical powers. If we *stir the fire*, the *poker* is a lever, the fulcrum being the bar of the grate on which the poker turns, and the force of the hand, pressing at one end, moving the coals at the other end of the poker.

A *pair of scales* is a lever, the fulcrum being the point on which the beam of the scales rests, and the weights in the scales are the two forces.

Many other examples are given in the next lesson.

## Proposition 9.

A weight suspended to a lever, at a point immediately under its own point of suspension, has no tendency to turn the lever round.




Suppose a bar, A B, perfectly balanced and suspended by a string, K C, attached to it at C.

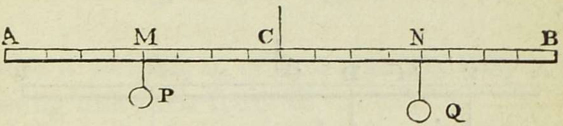
If any weight, R, be hung directly under C, the line of its direction passes through C and K; and, therefore, it can have no effect in causing any motion about C.

Also the pressure on K is equal to the weight of the lever together with the weight of R.

This is easily proved by experiment: for if C be supported by a string passing over a fixed pulley, and attached to a weight, T, which exactly supports the lever *alone*, and another weight, S, equal to R, be attached to T, the two weights, S and T, will be found exactly to balance the lever and R together.

## Proposition 10.

Two equal forces, applied perpendicularly to the arms of a straight lever, on opposite sides, and at equal distances from the fulcrum, and tending to move the lever in opposite directions, will keep each other at rest.




Let A B be a straight bar, suspended at C, so as to balance itself, and divided into inches and parts of an inch.

Then if two equal weights, P, Q, are hung upon it, at *any points* M, N, which are at *equal distances from* C, on either side, they will be found to balance each other.

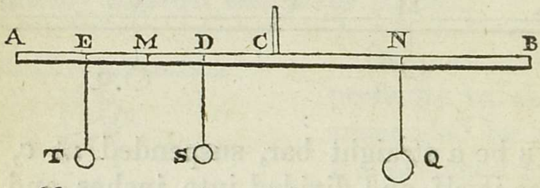
But if one of them, as P, be heavier than the other, Q, P is found to overbalance Q, when they are each at equal distances from C: or if one of the equal weights, as P, be moved to a greater distance from C than Q is, it then also overbalances Q.

It is indifferent what *units of weight and distance* are taken. In the following illustrations we shall usually assume an *ounce* as the unit of weight, and an *inch* as the unit of distance.

### Proposition 11.

Two equal forces, applied perpendicularly to a straight lever, produce the same effect as if they were *both applied together* at the middle point between them.

Let Q be hung upon the lever as before, and suppose it to be four ounces; and suppose C N to be six inches.



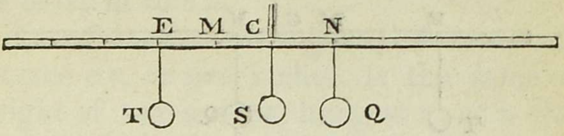

Then, if an equal weight were hung at M, C M being also six inches, it would just balance Q.

But instead of that *one* weight of four ounces, at M, let *two* weights, each of two ounces, be hung, one at D, and the other at E, on opposite sides of M, and equally distant from it. Then *those two weights* will be found exactly to balance the single weight Q; and, therefore, they produce the same effect as if they were to be applied together at the middle point between them.

It makes no difference in the effect of the two weights, S and T, upon the lever, *how far* from M they are hung, provided the distance D M, on one side of M, is equal to the distance E M, on the other side.

## PROPOSITION 12.

A FORCE of *one* ounce, at a distance of *two* inches, balances a force of *two* ounces at a distance of *one* inch.




Suppose a weight, Q, of two ounces, suspended at N, C N being one inch from C.

Then if C M is also one inch, *two* weights, S, T, of one ounce each, both suspended at M, would exactly balance Q.

And if S and T are moved in opposite directions from M, through equal spaces, the equilibrium will still continue, by Prop. 11.

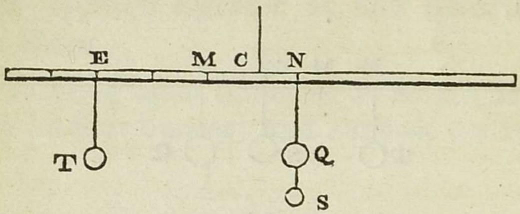
Suppose each of them moved through *one* inch.

Then the weight S will be hung exactly at the point C, on which the lever is suspended, and therefore will have *no effect* in turning the lever either one way or the other, by Prop. 9. And the weight T *alone* will balance Q, and it will be at a distance of *two* inches from C.

## PROPOSITION 13.

A FORCE of *one* ounce, at a distance of *three* inches, balances a force of *three* ounces at a distance of *one* inch.

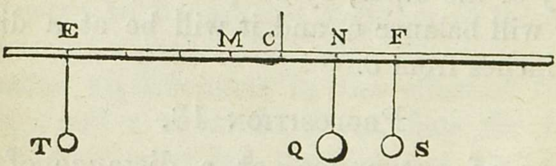
Suppose the weight Q, as before, to be *two* ounces, hung at N, and that the weights T and S, of one ounce each, are moved each to the distance of two inches on either side of M. The whole will still




balance. But the weight S will be moved exactly as far as N, and will now be *added* to the weight Q, the two together making *three ounces;* and the weight T, of one ounce, will be at the distance of *three* inches from C, and exactly balance the *three* ounces hung at a distance of *one* inch from C.

## Proposition 14.

A force of *one* ounce, at a distance of *four* inches, balances a force of *four* ounces at a distance of *one* inch.




Suppose the weight Q, as before, to be *two* ounces, hung at N; and that the two weights, T and S, each of one ounce, are moved to a distance of *three* inches on either side of M. Then the three weights will still balance, because the effect of the two weights, T and S, is the same as if they were hung together at M, the middle point between them, by Prop. 11, and C M is equal to C N.

But the effect of the weight S, of *one* ounce, at the distance C F, or *two* inches, is the same as that of a weight of *two* ounces, hung at N, at a distance, C N, of *one* inch, by Prop. 12.

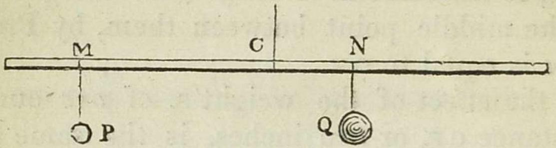
Suppose, therefore, S to be removed, and a weight of *two* ounces to be added to Q, making, together, *four* ounces hung at N; the whole system will still balance. That is, the weight T, of one ounce, at a distance C E, or *four* inches, balances a weight of *four* ounces at a distance of *one* inch.

In all these instances, we see that the number of *ounces* in the weights, multiplied by the number of *inches* in the distances, are equal on each side of the fulcrum C. Thus, one multiplied by four is equal to four multiplied by one; and the same is plainly seen in the other instances.

This leads us to a general property of the straight lever.

## Proposition 15.

Any two weights, tending to move a straight lever in different directions, and acting perpendicularly upon the arms, will balance each other, provided the product of the numbers, representing the *weights* and *distances* on each side of the fulcrum, is the same.




Thus, suppose we have a weight, Q, of twelve ounces, hung at a distance, C N, of four inches from C. Then the product of the *weight* and *distance*, on the side on which Q is hung, is four times twelve, or forty-eight.

And if P be *any* other weight, hung at a distance, C M, such that the corresponding product of the *weight* and *distance*, on the side on which P is hung, is also equal to 48, P is found to balance Q.

Thus, if P is one, C M must be 48, since one multiplied by 48 makes 48.

If P is 2, C M must be 24, since twice 24 are 48.

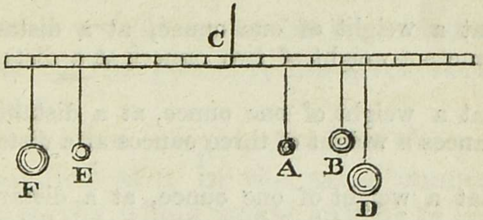
If P is 3, C M must be 16, since three times 16 are 48; and so on.

This property is often expressed by saying, that a weight, P, is in the same proportion to the weight Q, as C N is to C M, or that the weights are to each other inversely as their respective distances from C.

The product of the *force* multiplied by the *distance* at which it acts, is called the *moment* of the force. Hence any two forces balance on a straight lever, when the *moments* of the two forces on each side are the same.

In like manner *any* number of forces, acting on a straight lever, will keep it at rest, provided the sum of the *moments* of all the forces, tending to turn it in one direction, is equal to the sum of the *moments* of all the forces tending to turn it in another direction.

Thus, suppose on one side there are three weights, A of two ounces, at a distance of two inches; B of three ounces, at a distance of four inches; D of four ounces, at a distance of five inches. Then, the *moment* of A is twice two, or four: the *moment* of B is three times four, or twelve: the *moment* of C is four times five, or twenty; and the *sum* of all the moments is thirty-six.




Now, suppose that on the other side of the lever we have two weights; E of two ounces, at a distance of six inches; and F of three ounces, at a distance of eight inches.

Then the *moment* of E is twice six, or twelve; and the *moment* of F is three times eight, or twenty-four.

And the *sum* of the moments is thirty-six, as it was also on the other side.

Hence the two weights, E, F, so placed, will exactly balance the three first weights: as may easily be verified by experiment.

QUESTIONS.

What are the mechanical powers?

What is a lever?

What is a fulcrum?

Give instances of a lever.

Show that a weight suspended to a lever at a point, directly under its own point of suspension, has no tendency to turn the lever round.

How must two equal forces be applied perpendicularly to the arms of a straight lever on opposite sides, so as to keep it at rest?

How is it shown that two equal forces, applied perpendicularly to the arms of a straight lever, produce the same effect as if both were applied together at the middle point between them?

Show that a weight of one ounce, at a distance of two inches, balances a weight of two ounces at a distance of one inch.

Show that a weight of one ounce, at a distance of three inches, balances a weight of three ounces at a distance of one inch.

Show that a weight of one ounce, at a distance of four inches, balances a weight of four ounces at a distance of one inch.

When will any two weights balance one another, acting perpendicularly on opposite sides of a straight line?

What is meant by the moment of a force?

How do you know when any number of forces, tending to turn a straight lever in one direction, will balance any number of forces tending to turn it in the opposite direction?

## LESSON VI.

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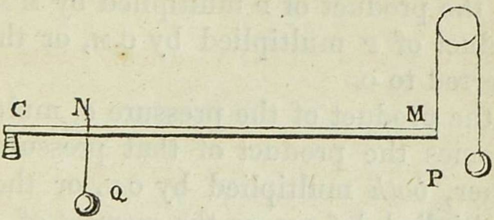
### Proposition 16.

In *any* straight lever, in equilibrio, the *moment* of the forces tending to move the lever in opposite directions, and referred to the centre of motion, is the same.




Suppose a weight, Q, to be sustained upon the lever C N M by the power P, the lever resting upon the fulcum C.

Then if another weight, equal to the pressure at C, were applied at C, in the same manner as P is applied at M, the whole would still be in equilibrio.

And, in this case, the bar C N M is a straight lever *inverted*, the forces at the two ends being P and the pressure at C, and the force in the opposite direction, corresponding with the pressure on the fulcrum in Proposition 15, being the weight Q.

Hence, by Proposition 15, the *moment* of the force P, referred to N, or the product of the weight

P, multiplied by the distance M N, is equal to the *moment* of the force C, referred to N, or to the product of the pressure on C, multiplied by the distance C N.

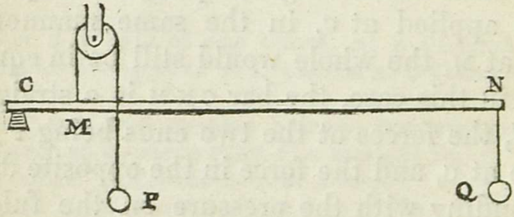
Also the weight Q is equal to the *sum* of the power P and the pressure on C, if Q is between M and C; since the whole of the weight Q is sustained at the points M and C.

Suppose that to each of these equal *moments* we add the product of P, multiplied by the distance C N.

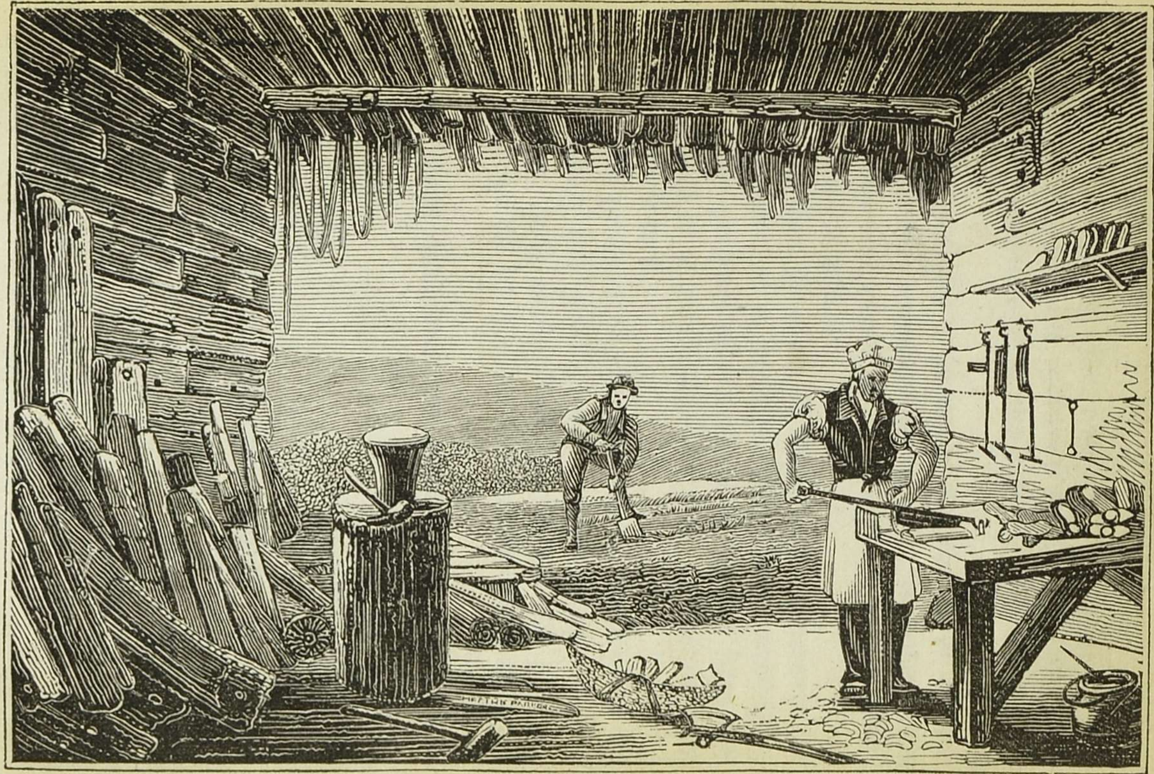
Then the product of P multiplied by M N becomes the product of P multiplied by C M, or the *moment* of P referred to C.

And the product of the pressure C, multiplied by C N, becomes the product of that pressure C and of P together, *both* multiplied by C N, or the product of Q multiplied by C N, or the *moment* of Q referred to C.

And these moments are, therefore, equal to one another.




For example, suppose P to be 2 ounces, Q 12 ounces, C M 6 inches, C N 1 inch.



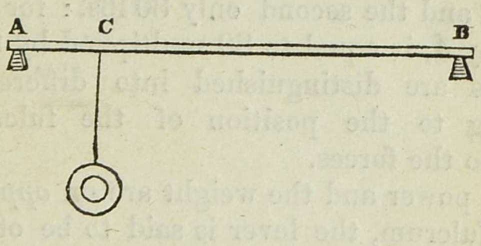
Levers of the First and Second Kind.

Then the moment of P, referred to C, is twice six, or twelve. The moment of Q, referred to C, is twelve multiplied by one, or twelve. And the two forces tend to turn the lever in opposite directions about C: therefore they will balance one another.

If P is nearer to C than Q is, P must be greater than Q.

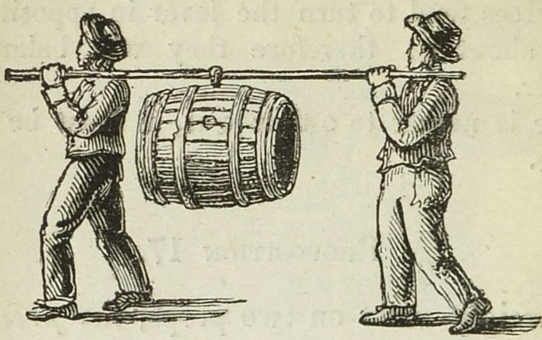
## PROPOSITION 17.

IF a weight rests on two props, the *pressure* on each is such, that the *pressure* multiplied by the *distance* from the weight, is the same on each side.




For the bar A C B may be considered as a lever turned upside down, the two *pressures* on A and B being equal to two weights which would balance themselves on a fulcrum placed at C.

Suppose, for instance, two porters, twelve feet from one another, carry a cask, weighing 240 lbs., upon a pole; and that the point at which the cask is hung, is four feet from the shoulder of the first, and eight feet from the shoulder of the second.



Then the first man will sustain a pressure *twice* as great as the second man does: the first carrying 160 lbs., and the second only 80 lbs.: for 160 multiplied by 4, is equal to 80 multiplied by 8.

Levers are distinguished into different *kinds*, according to the position of the fulcrum with respect to the forces.

If the power and the weight are on *opposite* sides of the fulcrum, the lever is said to be of the *first kind*.

Thus, when a man turns up earth with *a spade*, the *fulcrum* is the part of the spade which rests on the ground, the *power* is applied at the *handle* of the spade, and the *weight* to be moved is the earth at the lower end of the spade.

A pair of scissors is a lever of the first kind; the

Levers of the Third Kind.

power being the pressure of the finger and thumb, and the fulcrum the rivet of the scissors.

The lever is of the *second kind*, when the power and weight are on the same side of the fulcrum, and the power is *farther* from the fulcrum than the weight is.

The knife used by patten-makers, is a lever of this kind. A pair of common nut-crackers is another instance.

The power of the hand is applied near the extremity of the nut-crackers, the joint is the fulcrum, and the fruit to be crushed is placed between the joint and the hand.

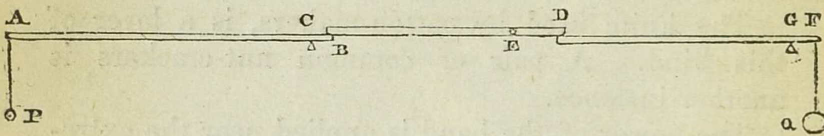
The lever is of the *third* kind, when the power and weight are on the same side of the fulcrum, and the power is *nearer* the fulcrum than the weight.

A pair of shears, used for clipping sheep, is an instance of this kind of lever.

An oar is also a lever of the third kind. For the *weight* to be moved is the *boat*: the *power* is the force of the man pulling at the end of the oar; and the *fulcrum* is that part of the *water* against which the blade of the oar presses.

## Proposition 18.

In a combination of straight levers, A B, B D, D F, of which the centres of motion are C, E, G, there will be an equilibrium between two forces, P and Q, perpendicularly applied, when P is in the same proportion to Q, as the product of all the distances, C B, E D, G F, to the product of all the distances, C A, B E, D G.




Thus, suppose A C is 12 inches, B C 1 inch; B E 7 inches, E D 2 inches; D G 9 inches, G F 1 inch: then P is to to the pressure at B, as A C to C B, or 12 to 1, by Prop. 15.

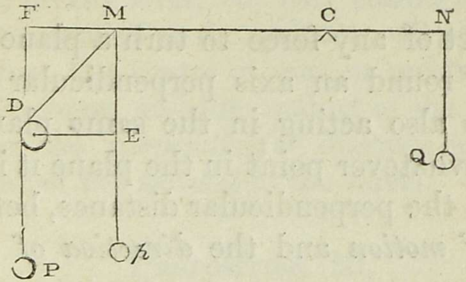
The pressure at B is to the pressure at D, as B E to E D, or 7 to 2.

The pressure at D is to the weight Q, as D G to G F, or 9 to 1; and therefore, by the general principle of proportional quantities, P is to Q as the product of A C, B E, D G, to the product of C B, E D, and G F; or, in this instance, as the product of 12, 7, and 9, to the product of 1, 2, and 1; or as 756 to 2; or as 378 to 1.

The same principle may be extended to any combination of straight levers.

In any straight lever, if either of the forces, as P, does not act perpendicularly upon the arm C M at M, but in some other direction, as M D, if we draw M E perpendicular to C M, and D E parallel to

C M, and take M D to represent the force of P, M D is equivalent to two forces applied at M, one represented by M E, perpendicular to C M, the other equal to E D, or M F, applied at M in the direction M F.




The force M F acting *from* C, can not tend to cause motion *about* C, and the effect of P upon the lever will be the same as if a force, *p*, represented in magnitude by M E, were applied perpendicularly at M.

By these means, the effect of any oblique forces upon a straight lever, may be at once reduced to that of forces perpendicularly applied.

QUESTIONS.

How do you find the proportion of two forces which balance each other on any straight lever?

If a weight rests on two props, how do you find the pressure on each?

How many kinds of lever are there?

Give instances of levers of each kind.

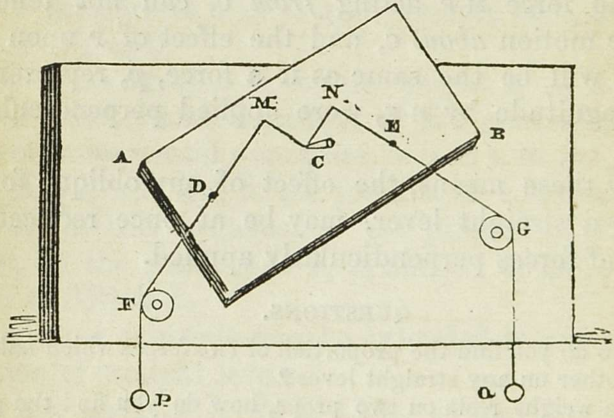
How do you find the proportion between two forces which balance each other by means of a combination of straight levers?

# LESSON VII.

## ON THE BENT LEVER.

### Proposition 19.

The effect of any force to turn a plane body, as a plank, round an axis perpendicular to itself, the force also acting in the same plane, is the same at whatever point in the plane it is applied, provided the perpendicular distance, between the *centre of motion* and the *directioa of the force*, be the same.




Let A B be a plane moveable about a horizontal axis, C, upon which it is found to balance itself: and, suppose two equal weights, P and Q, are suspended from two points of A B, namely, D and E, by cords which pass over two pulleys, F and G.

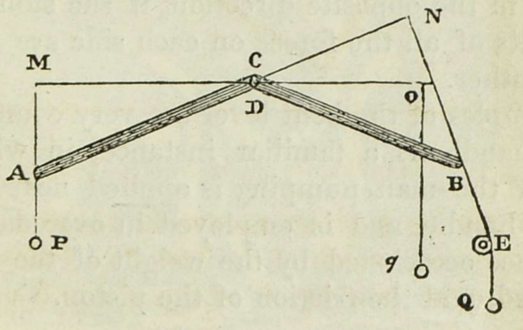
Then, if C M and C N, the perpendiculars from C, upon the directions F D M, G E N, are equal, the whole system will be found to balance itself.

Hence, in any lever, we may consider any force as P, to be applied at the point M, where a perpendicular from the centre of motion meets the direction of the force: and the product of the force, multiplied by the *perpendicular* distance at which it acts, is called the *moment* of the force.

## Proposition 20.

Any two forces tending to turn a *bent* lever in different directions, will balance each other, provided the *moment* of the two forces is the same.

Suppose a bar, A C B, which balances itself on C, has two forces, P and Q, applied to it at the points A and B, in the directions A P, B E.



Suppose C M, C N, are perpendicular to P A M, E B N.

Then the effect is the same as if the force P were applied perpendicularly to C M at the point M, and the force Q applied perpendicularly to C N at the point N.

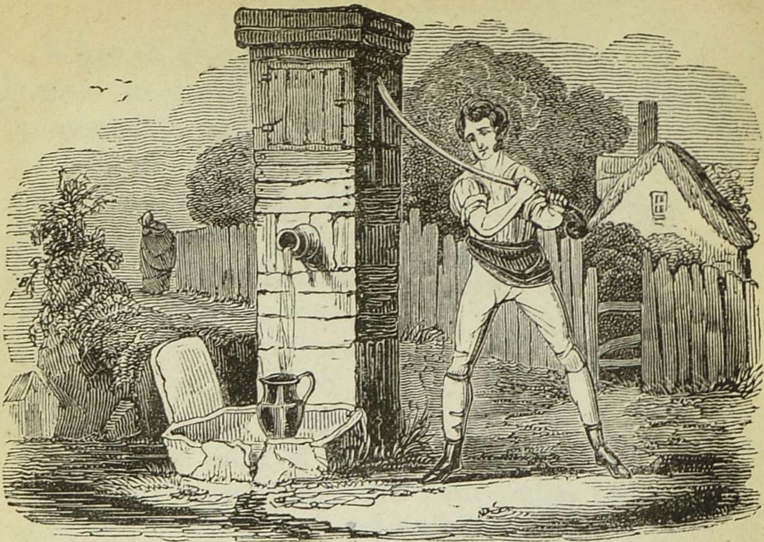
Now take C O in the same straight line as M C, and equal C N.

Then, by Proposition 19, the effect on the system is the same, if the weight Q be removed, and another equal weight, $q$, be applied perpendicularly to C O at the point O: so that the *moment* of $q$ is equal to the *moment* Q.

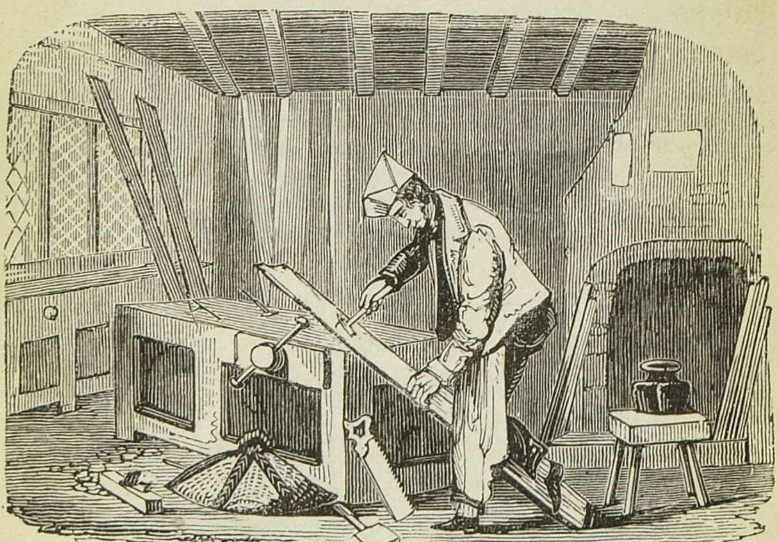
Hence we may consider M C O as a *straight* lever and, by Prop. 15, p. 42, the two forces, P, $q$, will balance when the *moment* of P is equal to the *moment* of $q$; and therefore is equal to the moment of Q.

In like manner it may be shown, that any number of forces tending to turn any lever in one direction will balance any number of forces tending to turn it in the opposite direction, if the sums of the moments of all the forces on each side are equal to one another.

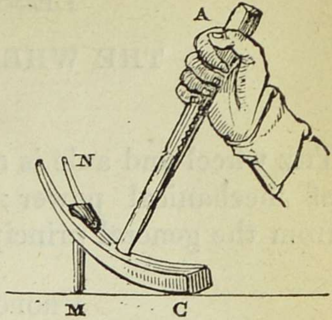
Examples of the bent lever are very common. A pump-handle is a familiar instance, in which the force of the man pumping is applied near the end of the handle, and is employed in overcoming the resistance occasioned by the weight of the water to be raised, and the friction of the piston.



Examples of the Bent Lever.



When a clawed-hammer is used to draw a nail, the hammer is a bent lever, the fulcrum of which is the point C, on which the hammer presses, the power is applied upon the handle of the hammer at A, and the resistance to be overcome is the force with which the nail sticks in the wood.




In this instance, if the length of the handle, and the direction in which the man pulls, remain unaltered, the force exerted will be greater in proportion as the distance, C M, is diminished.

## QUESTIONS.

How do you show that the effect of a force to move any plane is the same at whatever point in the plane it is applied, provided the perpendicular distance between the centre of motion and the direction of the force is the same?

When will two forces balance one another on a bent lever?

Give some examples of a bent lever.

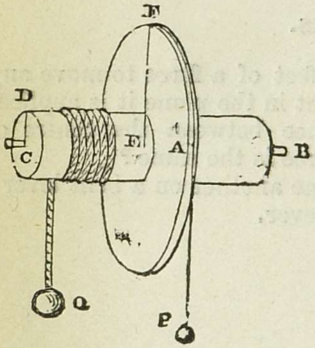
# LESSON VIII.

## THE WHEEL AND AXLE.

The wheel and axle is a very convenient application of mechanical power; and is easily understood from the general principles of the lever.

### Proposition 21.

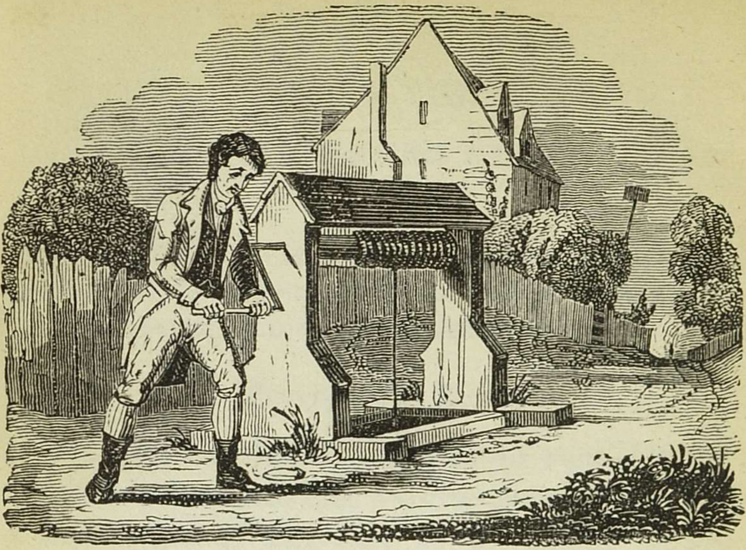
In the wheel and axle there is an equilibrium, when the power and the weight are to each other as the radius of the axle to the radius of the wheel.




Suppose a wheel, A F, and an axle, E D, to be moveable about the same centres of motion, C, B: and that a weight, P, is suspended to the wheel, and a weight, Q, to the axle.

Then the weight P acts perpendicularly at the point A, where the string leaves the wheel, and at a distance, E A, from the axis of motion, C B; E A being equal to E F, the radius of the wheel.

The weight Q also acts perpendicularly on the



Wheel and Axle.

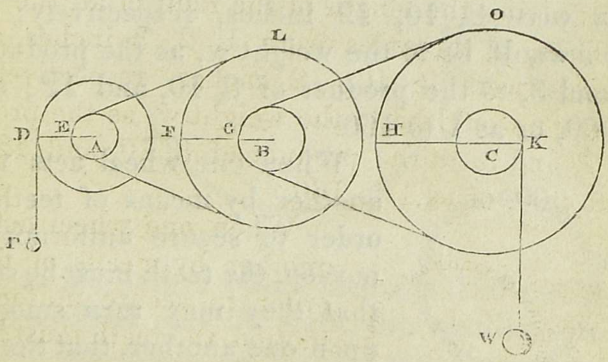
*axle*, and at a distance, equal to C D, the radius of the axle. Hence the two weights will balance each other, when the weight P is in the same proportion to the weight Q, as the radius of the axle, C D, to the radius of the wheel, E F, by the property of the lever.

Thus, if C F is 12 inches, and C D one inch, Q will be twelve times as heavy as P.

A common winch, such as that used to draw water from a well, is an example of the wheel and axle.

A windlass, in which men, acting by means of bars, draw a weight, sustained by a cord wound round an axle, is another instance.

Sometimes the axle of one wheel is made to act upon the circumference of another wheel, the axle

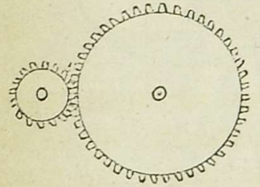



of which again acts upon the circumference of a third wheel; and so on to any number of wheels. This is effected either by means of a strap passing round each wheel and axle, or by teeth cut in each of them.

In such a combination, the force of P is to the force exerted on the strap, E L, as the radius of the axle A E, to the radius of the wheel A D. The force on the strap E L is again to the force on the strap G O, as the radius of the second axle, G B, to the radius of the wheel B F; and this third force on G O is to the force at K, or the weight W, as the radius of the third axle, C K, to the radius of the third wheel, E H.

Hence, in any such combination, the power P will be to the weight W, in the same proportion as the number obtained by multiplying the length of the radii of all the axles is to the number obtained by multiplying the length of the radii of all the wheels.

Thus, if the axles were 1, 2, 3 inches, and the wheels were 8, 10, 12 inches, respectively; the power P would be to the weight W, as the product of 1, 2, and 3, to the product of 8, 10, and 12; or as 6 to 960, or as 1 to 160.



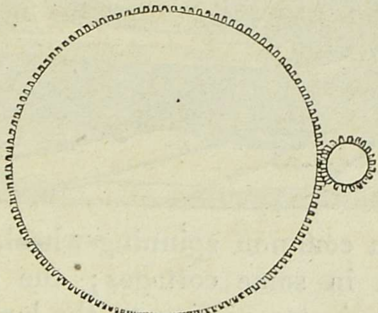
When one wheel acts upon another by means of teeth, in order to secure uniformity of motion, the teeth must be so cut that they may turn smoothly upon one another, that the surfaces in contact may be perpendicular to one another, and that the proportion of the power and weight necessary for equilibrium should not be altered as the wheels revolve.

The means by which these conditions are secured.

and the different kinds of toothed-wheels commonly used, would require for their explanation more detail than can be entered upon here.

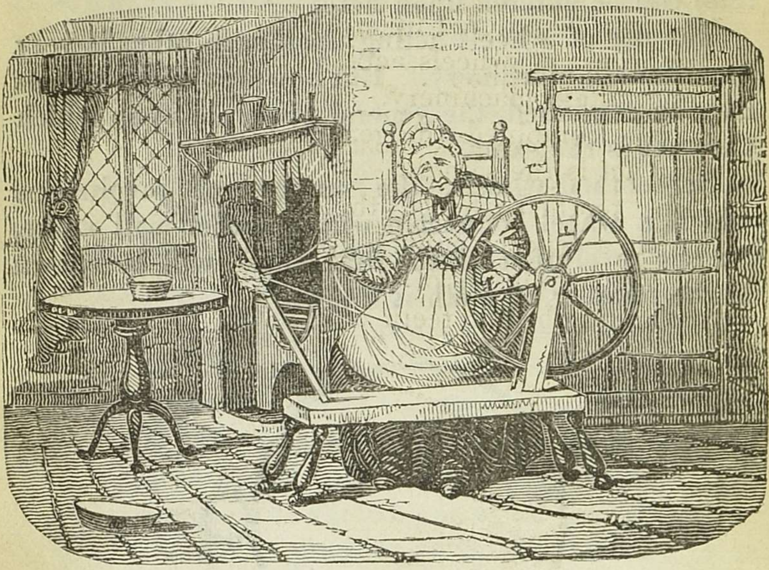
The use of wheels acting upon each other is very important in machinery. But however many wheels may be combined, the general principle upon which they act is the same.

In toothed-wheels, the number of revolutions which a wheel connected with another will make, for one revolution of the first wheel, will be known if the number of teeth is known.



Thus if a larger wheel contains 120 teeth, and a smaller wheel only 20 teeth, the smaller wheel will revolve six times as fast as the larger one: and thus angular motion can be communicated, and at the same time increased or diminished in any required proportion. This is constantly seen in clockwork.

A very rapid motion may readily be communicated to a spindle, by means of a strap passing round a larger wheel.



Thus, in a common spinning-wheel, which may still be seen in some cottages; the woman who spins communicates motion to the large wheel, by pressing upon the spokes of the wheel near the centre, thus giving a rotatory motion to the circumference of that wheel, much greater than that of her own hand at the time of communicating the motion. This motion is communicated to the small spindle on which the thread is wound, by means of a band: the spindle revolving, perhaps, five hundred times for every revolution of the larger wheel.

QUESTIONS.

What is the wheel and axle?

What is the proportion of the power and weight which balance each other by means of the wheel and axle?

How may motion be communicated from one wheel to another?

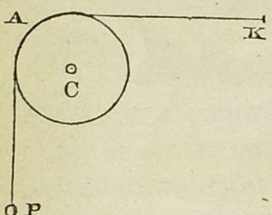
How can the number of revolutions made by a wheel, connected with another wheel in motion, be discovered?

# LESSON IX.

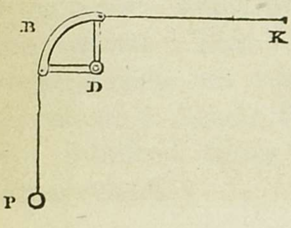
## THE PULLEY.

A PULLEY is a wheel, with a groove in the circumference, in which a string or chain passes. If the centre of motion of the pulley is itself immoveable, the pulley is said to be a fixed pulley. If not, the pulley is said to be moveable.

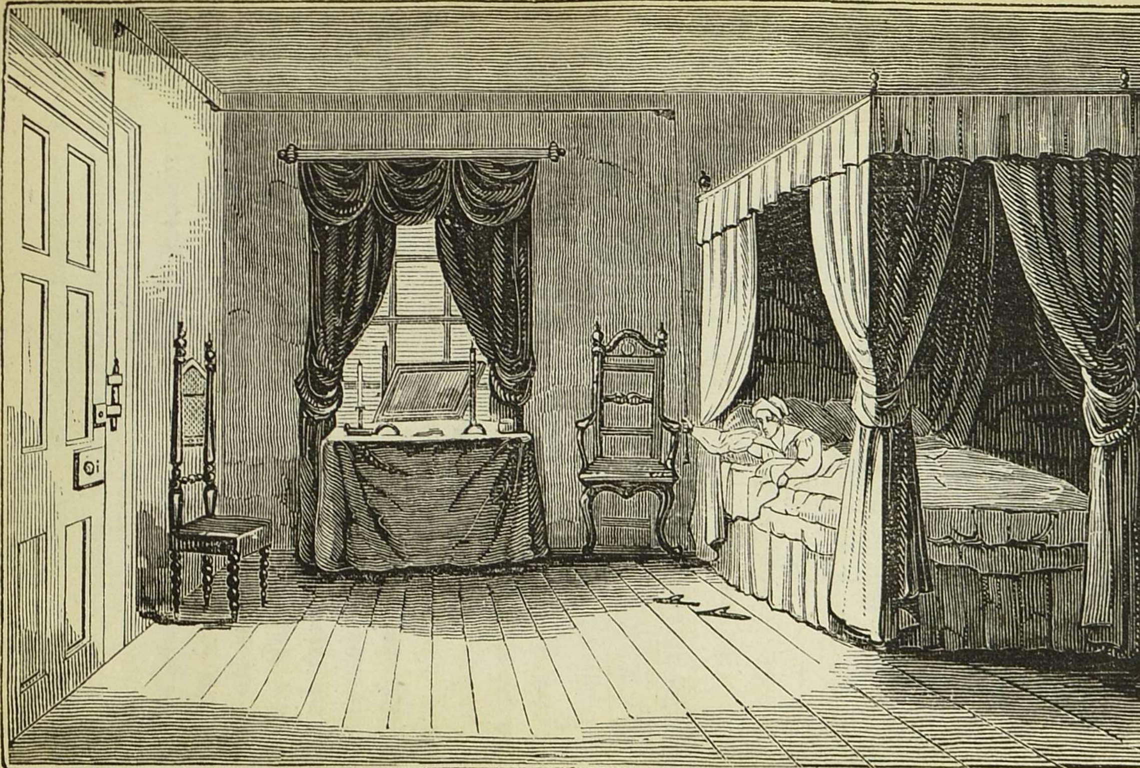
One of the most common uses of a fixed pulley is to change the direction of a force. Thus, if a person wishes to lift the bolt of a door, without getting out of bed, a string may be fastened to the bolt, and, passing over as many pulleys as may be necessary, may be brought within reach of his hand.




It is plain, that the same effect may be produced by a pulley, A, moveable about a fixed centre, C, or by a *crank*, B, (next figure,) moveable about a fixed centre, D. Thus the greater part of the various contrivances employed in bell-hanging may be referred to the fixed pulley.




In those instances, the force employed at P is exactly equal to the pressure produced at K, the other extremity of the line, supposing that the line is not capable of being stretched.



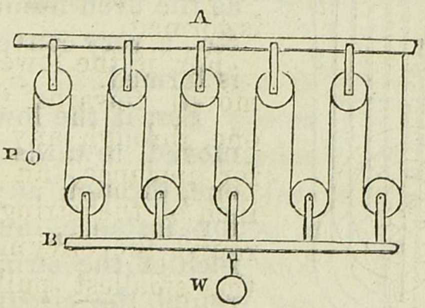
The Pulley.

But when many strings are employed in supporting a weight, or sustaining a pressure, while only one of the strings is connected with the power, the weight supported may be much greater than the power.

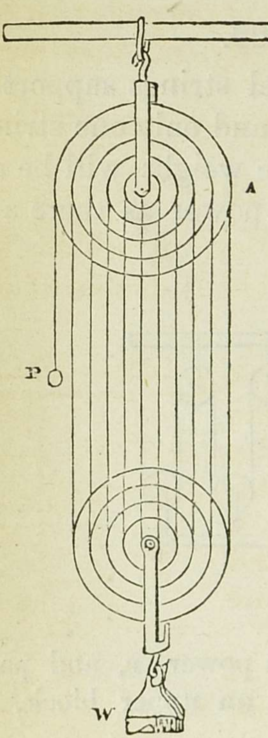
In all such cases it will be observed, that the part of the weight, not sustained by the power, is supported upon the points on which the pulleys themselves rest, or to which the remaining part of the strings are attached.

### Proposition 22.

When any number of parallel strings support a weight, by means of pulleys, and only one string is attached to the power, the weight will be as many times greater than the power as there are strings at the lower block.




Let a string be attached to a power, P, and pass several times over pulleys in an upper block, A, and under pulleys in a lower block, B, and let the other end of the string be made fast. Then, if a weight, W, be attached to the lower block, and all the strings are parallel, every part of the string sustains a pressure equal to the weight of P; and neglecting the weights of the pulleys themselves, the weight W must be as many times greater than P as there are strings by which it is supported. So that if there be five pulleys, and therefore ten strings, at the lower block, W will be ten times as great as P.




If all the upper pulleys revolve upon a common axis, and all the lower pulleys upon another axis, and the radii of the lower pulleys are to one another as the odd numbers 1, 3, 5, &c.; and those of the upper pulleys as the even numbers 2, 4, 6, &c., a very convenient block is formed.

For, if the lower block be moved towards the upper one, through any space, as, for instance, an inch, one inch of the string will pass round the circumference of the smallest pulley at the lower block.

And *each* of the strings

which pass under that pulley, being shortened *one* inch, there will be *two* inches of the string passing over the smallest pulley of the *upper* block.

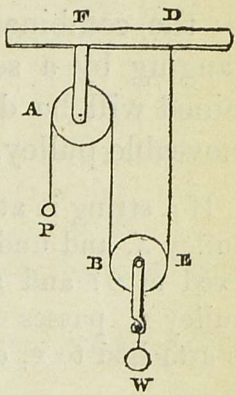
In like manner, it will be seen that the length of string passing round the other *lower* pulleys, will be successively three inches, five inches, seven inches, and nine inches: and the length of string passing over the other upper pulleys will be four inches, six inches, eight inches, and ten inches respectively.

And the size of the pulleys having been made in the same proportion, the whole of the pulleys at the upper and lower block respectively, will turn round together, at the same rate.

This kind of block acts with very little friction.

## Proposition 23.

In the single moveable pulley, in which the strings are parallel, the weight is double of the power.




Suppose a string P A B E D is attached to a weight P, passes over a fixed pulley A, and under a moveable pulley B, and is fastened at D: and suppose also a weight, W, is suspended to B, and that the strings P A, A B, E D, are all parallel.

Then, neglecting the weight of the pulleys themselves, it is clear that every part of the string P A B E D sustains the same degree of tension, which is exactly equal to the weight P, since the tension of the string *at* P just sustains that weight.

And since *each* of the strings, A B, E D, sustains a force equal to the weight of P, the two *together* will sustain twice that force. But the *upward* force of these two strings is exactly balanced by the *downward* force of W. Hence W must be exactly double of P: or a weight of *one* ounce may be made to support a weight of *two* ounces by means of a single moveable pulley.

It will be seen that the point F sustains a pressure equal to that of the two strings A P, A B, or equal to twice the weight of P, and the point D sustains a pressure equal to the weight of P, the weights of the pulleys being neglected.

## Proposition 24.

In the combination of moveable pulleys, each hanging by a separate string, the weight sustained will be doubled, by the addition of each moveable pulley.

If a string is attached to P, passes over the fixed pulley A, and under the moveable pulley B, and is fixed at D: and another string is attached to the pulley B, passes under the moveable pulley C, and is attached to E, all the strings being parallel; and a weight, W, is hung to C; the weights of the pulleys being neglected:

Then each of the strings which support B, sustains a pressure equal to the weight of P; and therefore the string attached to B sustains a pressure equal to twice P: and the part of the string attached at E must sustain the same pressure: and the two together must sustain a pressure equal to four times P. And the pressure on those two strings must exactly equal the weight of W. (*See fig.* 1.)

*Fig.* 1.

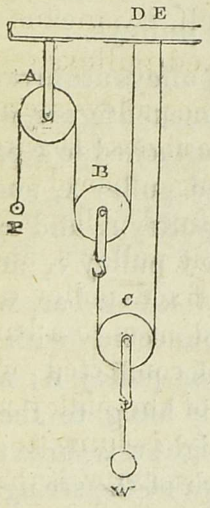
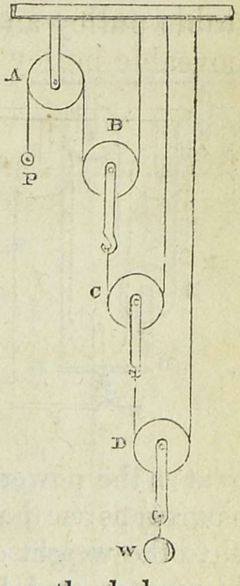



*Fig.* 2.

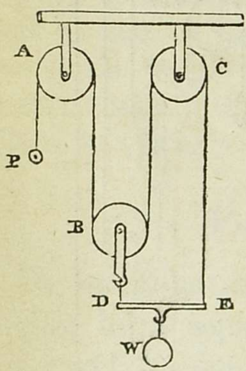



Or one ounce at P will exactly balance four ounces at W.

In like manner, if there be continually added other moveable pulleys, and all the strings are parallel, the weight sustained at W (*fig.* 2, p. 77) will be doubled by the addition of each moveable pulley; so that if there be *one* moveable pulley, W is equal to *twice* P: if there are *two* moveable pulleys, W is equal to *four* times P: if there be *three* moveable pulleys, W is equal to *eight* times P; and so on continually.

### Proposition 25.

A WEIGHT may be made to support another three times as great as itself, by means of one moveable pulley and two fixed pulleys.



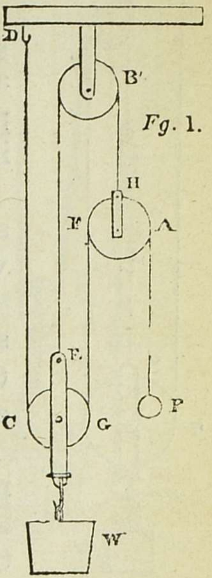

If three pulleys are arranged as in the annexed figure, where the string attached to P passes over a fixed pulley A, under a moveable pulley B, and over a second fixed pulley C, and is fastened at E to a bar, which is also connected with the centre of the pulley B; and a weight W is hung to the bar D E; the weight W is three times as great as the power P. For each of the strings A B, B C, C E, supports the same tension, namely, a force equal to the weight of P. And the tension of them all is counteracted by the weight of W; the weight of the pulley B, and of the bar D E, being neglected, or considered as part of the weight supported.

Since the bar D E acts as a lever, having a force at D double of the force at E, the weight W must be suspended at a point which is twice as far distant from E as it is from D, in order that the bar may rest in a horizontal position.

There are many other combinations of pulleys used: but the explanation of all depends upon the same general principle: and the results may easily be verified by experiment, allowance being made for the weights of the pulleys themselves.

## Proposition 26.

A weight may be made to support another weight four times as great as itself, or five times as great as itself, by means of two cords and two moveable pulleys, called Spanish Bartons.



*Fg.* 1.

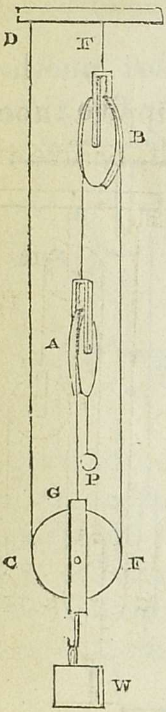
A string attached to D, (*fig.* 1,) passes under the moveable pulley, C, over the moveable pulley, A, and is attached to the power, P. Another string, E B A, is attached to the axis of the pulley C, passes over the fixed pulley B, and is attached to the axis of the pulley A.

Here, each of the strings, P A, F G, C D, supports a pressure equal to the weight P: and each of the strings, H B, B E, supports a pressure equal to twice the weight of P.

And the weight W counteracts the pressures upon the three strings, D C, B E, G F, and must, therefore, be four times as great as P.

In this system, the weights of the pulleys will have no effect in disturbing the equilibrium, if the weight of the pulley C G be equal to the weight of the pulley A: for they will then exactly balance each other over the pulley B.

*Fig.* 2.




In *fig.* 2, a string is attached at D, passes under the moveable pulley C, and over the fixed pulley B, and is then attached to the axis of the moveable pulley A.

Another string is attached to P, passes over the moveable pulley A, and is fixed at G to the axis of the pulley C, to which also the weight W is hung.

Thus each of the strings, P A, A G, sustains a pressure equal to the weight of P.

And *each* of the strings, D C, F B, sustains a pressure equal to that at the *axis* of the pulley A, or a pressure equal to *twice* the weight of P.

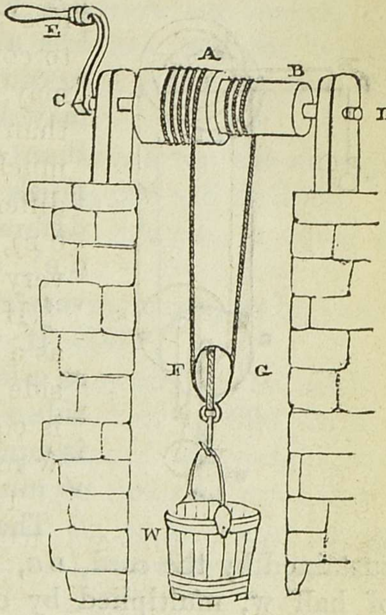
And the weight W counteracts the pressure upon the three strings, D C, F B, A G: and must, therefore, be *five* times as great as P.

The pulleys are supposed to be so arranged that all the strings are parallel.

In this system, the weights of the pulleys will have no effect in disturbing the equilibrium, if the pulley A be *half* the weight of the lower pulley, C; for the remaining half of the weight of C will be supported by the tension of the string, C D.

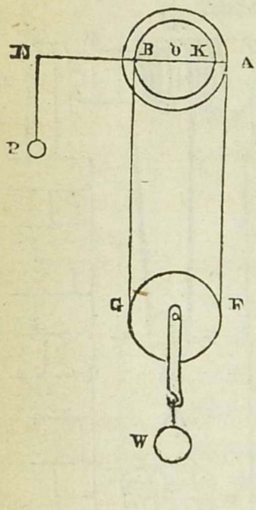
## Proposition 27.

A very small force may be made to sustain a very large one, by means of a cord passing round two cylinders of nearly equal diameters on a common axis.




Suppose two cylinders, A, B, of nearly equal diameters, are connected firmly together, and moveable about the same axis C D, to which a crank, E, is attached. Suppose also a cord is wound round the larger cylinder, A, in one direction, that the part A F passes under the moveable pulley F G, and, after reaching the cylinder B, is wound round it in the direction opposite to that in which it was wound round A. Then a very small force applied at E may be made to balance a very large weight suspended to the pulley F G.

For suppose fig. 2 represents a section of the cylinders, by a plane perpendicular to the axis. Then each of the strings A F, B G, supports the same weight, namely half of the weight W, and of the pulley F G. And if both strings acted perpendicularly at the same distance from the centre of motion O, they would exactly balance each other: and they will *nearly* balance if O A is nearly equal to O B. But A F, being at a somewhat greater distance from O than B G, would preponderate, unless a force were applied at E to counteract it: and since O E may be made much greater than O A, and therefore very much greater than A K, (the difference between O A and O B), the force at E may be very small.




If we consider the system as a lever, the *moment*, on the side O A, (see Lesson V., p. 43,) is equal to the product of half W multiplied by O A.

The *moment* of the force sustained by the cord, B G, is equal to the product of half W, multiplied by O B, or by O K, which is equal to O B. And this moment is opposed to that on the side O A.

Hence, if there were no other force than that of W, there would remain on the side, O A, a *moment* equal to the product of half W, multiplied by the *difference* of O A and O K, or by K A.

And that there may be an equilibrium, there must be an equal moment on the other side, produced by some power, P, applied at E. And in that case, P multiplied by O E must equal half W multiplied by A K, or P is in the same proportion to *half* W that A K is to O E; and therefore P is to W in the same proportion that A K is to twice O E.

Suppose, for instance, O A were six inches, O B five inches, O E twenty-four inches, and the weight of W and the pulley 1200 lbs: and therefore the weight on each string 600 lbs.

If we consider the system as a lever, the moment of A on the side O A is equal to $600 \times 6$ or 3600.

The moment of B, on the side O B, is equal to $600 \times 5$ or 3000.

And the *difference* of these moments, to be counteracted by the force at E, is 600.

Hence the *weight* in lbs. necessary to be applied at E, O E being twenty-four inches, is 600 divided by 24, or 25 lbs. In this instance the weight sustained, 1200 lbs., is forty-eight times as great as the power employed to balance it; as it ought to be, since A K is to twice O E, as 1 to twice 24, or as 1 to 48.

## QUESTIONS.

What is a pulley ?

Distinguish a fixed pulley from a moveable pulley.

How can a pulley be employed to change the direction of a force ?

When the same string passes round any number of pulleys, and the strings at the lower block are all parallel, what is the proportion between the power and the weight ?

What advantage is gained by having all the upper and lower pulleys moveable together ; and what must be the proportion of the size of the pulleys to secure that effect ?

In the single moveable pulley, what weight will a power of one ounce support ?

What is the proportion between the power and the weight, when several moveable pulleys are combined ?

How can two fixed pulleys and one moveable pulley be combined, so that a weight may be made to support another weight three times as great ?

How may two moveable pulleys and one fixed pulley be arranged, with two cords, so that a weight may be made to support another weight four or five times as great as itself ?

How can two cylinders, of nearly equal diameter, and moveable on a common axis, be employed so as to enable a weight to sustain another weight far greater ?

How can we ascertain, in this case, the proportion between the power and the weight sustained ?

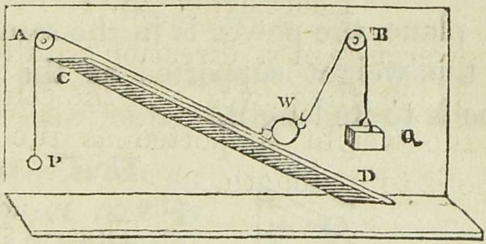
# LESSON X.

## ON THE INCLINED PLANE.

When a body rests on a horizontal plane, the pressure acts in a direction perpendicular to the plane; for if it were not, we should find a body *run about* upon a perfectly smooth horizontal plane, which is contrary to experience.

When a body is partially supported upon a plane inclined to the horizon, the pressure upon the plane is also perpendicular to the surface of the plane.

This may be practically shown as follows.




Let a body, W, be sustained on the inclined plane C D, by a sufficient weight, P, connected with W by a string passing over a pulley A.

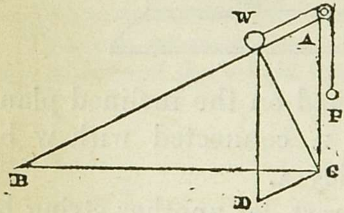
When the whole is at rest, let another string be fastened to W, and pass over a pulley B, in such a way that the part, W B, may be perpendicular to the plane C D: and let small weights be put into a box Q, attached to the string Q B W, until W *just* ceases to press upon C D.

Then, if the plane C D be removed, P and W will be found still to remain in their former positions; the tension of the string W B being exactly equal to the *reaction* of the plane C D, and being perpendicular to that plane; or in the direction of the line W B. Hence the *action* of W upon the plane C D, or its pressure upon it, must be perpendicular to that plane.

The most advantageous way of employing a power to sustain a weight on an inclined plane, is when the power acts in a direction parallel to the surface of the plane.

## Proposition 28.

If the power acts in a direction parallel to the inclined plane, the power is in the same proportion to the weight supported as the height of the plane is to its length.




Thus, suppose a power, P, acts in the direction W A, and sustains a weight W on the inclined plane A B.

Then, if C W is perpendicular to A B, and W D is drawn parallel to A C, or perpendicular to the horizon, and C D parallel to the plane A B, the weight W acts in the direction W D, and the power P in the direction W A. These two forces, together compound a force which is equal to the *pressure* of W on the plane, and therefore is perpendicular to A B, or is in the direction W C.

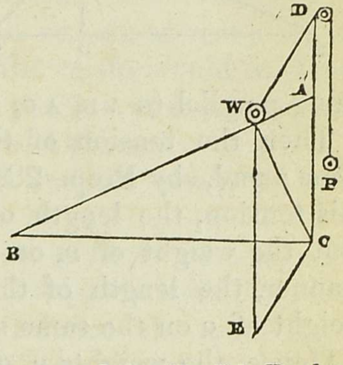
And W C is the *diagonal* of a parallelogram, of which W A, W D are the sides.

Therefore, by the principle of the composition of forces (see Lesson III.) the three lines W A, W D, W C, are proportional to the forces in those directions. Hence the power, P, is in the same proportion to the weight, W, as W A is to W D, or as W A is to A C; or as A C is to A B*.

Thus, if the height of a plane is one foot, and its length a hundred feet, a force of one pound will support upon it a hundred pounds.

## Proposition 29.

### To find the proportion of the power to the weight, when the power acts in any direction.




If the power on an inclined plane acts in any other direction, as for instance in the direction W D, the proportion of the power P to the weight W may be found by drawing W C perpendicular to the

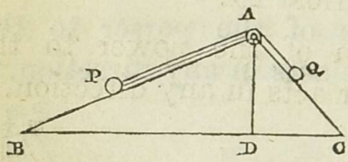
* That W A : A C : : A C : A B is proved in *Euclid*, Book vi. Prop. 8.

plane A B, W E perpendicular to the horizon, and E C parallel to W D.

For the two forces represented by W D, W E will be proportional to the forces P, W, respectively, and will compound a force W C, which represents, on the same scale, the pressure on the plane.

## PROPOSITION 30.

IF there be two inclined planes, of the same altitude, and two weights resting upon the planes sustain each other by means of a string acting parallel to each plane, the weights are proportional to the lengths of the planes on which they rest.



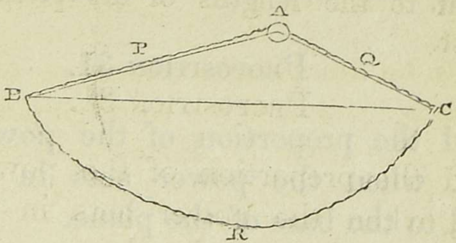

Let P, Q, be two weights, resting upon the planes A B, A C, having the same altitude A D, and the parts of the cord P A, A Q, being parallel to A B, A C, respectively.

Then the tension of the cord is everywhere the same; and, by Prop. 28, if the line A D represents this tension, the length of the plane A B will represent the weight of P, on the same scale. In like manner, the length of the plane A C represents the weight of Q on the same scale.

Hence, the weights P, Q, are to one another in the same proportion as A B, A C.

For instance, if A B is twelve inches, A C six inches, and P ten pounds; Q will be five pounds.

This property of the inclined plane was first discovered by Simon Stevin, a Flemish mathematician of Bruges: and his method of proof is so simple and ingenious that it may be introduced here.




Suppose two inclined planes, A B, A C, have a common altitude, the base, B C, being horizontal; and that a uniform chain, A P B R C Q, is hung upon it, the parts P, Q, resting upon the planes A B, A C; and the part B R C hanging freely.

Thus the whole chain will remain at rest; for if any motion *did* take place, the chain would still be situated in the same relative position as it was before, all parts being supposed to be uniform.

Also the part B R C will support itself, if the points, B, C, are supported.

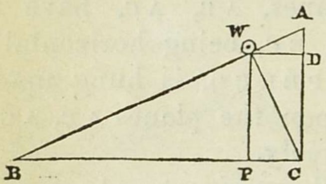
Suppose, therefore, the part B R C to be removed. Then the part B A C will still support itself; or the part A P B upon A B, will balance the part A Q C upon A C.

And the weight of the part P will be to the weight of the part Q in the same proportion as the lengths of A B, A C, respectively.

If therefore we substitute for the *chains* P, Q, single weights P, Q, as in the *fig.* in p. 85, the equilibrium will still continue; the weights being proportional to the lengths of the planes on which they rest.

## Proposition 31.

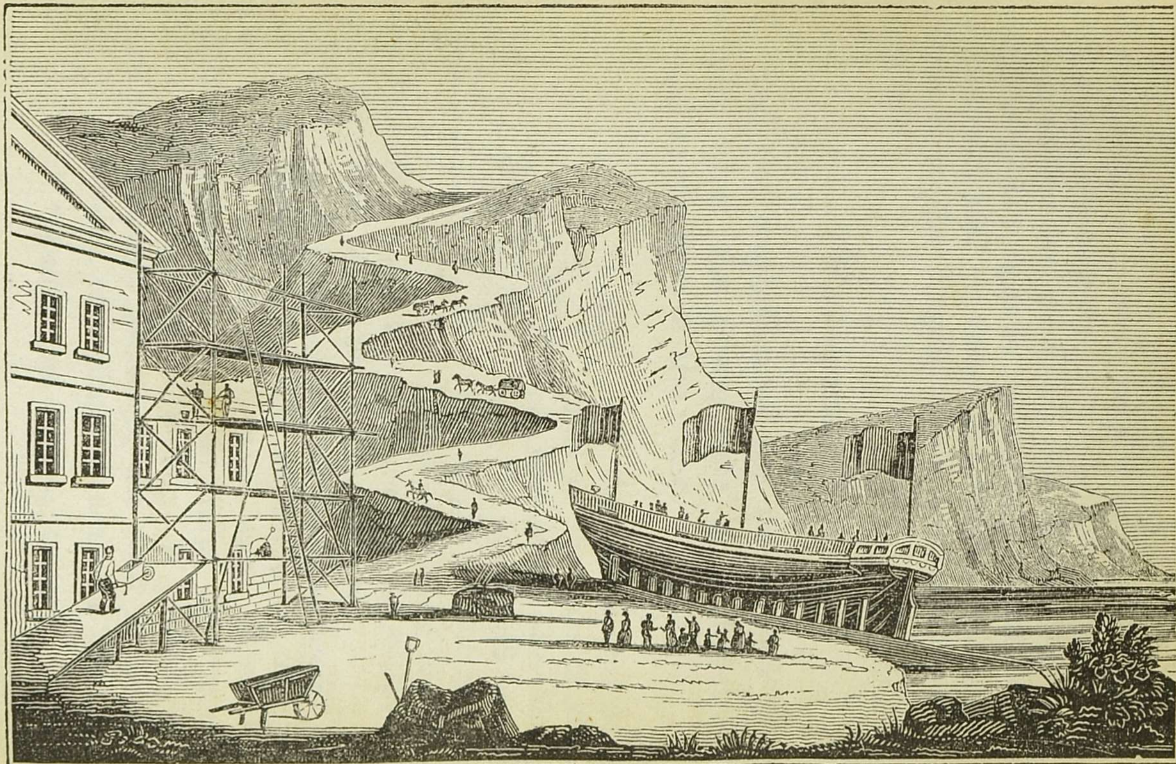
To find the proportion of the power and the weight, when the power acts in a direction parallel to the base of the plane.




If the power acts parallel to the *base* of the plane, or in the direction W D, the power will be to the weight in the same proportion as W D to W F, (see Prop. 29,) or as W D to D C, or as A C to B C*, or as the *height* of the plane is to its *base*.

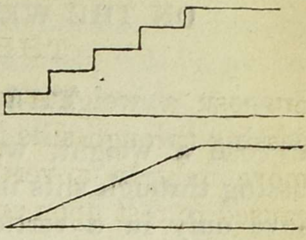
The inclined plane is one of the most useful of the mechanical powers. By means of it we are able to raise weights with great ease. Thus, in building, a wheelbarrow can be driven up a plank to the higher stage of scaffolding. If a hill is too steep to be readily ascended, the road is broken into a succession of inclined planes, and made to

* *Euclid*, Book vi., Prop. 8.



The Inclined Plane.

advance in a zig-zag form. A flight of stairs is little else than an inclined plane, broken into successive steps for the convenience of affording a firmer footing than could be obtained if the plane were not so divided.



An inclined plane is also often used to produce a gradual *descent*, as in launching a ship. Upon a rail-road, an inclined plane is frequently employed, the loaded carriages which run down, being attached to a chain or rope which passes round a wheel, and is again attached to the empty carriages which are to be drawn up.

### QUESTIONS.

When a body rests upon an inclined plane, in what direction does the pressure on the surface of the plane take place?

If the power acts parallel to the plane, what is the proportion between the power and the weight?

How is that proportion found, when the power acts in any other direction?

If there be two inclined planes, of equal height, and two weights are sustained on each by a string, acting, in each case, parallel to the planes, what is the proportion between the weights?

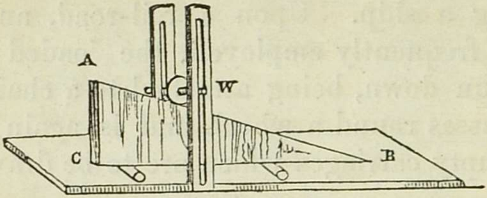
Give some examples of the application of the inclined plane.

# LESSON XI.

## ON THE WEDGE AND SCREW.

### THE WEDGE.

Suppose a weight, W, to be so confined, by pins passing through slits in two upright bars, that it can move only in a vertical direction, and that it is caused to rest upon an inclined plane, A B.



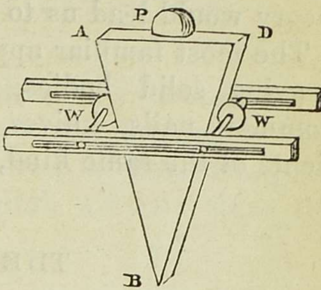

Then, if a force were applied to W, in a direction parallel to B C, and having the same proportion to W that A C has to B C, the weight W would be just supported, and have no tendency to ascend or descend: by Prop. 31.

But if the plane, A B C, itself were perfectly free to move, being placed on rollers to take off the friction, such a pressure would cause the *plane* to move in the direction B C.

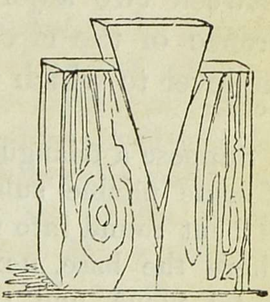
But if we suppose an equal force, applied perpendicularly to the back of the inclined plane, A C, the whole system will be kept at rest; and if the force first applied to W is removed, it will be supplied by the pressure upon the pins passing through the upright bars.

Now if the pressure upon the back of this inclined plane be increased, the plane itself will be pushed forward in the direction C B, and the weight W *raised.*

When a plane is thus employed it is called a *wedge.*




A more common form of the wedge is A B D, having two equal sides, A B, B D. A force, P, applied at the back of the wedge is employed to separate two bodies, W, W, which are pressed against the sides of the wedge by a force of any kind. If the direction, in which the motion of W, W must take place is, parallel to A D, this case is exactly like the preceding; the force, P, necessary to balance the two forces, W, W, being double of that necessary to balance one of them.



But when a wedge is employed to split timber, or any other body of that kind, the direction in which the motion of the parts separated would take place must be ascertained, and the proportion between the power applied and the pressure produced can then be determined.

The method, however, of finding the proportion of these forces, even in the simplest case, is rather too difficult to be here introduced; and it is of little practical use, for the wedge generally acts not by *pressure*, but by *impact*, or a violent blow, by which means the effects produced are far greater than the theory would lead us to expect.

The most familiar application of the wedge is in cleaving solid bodies, such as wood or stones. Common nails, knives, chisels, and other instruments of the same kind, also acts as wedges.

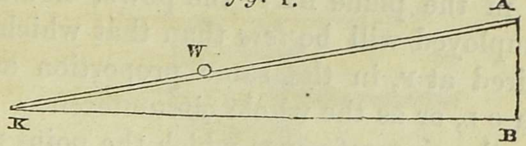
## THE SCREW.

### Proposition 32.

If a power, P, acting perpendicularly at the end of a lever, sustains a weight, W, upon the thread of a screw, the forces will be balanced when P is to W in the same proportion as the distance between two adjoining threads to the circumference of the circle described by that point of the lever to which P is attached.

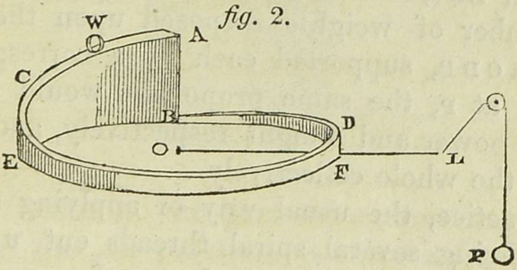
Suppose a triangular plane, ABK, *fig.* 1, formed of some pliable substance, such as card paper, to be bent round into the form represented in *fig.* 2, where the base, BEFB, is *circular*, and in a horizontal plane, and the upper surface of the plane, A K, in *fig.* 1, takes the form of a *spiral* surface, A C D B, in *fig.* 2.

*fig.* 1.




Then, the same force which would be required to keep a body, W, at rest upon the inclined plane, A K, would be required to keep a similar body at rest upon a part of the spiral, A C D B; since the

*fig.* 2.




inclination of every part of the spiral to the horizon is the same as the inclination of the plane, A K.

And if the force necessary to keep W at rest on the plane were applied in a direction parallel to the base of the plane, such a force bears the same proportion to the weight W that A B, the height of the plane, bears to the base, B K (Prop. 31) or that A B bears to the circumference, B E F B, in *fig.* 2.

H

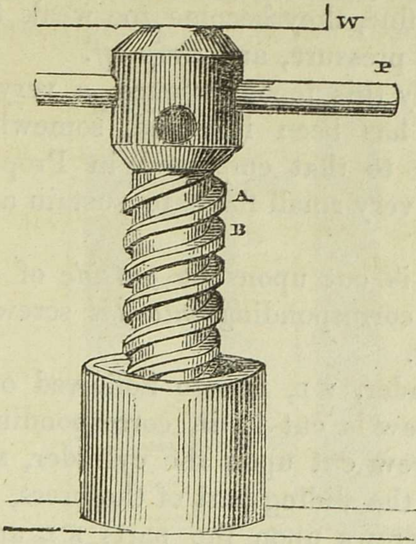
And if, instead of employing a power at F, a point in the circumference, F E B, we employ a power, P, acting perpendicularly at the extremity of a bar, O L, and in the plane B E F, the power necessary to be so employed will be *less* than that which would be required at F, in the same proportion as O F is less than O L, or as the whole circumference, B F E, is less than the circumference which the point L would describe in a whole revolution.

Hence the power P is to the weight W in the same proportion as A B is to the circumference, which the point L would perform in a whole revolution.

And if instead of having *one* weight W, we had any number of weights disposed upon the spiral thread A C D B, supported each by a corresponding pressure at P, the same proportion would be true for each power and weight respectively, and therefore for the whole collectively.

In practice, the usual way of applying a screw is by having several spiral threads cut upon the external surface of a cylinder, and a number of threads, of the same size, cut in the internal surface of another cylinder. A power is then applied at the end of a bar, as at P, and sustains a weight W, or a pressure of any kind.

Suppose, for instance, the distance between two threads of the screw A B, is one inch, and the circumference which the point P would describe in a whole revolution would be nine feet, or 108 inches, then a pressure of *one* pound, at P, would sustain 108 lbs. at W.




The friction of the parts of a screw is, however, so great that, in practice, its effect is far less than the theory would lead us to expect.

It will be observed that, in the screw, the weight which can be supported by a given power depends upon the proportion between the circumference which the power describes and the distance between two contiguous threads of the screw. Hence, by lengthening the lever by which the power acts, or by cutting the threads sufficiently fine, the effect of the screw would appear capable of being increased to any extent. It is, however, often practically inconvenient to increase the length of the lever employed; and, if the threads of the screw are cut too fine, they become too weak to support the required pressure, and *strip off*.

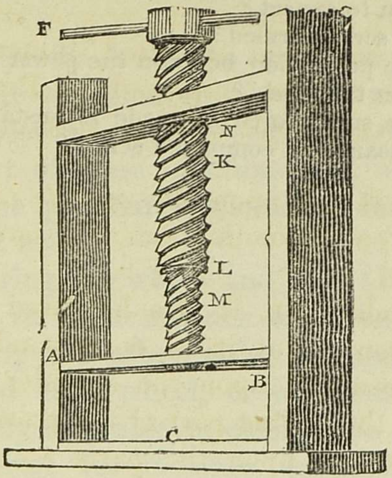
To remedy this inconvenience, a very ingenious contrivance has been invented, somewhat similar in principle to that employed in Proposition 27, to enable a very small force to sustain a very large one.

A screw is cut upon the *outside* of a cylinder, K L, and a corresponding *internal* screw is cut in the nut at N.

The cylinder, K L, is also hollowed out, and an *internal* screw is cut in it, corresponding with an *external* screw cut upon the cylinder, M, which is attached to the sliding part of the press, A B.

If the screws upon the parts K L and L M had precisely the *same* distance between two contiguous threads, and the upper screw were turned round by a power, P, applied to the lever, the sliding-board, A B, would neither ascend nor descend; for the part K L would be *depressed* at the nut N, precisely as much as the part M would be *raised* in the internal screw at L.

But if the distance between the threads in the part M is somewhat less than in the part K L, the board A B will be depressed, in each revolution of P, through a space equal to the difference of the distance between two threads in K L and two threads in L M: and will be under the same circumstances as if a simple screw were used, the threads of which were at a distance equal to the difference of the distances between the threads of the screws.




In this machine, then, there is an equilibrium when the power applied at P is to the pressure on A B, as the difference of the distances between the threads of the screws is to the circumference described by P. And this difference may be made as small as we please, without weakening the machine by diminishing the size of the threads cut upon the screws.

QUESTIONS.

What is a wedge ?

How is it usually employed ?

Is the effect greater or less than that which the theory would lead you to expect ?

How is the screw formed ?

What is the proportion between the power and the weight in equilibrio on the screw ?

How may a small force be made to sustain a very great pressure by means of a compound screw ?

## LESSON XII.

### PECULIAR MECHANICAL CONTRIVANCES.

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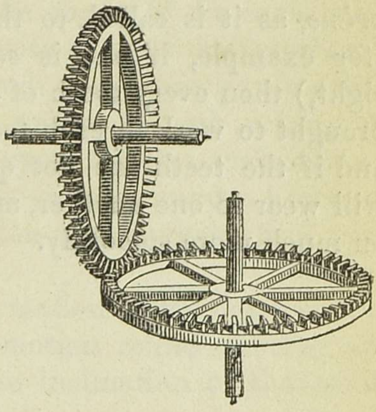
We have already seen the simplest mechanical powers, by the combination of which machines are composed. Before we proceed to mention some of the easiest of these combinations, we may take notice of some peculiar mechanical contrivances of very frequent use.

In considering the wheel and axle (Lesson viii.), we observed in what manner one wheel could be made to act upon another, either by means of bands passing round each wheel, or by means of teeth. It is frequently desirable to change the direction of circular motion; for instance, to produce motion about a vertical axis, by means of a motion about a horizontal axis. This is easily effected by having the teeth cut upon the *circumference* of one wheel, and upon the upper or under surface of the other. And the second wheel may be made to re-



volve faster or slower than the first, according to the number of teeth in each wheel.

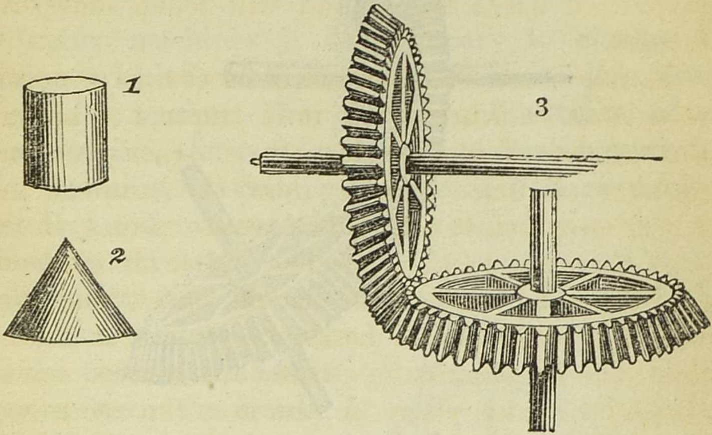
Thus, if the first wheel have forty-eight teeth, and the second only twelve, the first will make one revolution while the second will make four revolutions.

It may here be observed, however, as a general rule, that in wheel-work it is better not to have the number of teeth in one wheel exactly a certain number of times greater than the number in a wheel on which it acts. The reason is that, in that case, the same tooth of the first wheel always comes into the same tooth of the second wheel, after every complete revolution of both wheels: and if there is any little irregularity in cutting the wheels, such as is sure to take place in practice, the teeth wear unequally, and the machine goes irregularly, and soon gets out of order.

But if the number of the teeth in one wheel is *prime*, as it is called, to the number in the other, (for example, if one is seventeen and the other eight,) then every tooth of the first wheel is in time brought to work in each tooth of the second wheel, and if the teeth are not quite accurately cut, they will wear to one another, and the movement will go on much more smoothly.

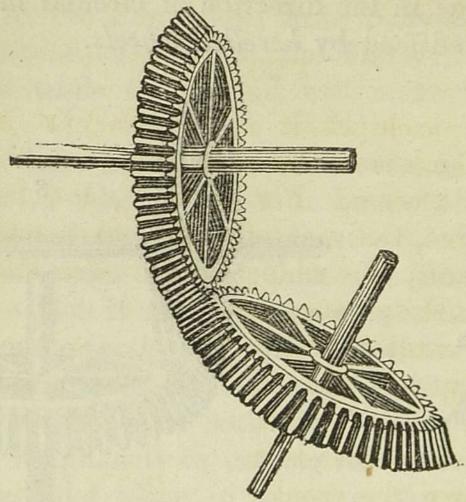
## BEVELLED WHEELS.

THE change in the direction of circular motion may also be produced by *bevelled wheels.*



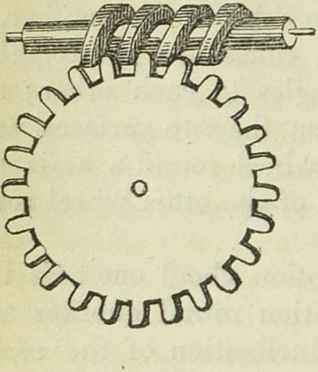

If the circumference of each wheel, instead of being a portion of a *cylinder* (*fig.* 1), be a portion of a *cone* (*fig.* 2), and the *angles* of the cone, or the inclination of two opposite sides to one another, be such that, when two such cones are put together, their *axes* are at right angles to each other; and teeth are properly cut upon the two surfaces, as in *fig.* 3, the motion of one wheel round a horizontal axis will produce a motion of the other wheel round a vertical axis.

By the same means, motion about one axis may be caused to produce motion round another axis, inclined to the first; the inclination of the conical surfaces, on which the teeth are cut being properly adjusted.



THE ENDLESS SCREW.

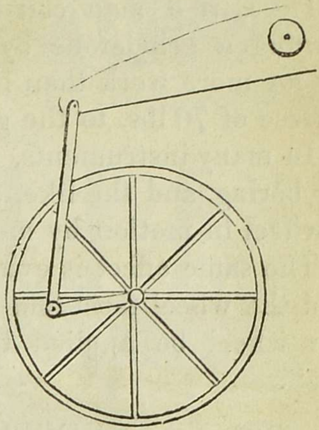
The endless screw is a combination of the screw



with the toothed wheel; and is very convenient for changing the direction of circular motion. The thread of the screw is so cut as to act upon the teeth of the wheel; and the cylinder, on which the screw is cut, being set in motion about a horizontal axis, for instance, produces a motion of the wheel about its own axis, which may be in any direction required.

## CRANKS.

In many machines it is necessary to change a motion, which is nearly or exactly in one line, into a circular motion. For example, the beam of a steam-engine, the end of which moves backwards and forwards, describing an arc of a large circle, may be used to cause a wheel to revolve. This may easily be effected by means of a crank. As the beam descends it presses down the crank; and lifts it again in ascending. In this case, there will be two opposite points, in which the direction of the force, exerted by the bar connecting the beam and the crank, passes through the axis of the wheel, and, therefore, has no tendency to create motion round that axis; but if the machine is in motion, it will be carried past those points by the continuance of that motion. It is plain that even if the force applied at the end of the beam is uniform, that applied to the wheel will not be so. For it acts sometimes *perpendicularly*



at the end of a lever, the length of which is the distance between the axis of the wheel and the point of application of the force to the crank; and sometimes *obliquely* on the same lever.

It is computed that, if the force of the beam acts directly up and down, the effect of a pressure of eleven pounds on the beam produces the same average effect as if a force of seven pounds acted perpendicularly on the crank during the whole of each revolution.

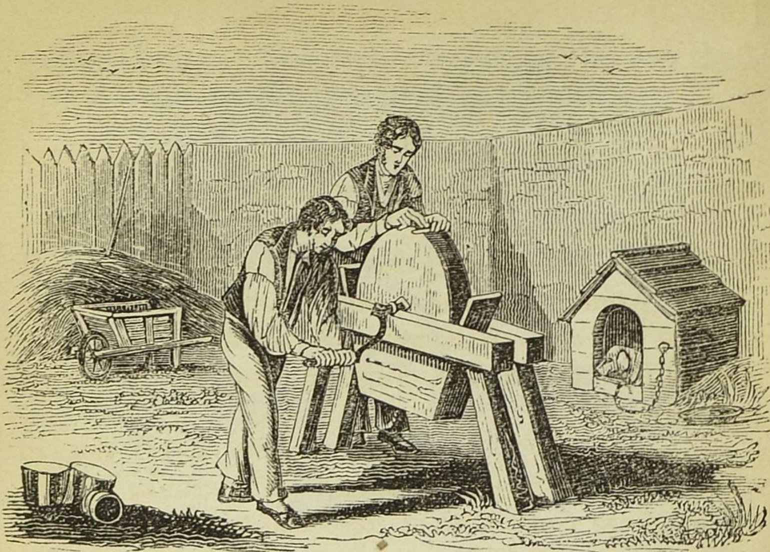
Thus, if a man can exert a force of 110 lbs., in turning a grindstone by means of a crank, he will do no more work than if he could constantly apply a force of 70 lbs. to the greatest advantage.

In many instruments, such as lathes, contrivances for boring, and the like, a treadle is employed to set a wheel in motion by means of a crank.

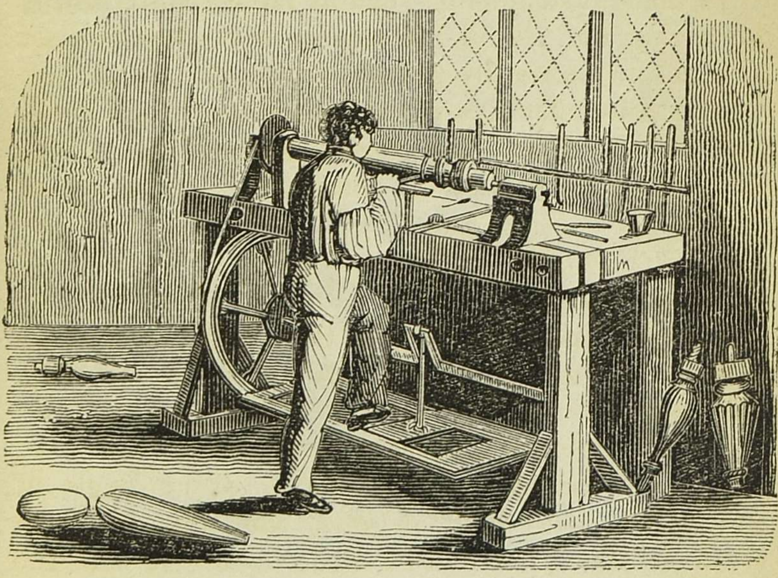
The same effect is evidently produced if the crank and the wheel form one piece; or the force to turn the wheel be applied to some point of the wheel itself.

## UNIVERSAL JOINT.

A CONTRIVANCE, known by the name of Hook's universal joint, is also employed to change the direction of rotatory motion. If two shafts be connected in the manner represented in *fig.* 1, p. 111, and inclined to one another at an angle of not more than forty degrees, and one of them revolves, the other will be made to revolve with the



Cranks.



same velocity. The cross connecting the two shafts moves freely on the pins at its extremities.

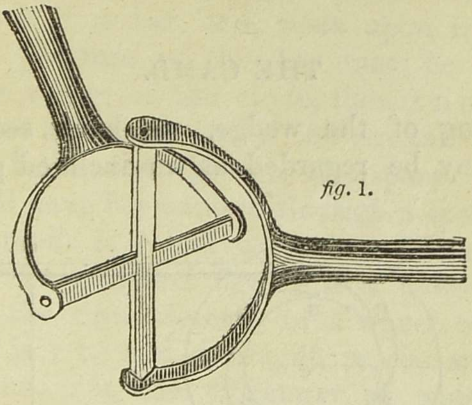


*fig.* 1.

This joint is extensively used in the machinery of cotton-mills.

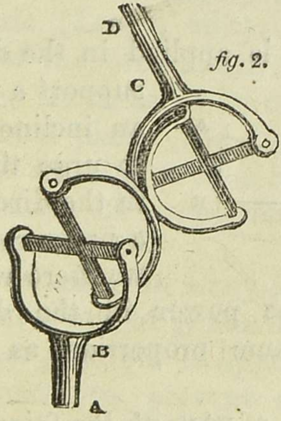
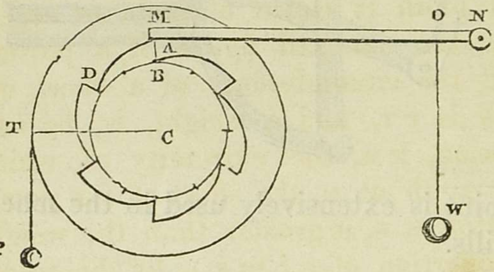



*fig.* 2.

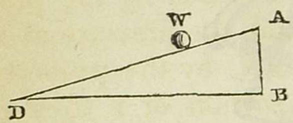
By combining two of these joints, as in *fig.* 2, motion may be communicated from one axis, A B, to another, C D, inclined to the first at any angle not greater than a right angle.

## THE CAMB.

In treating of the wedge, we have seen that a wedge may be regarded as an inclined plane, and



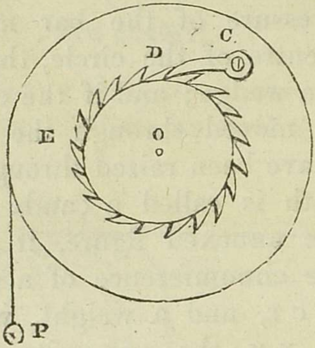

that if a power is applied in the direction B D, to support a weight, W, upon an inclined plane, A D, or to urge the wedge, A B D, in the direction B D, under a pressure represented by W, there will be an equilibrium when the power in the direction B D is to W in the same proportion as A B is to B D. Prop. 30.




Now suppose a part of the circumference of a wheel, C B, to be taken equal to B D, and a tooth, A B D, to be fixed upon it, of such a form that its greatest distance from the circumference of the wheel may be A B, and the surface, A D, be so formed, that if a bar, M N, rests upon it, at any point, the pressure of the bar may be directed towards the centre of the circle, then A D B may be considered as a wedge; and if the circumference of the wheel be moved through the space D B, the point M will have been raised through a space B A.

Such a tooth is called a camb, or an eccentric piece. In the annexed figure, if a power, P, be applied at the circumference of a wheel, of which the radius is C T, and a weight, W, be supported upon a beam, N M, the extremity of which rests upon the camb at M, the effect of the power, P, to cause motion at B, is greater than the weight of P, in the proportion of C T to C B, by the principle of the wheel and axle (Lesson viii.): the force at B in the direction of the circumference, D B, is to the pressure supported at M, as A B to D B, by the property of the inclined plane: and the pressure at M is to the weight W as O N is to M N, by the property of the lever. Hence the proportion of P to W is known.

The camb can be used, in many instances, when it is required to lift a weight gradually, and then let it fall. The same principle is applied in a *rachet* wheel, a contrivance to prevent a wheel from turning, except in one direction. A *catch*, C, plays into the teeth of the wheel, O D, and permits the wheel to revolve freely in the direction D E, while it immediately prevents any motion which would take place in the opposite direction.




QUESTIONS.

How may toothed wheels be formed, so as to change the direction of a circular motion from one axis to another at right angles to it?

What is the construction and use of bevelled wheels?

Describe the endless screw.

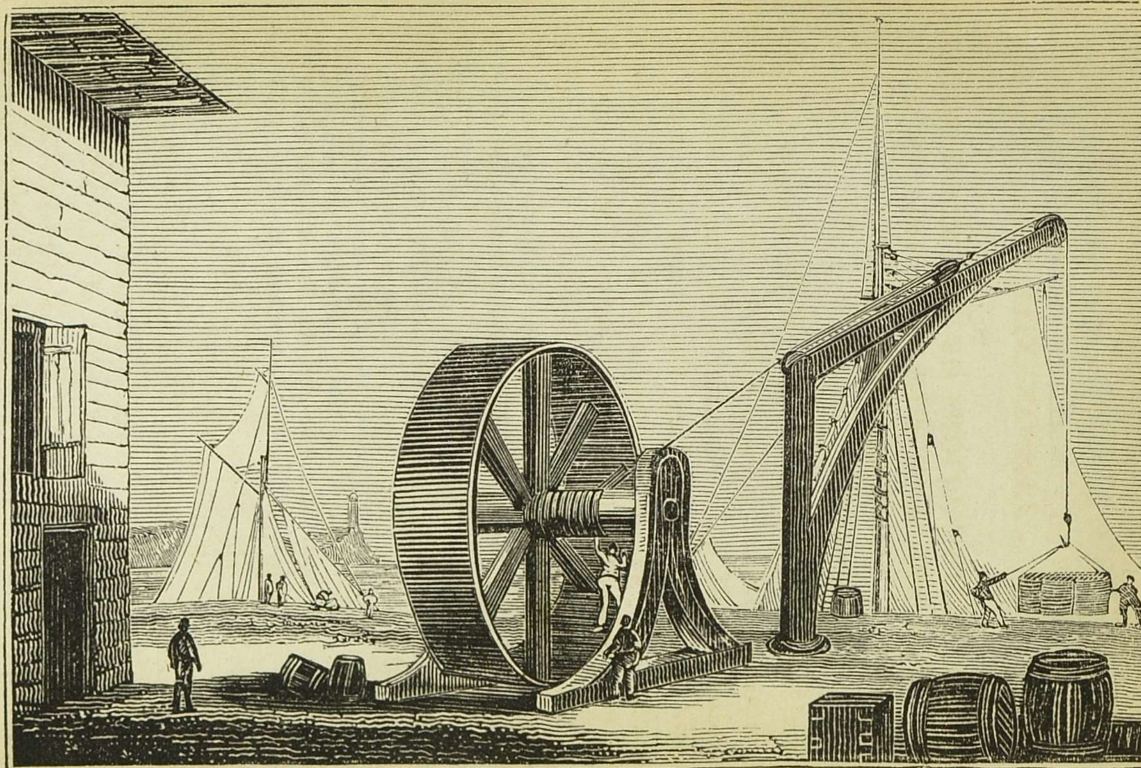
How is a crank applied?

How much power is lost by using a crank?

Describe the universal joint.

What is the use of a camb, or eccentric piece?

What is the application of a rachet wheel?



Combination of Mechanical Powers.

## LESSON XIII.

### ON THE COMBINATIONS OF THE MECHANICAL POWERS, AND THE PRINCIPLE OF VIRTUAL VELOCITIES.

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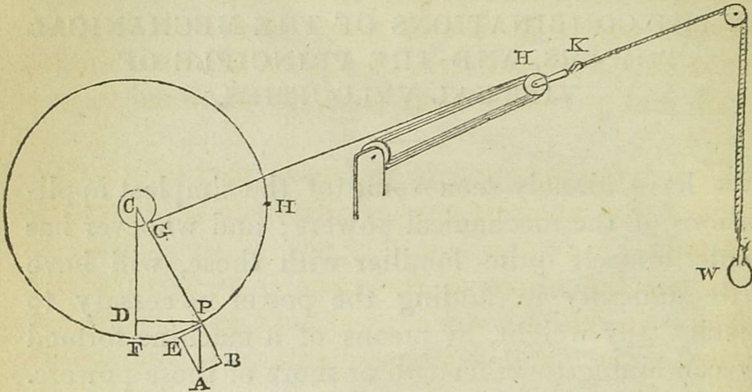
We have already seen some of the simplest applications of the mechanical powers; and whoever has made himself quite familiar with those, will have little difficulty in finding the power necessary to sustain any weight, by means of a machine formed by combining together two or more of those powers.

For instance, supose a wheel of twelve feet radius is constructed in such a manner as to be moved by the weight of men treading upon bars attached to its inner surface; that the axle of the wheel has a radius of six inches; and that the rope wound round the axle is attached to a system of pulleys, in which the same cord goes round six pulleys at the block nearest to the weight to be sustained: and that it is required to know what weight will be supported by the force of each man employed, the weight of a man being 140lbs.

In order to ascertain what portion of the man's weight is effective towards sustaining the weight, we must know at what *part* of the wheel it is applied. For it is plain, that if the man stood at the lowest point, F, his weight would have no effect whatever in *turning* the wheel: and that the effect of his force will increase as it is applied higher and higher, and would be most effective if it could be




applied at the point H, in the same horizontal lever as the centre of the wheel, C.

Suppose, however, that the weight of the man is applied at a point, P, such that if PD be drawn perpendicular to FC, the vertical line through C, PD is equal to half of PC*.

Through P draw PA vertical; and PE, AB, perpendicular to CPB.

Then, if PA be taken to represent the whole force of the man, or 140 lbs., we may resolve (see Lesson iii., Prop. 4, p. 14,) the force PA into two, PB, PE; of which PB acts *from* C, and PE acts perpendicularly to CP.

* This will be the case when the arc, FP, is 30°, or one-third of the quadrant, FH.

Also, the angle C P E being a right angle, as well as the angle D P A, if we take away from each of those angles the angle D P E, the remaining angle C P D will be equal to the remaining angle E P A: and the angles at D and E are both right angles: hence the triangles CPD, PEA, have all their angles equal, and therefore their sides proportional to one another*.

Hence the line P E is the same part of P A that P D is of P C; and this has been assumed to be exactly one-half.

Hence the effect of the man at P to turn the wheel will be the same as if half his weight, or 70 lbs., were applied perpendicularly to the radius of the wheel at P.

Again, the radius of the wheel, 12 *feet*, is 24 times as great as that of the axle, which is 6 *inches*.

Hence a force of 70 lbs. applied at P, causes a tension upon the string G H, 24 times as great, or 1680 lbs.

Again, there are supposed to be six pulleys, and therefore 12 parallel strings at the block H; and only one of those strings passes round the axle, C G.

Hence the force at K, or the weight W, which it supports, is 12 times 1680 lbs., or 20,160 lbs.; or 9 tons.

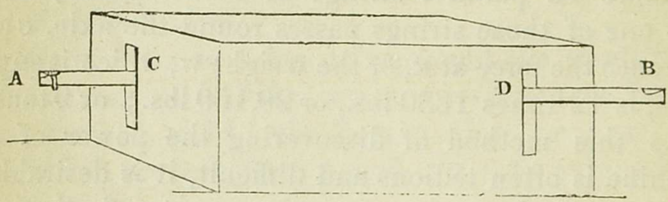
As this method of discovering the power of a machine is often tedious and difficult, it is desirable to avail ourselves of a general property of all machines, which is this:—

* This is proved in *Euclid*, vi. 4.

*If two forces,* P *and* W, *balance each other upon any machine, and the whole be set in motion through a very small space, and the distances, through which* P *and* W *respectively move in their own directions, be observed, the product of* P, *multiplied by the distance through which it is moved, is equal to the product of* W *multiplied by the distance through which it is moved.*

This is sometimes called the principle of *virtual velocities;* and furnishes a very easy method of ascertaining the power of a machine, or the proportion between two forces which would balance one another by means of it: for they will be to each other inversely as the spaces through which the forces move, in their respective directions, or inversely as the *velocities* with which they move.

Thus, suppose we had a box containing machinery, of which we knew nothing but that when the point A of the bar A C is pressed down through one inch, the point B of the bar B D is raised through the hundredth part of an inch.



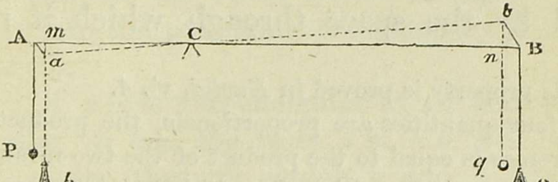

Then, assuming this general property of machinery to be true, we could at once conclude that a weight of *one* lb. hung on A would exactly balance a weight of a *hundred* lbs. hung on B.

In order, however, to establish this general property, it will be necessary to show that it is true in the different mechanical powers separately, and consequently in any combination of them.

## Proposition 33.

If two forces balance one another acting perpendicularly upon the straight lever, and the whole system be set in motion, the product of the force, multiplied by the space through which it moves, is the same on each side of the centre of motion.

Suppose P and Q are two weights, which balance one another upon the straight lever, A C B: and therefore, by Prop. 15, the product of P multiplied by the distance A C is equal to the product of Q multiplied by the distance C B: or P is to Q in the same proportion as C B to A C.




Now, suppose the lever is moved into the position $a$ C $b$; C $a$ being equal to C A, and C $b$ equal to C B.

Let $a\,m$, $b\,n$, be perpendicular to A C, C B. Then $m\,a$ is equal to the space through which P has been caused to *descend*, in the direction of its action; and $b\,n$ is equal to the space through which Q has been *raised*, in the direction of its action. And, since each of the angles at $m$, $n$, is a right angle, and the angle A C $a$ equal to the angle B C $b$, $m\,a$ is in the same proportion to $n\,b$ that C $a$ is to C $b$*; or that C A is to C B; or that Q is to P.

Hence, by the general property of proportional quantities†, the product of P multiplied by $m\,a$ is equal to the product of Q multiplied by $n\,b$.

For example, if P were 12 ounces, and Q 8 ounces, and $m\,a$ 2 inches, $b\,n$ would be 3 inches: and 2 multiplied by 12 is equal to 3 multiplied by 8.

## Proposition 34.

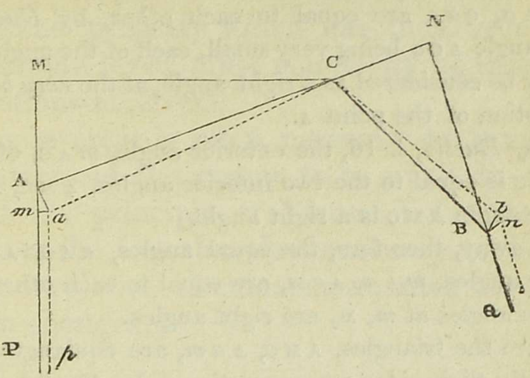
In any lever, if two forces balance one another, and the whole system be set in motion, through a very small space, the product of the force multiplied by the space through which it moves,

* This property is proved in *Euclid*, vi. 4.

† If four quantities are proportionals, the product of the two extremes is equal to the product of the two means: thus 3 : 5 : : 6 : 10; and 3 multiplied by 10 is equal to 5 multiplied by 6. This property is the foundation of the common Rule of Three.

is the same on each side of the centre of motion.

Suppose two forces, P, Q, acting in the directions M A P, N B Q, support each other upon any lever, A C B.




Then, by Prop. 20, the *moment* of the two forces, on each side of the fulcrum, C, is the same: or the product of P multiplied by C M is the same as the product of Q multiplied by C N: and therefore P is in the same proportion to Q that C N is to C M.

Now suppose the whole system is slightly moved about C, into the position represented by the dotted lines, *a* C *b*.

Then, if *a m* is perpendicular to A P and B *n* perpendicular to *b q*, the lines A *m*, *n b*, will be equal to the spaces through which P and Q have been respectively moved in the directions in which they act.

And it may be easily shown*, that A $m$ is in the same proportion to $b$ $n$ that C M is to C N, and therefore in the same proportion that Q is to P.

* The proportion may be proved thus :—

The two lines C A, C $a$, are equal to one another. Hence the angles C A $a$, C $a$ A are equal to each other, by *Euclid*, i. 5. And the angle A C $a$ being very small, each of the angles, C A $a$, C $a$ A, may be considered as a right angle, at the *very beginning* of the motion of the point A.

Now, by *Euclid*, i. 16, the exterior angle, $m$ A C, of the triangle A M C is equal to the two interior angles, A M C, A C M, of which the angle A M C is a right angle.

Taking away, therefore, the equal angles, $a$ A C, A M C, the remaining angles, $m$ A $a$, A C M, are equal to each other.

And the angles at $m$, M, are right angles.

Therefore the triangles, A M C, A $a$ $m$, are equiangular; and consequently their sides are proportional, by *Euclid*, vi. 4.

In like manner, the triangles, C B N, B $b$ $n$, are equiangular.

And the triangles A C $a$, B C $b$ are equiangular. For the angle A C $a$ is equal to the angle B C $b$, and the sides A C, C $a$; B C, C $b$ respectively equal. Hence the triangles are isosceles, and the angles, C A $a$, C $a$ A, C B $b$, C $b$ B, are equal to one another. Hence twice the angle C A $a$, together with the angle A C $a$, is equal to two right angles: and twice the angle C B $b$, together with the angle B C $b$, is also equal to two right angles. And therefore, taking away the equal angles A C $a$, B C $b$, the angle C A $a$ is equal to the angle C B $b$.

Hence the following proportions are true :

$$\text{A}m : \text{A}a :: \text{CM} : \text{AC}.$$
$$\text{A}a : \text{B}b :: \text{AC} : \text{CB}.$$
$$\text{B}b : bn :: \text{CB} : \text{CN}.$$

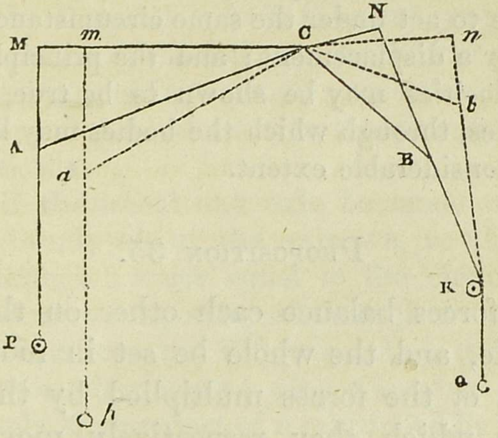
Therefore, $\text{A}m : bn :: \text{CM} : \text{CN}.$
$$:: \text{Q} : \text{P}.$$

Hence, by the nature of proportion, the product of P multiplied by A $m$ is equal to the product of Q multiplied by $b\,n$.

Thus, if P is 8 ounces, and Q 24 ounces, and A $m$ 3 inches, $b\,n$ is found to be 1 inch. And the product of 3 and 8 is equal to the product of 1 and 24.

The same property which has thus been proved of any simple lever, must manifestly be true for all combinations of levers.

In the lever, it is often necessary to suppose the displacement of the system to be small, in order to establish the principle of Virtual Velocities. This arises from the fact, that the forces may not act under the same relative circumstances, after the position has been changed.




For instance, suppose P, acting vertically, and suspended to the lever A C B at A, sustains Q acting over the fixed pulley, K, and attached to the lever at B.

Then, if C M, C N be perpendiculars from C upon P A M, K B N respectively, P must be to Q in the same proportion as C N to C M, by Prop. 20.

If now the lever be moved about C into any other position, as *a* C *b*; the force P, acting at *p*, acts at a perpendicular distance C *m*, *less* than C M, whereas Q acts at a perpendicular distance C *n*, *greater* than C N.

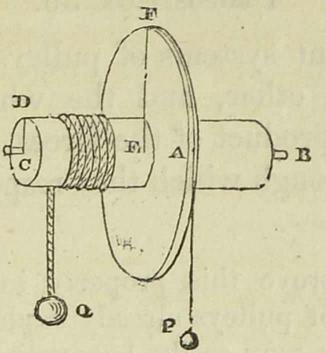
Hence the *moment* of P and Q on each side of C can no longer remain the same; and the equilibrium will be disturbed.

But in many of the mechanical powers, as in the wheel and axle, the pulley, and others, the forces continue to act under the same circumstances before and after a displacement; and the principle of Virtual Velocities may be shown to be true, although the spaces, through which the bodies may be moved, are of considerable extent.

## Proposition 35.

If two forces balance each other on the wheel and axle, and the whole be set in motion, the product of the forces multiplied by the spaces through which they respectively move is the same.

Suppose P suspended to the circumference of the wheel F A supports Q, suspended to the circumference of the axle.




Then, by Prop. 21, P is in the same proportion to Q as the radius of the axle C D, to the radius of the wheel E F: that is, in the same proportion as the *circumference* of the axle, D E, to the *circumference* of the wheel, F A.

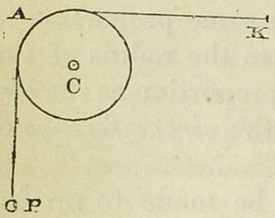
Now, if the wheel and axle be made to revolve once uniformly about the axis, C B, so that P descends through a space equal to the circumference of the *wheel*, Q will be raised through a space equal to the circumference of the axle. And, these spaces having been shown to be *proportional* to Q and P respectively, the product of P multiplied by the space through which it is moved is equal to the product of Q by the space through which it is moved.

The same property is manifestly true of all combinations of such wheels and axles: and may without difficulty be proved in the case of toothed wheels.

## Proposition 36.

In the different systems of pulleys, if two forces balance each other, and the whole be set in motion, the product of the forces, multiplied by the spaces through which they respectively move, is the same.

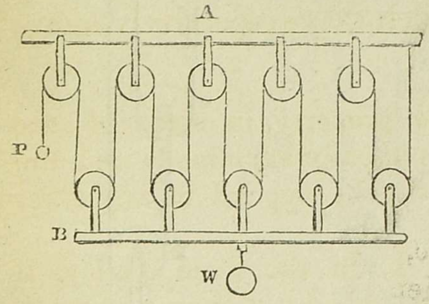
We shall prove this property in some of the combinations of pulleys already explained; and the same principles may easily be applied to any other instances.

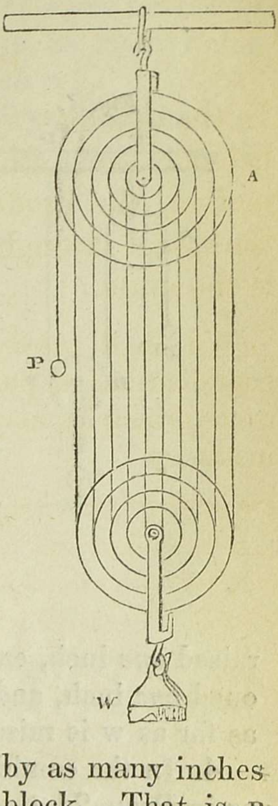



1. In the fixed pulley, if the weight P moves through any distance, in the direction of its action, the point, K, which is acted upon by a force equal to that of P, will move through an *equal* space.

2. If the same string passes round several pulleys, the parts of the strings being parallel, as in Prop. 26, and one string only is attached to P; W is as many times greater than P as there are strings at the lower block.

Now, suppose W is raised through one inch. Then each of the strings at the lower block is shortened one inch. And the string, to which P is attached, will be lengthened by a quantity equal to




the sum of those shortenings; or by as many inches as there are strings at the lower block. That is, P will descend as many times more than W is raised, as W is greater than P.

Hence the product of P, multiplied by the space through which it moves, is equal to the product of W, by the space through which it is moved.

3. In the single moveable pulley, *fig.* 1, in which W is twice as great as P, it is plain that if W is

*Fig.* 2.

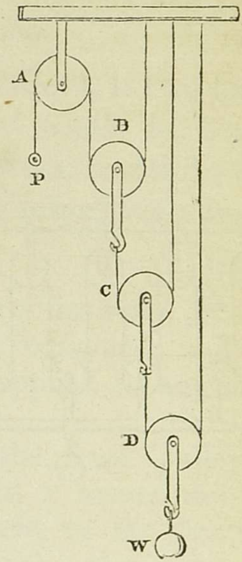
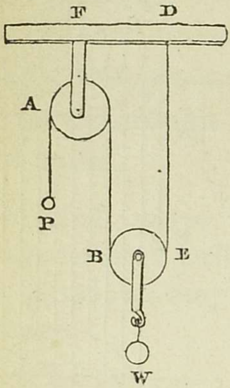



*Fig.* 1.

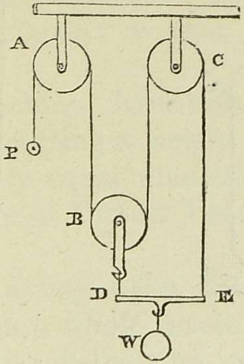



raised *one* inch, each of the strings at B, E is shortened *one* inch, and P descends *two* inches, or twice as far as W is raised.

4. In the combination of moveable pulleys, *fig.* 2, as in Prop. 24, the weight supported by P is doubled by the introduction of each moveable pulley. And if W is raised, as before, through one inch, each string at D will be shortened *one* inch, and the pulley, C, will be raised *two* inches.

Hence *each* string at C will be shortened *two* inches; and the pulley, B, will be raised *four* inches. And thus the distance through which P will descend, will be doubled by the addition of each moveable pulley: and therefore P will move as many times further than W, as W is greater than P.

5. In the combination of pulleys, supposed in Prop. 25, where W is three times as great as P; if

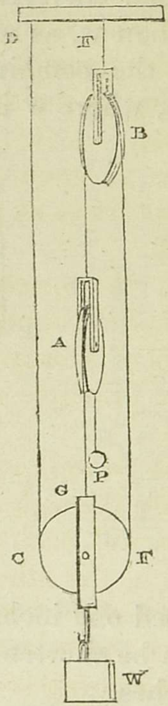



W is raised one inch, the three strings, E C, C B, B A, will each be shortened one inch, and P will descend *three* inches.

6. In the Spanish Bartons, described in Proposition 26, the property may be thus shown:—In *fig.* 1, where W is four times as great as P, suppose W raised one inch. Then each of the strings at C, G, and E, is shortened one inch. By the shortening of the string E B, the centre of the pulley, A F, is depressed *one* inch, and each of the strings, G F,

A P, is shortened one inch; and from *this* cause P descends *two* inches.

Fig. 2.



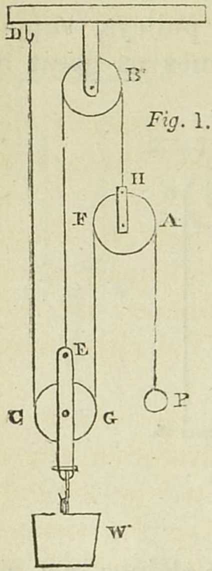




Fig. 1.

Again, by the shortening of the two strings, D C, G F, P descends two inches also: so that on the whole, P descends *four* inches for every inch that W is raised; or P moves four times as fast as W.

In *fig.* 2, where W is *five* times as great as P, suppose, as before, that W is raised one inch. Then each of the strings, D C, F B, is shortened one inch: and from this cause the centre of the pulley, A, descends two inches, and each of the strings, G A, A P, is shortened two inches; and consequently P descends *four* inches.

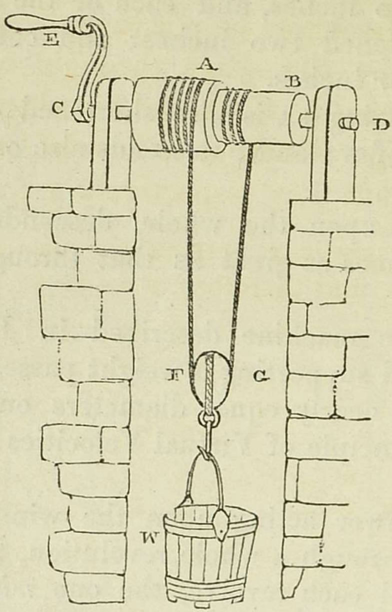
But the string G A is also shortened one inch by the raising of W; and P descends also one inch from this cause.

Hence P, upon the whole, descends through a space *five* times as great as that through which W is raised.

7. In the machine described in Prop. 27, in which a cord supporting a weight passes round two cylinders of nearly equal diameters on a common axis, the principle of Virtual Velocities may be thus proved.

If the power acting upon the winch is moved uniformly through a whole revolution, the cylinders A and B will each revolve, the one *winding up* the string A F, the other *unwinding* the string G. The string A F will therefore be *shortened* by a quantity equal to the circumference of the larger cylinder, while the string G will be *lengthened* by a quantity equal to the circumference of the smaller cylinder. Hence the whole string between the points of suspension will be shortened by a quantity equal to the *difference* of the circumferences of the two cylinders. And, this shortening being equally divided between the two strings, the weight W will be raised through a space equal to *half* that difference.

And, since the circumferences of circles are in the same proportion as their radii, (*Euclid*, vi. 33,)



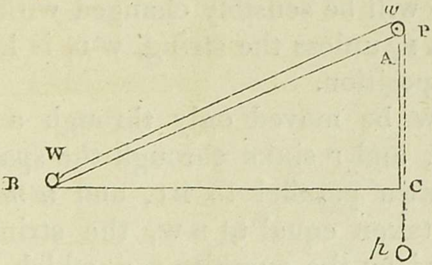

the space through which the *power* is moved is to the space through which the *weight* is moved as the *radius* of the winch, C E, to half the difference of the *radii* of the cylinders; or as *twice* the radius of the winch C E is to the difference of the radii of the cylinders; which is the proportion of the *weight* to the *power* in equilibrio.

Hence the power multiplied by the space through which it moves is equal to the weight multiplied by the space through which it moves.

## Proposition 37.

On the inclined plane, if two forces balance each other, and the whole be put in motion, the power multiplied by the space through which it is moved is equal to the weight multiplied by the space through which it is moved, in the directions in which they respectively act.

1. If the power, P, acts in the direction parallel to the plane, and therefore P is to W in the same




proportion as A C is to A B, by Prop. 28; suppose P and W to be first in the position represented in the figure; and then that W is raised uniformly along the plane from W to *w*, while P descends from P to *p*.

Then, the length of the string between W and P being still the same, W will have been raised, *in the direction of gravity*, or in the direction of its action, through a space equal to A C, above its first level, B C; and P will have moved through a space, P *p*, equal to B A, below its first level.

Hence, P multiplied by the space through which it moves is equal to W multiplied by the space through which it is moved.

Thus, if the height of the plane be one foot, and its length a hundred feet, and therefore W is a hundred times as great as P, P will have to descend a hundred feet while W is raised through a space of one foot.

2. If the power acts in any other direction, as W D, (figure p. 137,) and therefore, by Prop. 29, P is to W as W D is to D C, the *direction* in which the power acts will be sensibly changed with relation to the plane A B, unless the string, W D, is kept parallel to its first position.

But if W be moved only through a very small space, W *w*, and P sinks through the space P *p*, and W *m* is drawn parallel to B C, and *w m* vertically; and D *n* is taken equal to D W, the string W D will be shortened by the quantity *w n*, which is therefore the space through which P descends: and the weight W will be raised in the same time above the level of W, through the space *m w*.

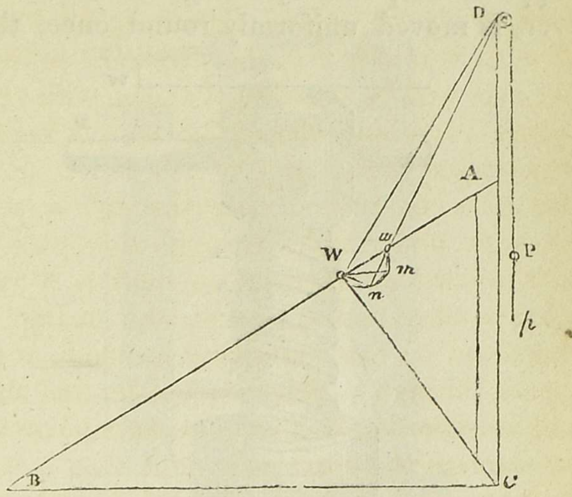
And it can be easily shown* that the triangle, *w n m*, is equiangular to the triangle D W C, and that *w n*, the space through which P is moved, is in the

* W is supposed to be *just* set in motion, through the very small space W *w*. And, since the angle W D *w* is supposed very small, each of the angles, D *n* W, D W *n*, may be taken for a right angle.

Hence a semicircle, described upon W *w*, will pass through same proportion to $w\,m$, the space through which W is moved, as D C is to D W, and therefore as W is to P.

Hence, P multiplied by the space through which it is moved, is equal to W multiplied by the space through which it is moved, in the directions in which they respectively act.

the points $m$ and $n$: and the angle $w\,n\,m$ will be equal to the angle $w$ W $m$ in the same segment, by *Euclid*, iii. 21; and




therefore to the angle A B C, by *Euclid*, i. 29, since B C is parallel to W $m$.

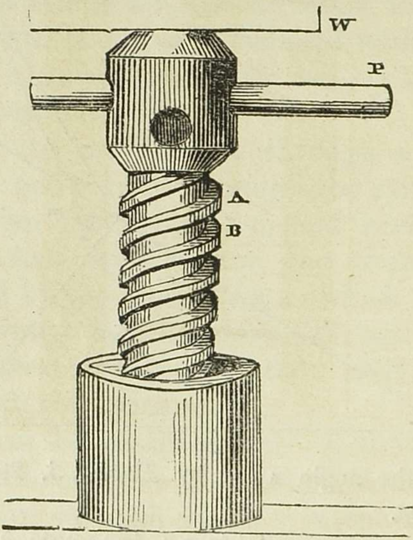
And, since $w\,m$ is parallel to D C, the angle $n\,w\,m$ is equal to the interior angle $w$ D C, which, by the supposition, does not sensibly differ from W D C, at the *commencement* of the motion of W.

Hence the triangles, $w\,n\,m$, D W C, are equiangular; and $w\,n : w\,m ::$ D C : W D :: W : P.

## Proposition 38.

In the screw, if two weights support each other, and the whole be put in motion, the power multiplied by the space through which it is moved, is equal to the weight multiplied by the space through which it is moved.

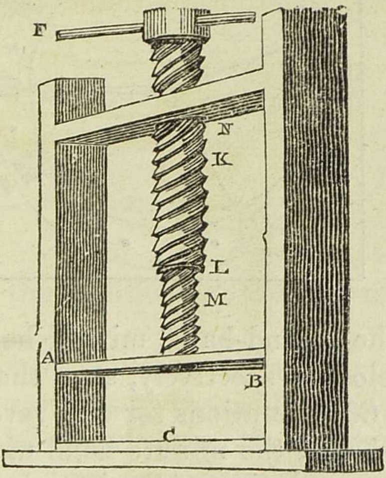
Suppose the power P, acting at the extremity of a lever, is moved uniformly round once, through a




space equal to the circumference of the circle described by P, then the weight will be uniformly moved through a space equal to the distance between two contiguous threads: and by Prop. 32, these spaces are to one another respectively as the weight is to the power.

Hence, by the property of proportional quantities, the power multiplied by the space through which it is moved, is equal to the weight multiplied by the space through which it is moved.

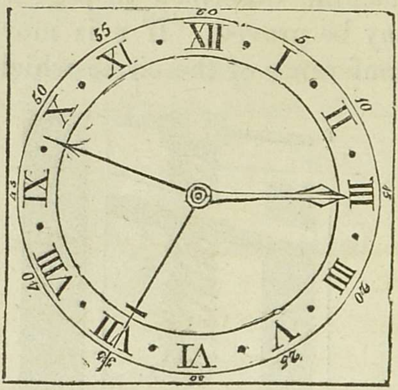
In the machine described in page 101, the same property may be proved. If P is moved through a whole circumference of the circle which it describes,




the part A B, which acts directly upon the weight, is moved through a space equal to the *difference* of two contiguous threads, in the screws K L, and M. And these spaces are to each other respectively as W to P.

Since all combinations of mechanical powers depend upon the principles established in the simple powers, we may conclude that in all machines the principle of Virtual Velocities exists: and we may apply it, at pleasure, to discover the mechanical *power*, as it is called, of any machine.

For instance, if machinery were so constructed that three bars were connected, moveable in the

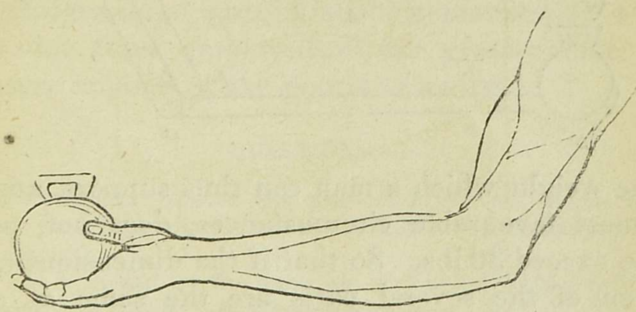


manner of the second-hand, minute-hand, and hour-hand of a clock respectively, such that the second-hand made 60 revolutions for one revolution of the minute-hand, and the minute-hand made 12 revolutions for one revolution of the hour-hand.

Then, if the motion of one of these hands could not take place without setting the others in motion, the principle of virtual velocities would enable us at once to determine that a force of one ounce, applied perpendicularly to the second-hand at any distance, would balance a force of 60 ounces applied perpendicularly, at the same distance, to the *minute-*

hand, or a force 12 times as great as the last, or a force of 720 ounces, applied perpendicularly, at the same distance, to the *hour*-hand.

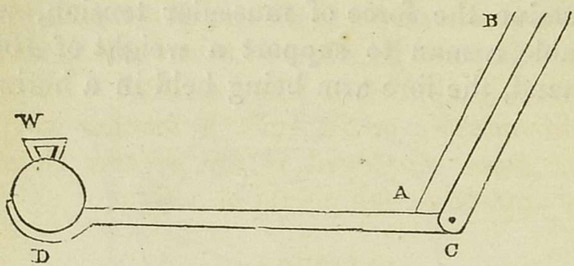
As another instance, suppose it were required to determine the force of muscular tension, which will enable a man to support a weight of 100 lbs. in his hand, the fore-arm being held in a horizontal



position, and the position of the upper arm being known.

The fore-arm is moveable about the elbow-joint, and is drawn upwards by the contraction of a muscle situated in the upper arm, the tendon being attached to the *radius*, one of the two bones of which the fore-arm is composed.

The position, therefore, of the parts, is the same as that represented in p. 142. Suppose it were found by experiment, that if D be raised one inch, the tendon, A B, is shortened the twentieth part of an inch. Then, since the space through which the weight is moved in the direction in which it acts, is 20 times as great as that through which the power is moved, we should conclude that the force of contraction of the muscle was 20 times as great as the weight supported, or equal to a weight of 2000 lbs.




The weight which a man can thus support, under the most favourable circumstances, does not, probably, exceed 30 lbs. So that if the dimensions and position of the several parts are the same as are here supposed, the greatest muscular force which the arm can exert, will be 20 times as great as the weight supported, or 600 lbs.

It is hence evident, that in all machinery as much is lost in rapidity of motion as is gained in power: so that all the advantage which can be obtained by such means is to substitute a quick motion for a slow one, or a slow motion for a rapid one. This, however, is in itself a great convenience. The strength of man and other animals being limited, there are many effects which they could never produce, unless they were assisted by artificial means. And the more usual application of mechanical agency is to enable us to exert a pressure greater than the natural animal force. On the other hand, the more usual application of mechanical agency in the different parts of the animal fabric itself, is by means of a large force, acting through a small space, to cause a smaller force which may act with greater rapidity. The moving forces in the limbs of animals are commonly applied at what is called a mechanical *disadvantage;* or in such a manner that the muscular force employed is far greater than the pressure applied at the points of action.

## QUESTIONS.

How can we discover the proportion of two forces which sustain each other by means of a combination of the mechanical powers?

What is meant by the principle of Virtual Velocities?

Show that it is true in the straight lever.

Show that it is true in the bent lever.

Show that it is true in the wheel and axle.

Show that it is true in the different systems of pulleys.

Show that it is true in the inclined plane.

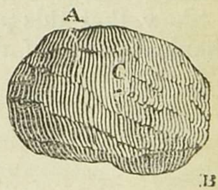
Show that it is true in the screw.

Show that this principle will enable us to discover the mechanical power of a machine.

## LESSON XIV.

### ON THE CENTRE OF GRAVITY.

In finding the proportion of forces which balance each other, we have seldom taken any account of the *size* of the bodies, where weights are used; but have considered them as *points*; or, at least, that their effect is the same as if they were each *applied* at a given point. Now, all the bodies, with which we can make any experiments, have a certain size and form: and it is desirable to show, that there really is a point in which, if the whole matter of any body or system of bodies were collected, the mechanical effect produced by its weight would be the same as when the several particles, of which the body or system is composed, are each in their respective positions. That point is called *the centre of gravity* of the body or system.




Suppose A B to be a body of any shape, however irregular, and that C, some point within it, is the centre of gravity, according to the above definition.

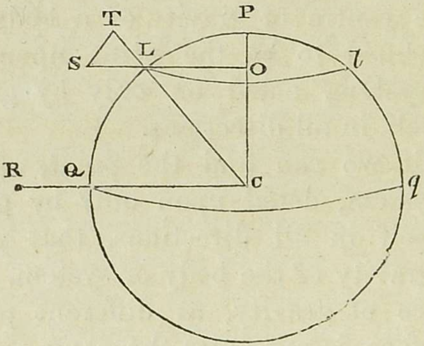
Then, if a single material point were placed at C, equal in weight to the whole body A B, and that point were *supported*, it would remain at rest. And by the supposition, the effect would not be altered, if, while C were thus supported, all the particles of the body were removed each to their respective positions. That is, the body would still balance itself upon C. And, since this is quite independent of the *direction* in which we suppose the body to be placed, the centre of gravity of a body or system may be defined to be the point upon which the body or system, acted on only by gravity, will balance itself in all directions.

Hence, if we can find the point, upon which a body or system, acted upon only by gravity, will balance itself in all directions, that point is the centre of gravity of the body or system.

The force of gravity, at different parts of the earth's surface, is not quite the same. Every body which moves in a curve, has a tendency to fly off from the centre about which the motion takes place. This force is called a centrifugal force, or a force by which a body would tend to go away from the centre of motion. This is the force by which a stone flies away, after having been rapidly whirled round in a sling: and may be easily shown to exist, by hanging a vessel of water by a string which is twisted, and suffering the string to unwind itself. The water in the vessel rises up at the sides, and if the motion becomes sufficiently rapid, will be thrown over at the edges of the vessel. We shall have occasion to notice the cause of this, when we come to treat of the laws of motion. At present, we may take the fact to be proved by experiment;

and that the more rapidly a body is moved round, the greater the centrifugal force becomes.

Thus, suppose the annexed figure represents the earth, and the point L, the position of any place,




as London: and L $l$, its parallel of latitude. Let Q $q$ be the equator, and P the north-pole. Draw L O, Q C perpendicular to P C, the radius of the earth passing through P.

Then, as the earth revolves once in 24 hours, the point L revolves through a circle of which the radius is L O: whereas, the point Q revolves through a larger circle, of which the radius is Q C.

Suppose Q R to be taken to represent the centrifugal force at the equator Q $q$; or the space through which a body would move uniformly in a small given time when acted upon by that force. Then a body at L would be moved uniformly in the same time by the centrifugal force at L, in the direction L S, only through a space L S, having the same proportion to

QR that LO has to QC, since the centrifugal force is proportional to the velocities with which the points Q and L respectively move round C and O.

Now join CL; and draw ST perpendicular to CLT. Then, if LS be taken to represent the whole centrifugal force at L, LT will represent that part of the centrifugal force which is directly opposed to the gravity of a body at L*. And this part is again less than LS in the proportion of LO to QC. And from both these causes, the part of the centrifugal force opposed to gravity is less at L than at Q.

Consequently, if a body, the weight of which is ascertained by the degree in which it bends a spring in London, is carried towards the equator, it will be found to bend the spring less and less, or to be less heavy, as it approaches the equator. This change, however, in the absolute weight of the body, does not alter the position of its centre of gravity. For all the parts of the body lose a quantity of their

* Since the angles at T and O are right angles, and the angle TLS is equal to the vertical angle CLO (*Euclid*, i. 15), the triangles TLS, LOC, are equiangular, and

$$\begin{array}{lll} & LT : LS :: LO : LC & :: LO : QC. \\ \text{and,} & LS : QR & :: LO : QC. \\ \hline \therefore & LT : QR & :: LO^2 : QC^2. \end{array}$$

Hence the diminution of gravity in different latitudes, arising from the centrifugal force of rotation, is as the square of the cosine of latitude, the earth being considered as a sphere.

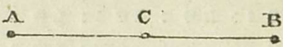
weight *proportional* to their whole weight: and therefore, the *relative* weight of all the several parts is still the same.

In like manner, if any forces act in parallel lines, and in the same direction, upon the several parts of a body or system, and *proportional to the weights of the several parts*, the body or system will still balance itself upon the centre of gravity: or the effect produced is the same as if a single force acted at that point. Hence, the centre of gravity has been called *the Centre of Parallel Forces**.

## Proposition 39.

To find the centre of gravity of any number of material points.

Suppose two material points A, B, of equal weight, to be united by same rigid substance without weight. Then, if A B be bisected in C, the two equal bodies, A, B, will balance one another in all positions about C, if that point be supported. (Prop. 10, p. 37.)



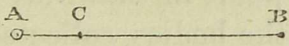

Hence, C is the centre of gravity of A and B.

Also the pressure upon C is equal to the weights

* This is the supposition made in Lesson IV., Ex. 2, p. 29.

of A and B together; or equal to twice the weight of A.

If A, B, be unequal, divide A B in C, so that A C is in the same proportion to C B that B is to A.




Therefore the product of A, multiplied by A C, or the moment of A referred to C, is the same as the product of B multiplied by B C, or the moment of B referred to C.

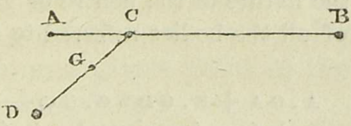
Therefore, by Prop. 15, A and B will balance one another upon C, if that point be supported.

Therefore, C is the centre of gravity of A and B.

Also, the pressure upon C is equal to the weights of A and B together.

Again, suppose there are three bodies, A, B, D.

Find, as before, C, the centre of gravity of A and B. Then, if D, C, be joined, and C D be divided in G, so that D G is to G C as the sum of A and B is to




D; the whole will balance upon G: and, therefore, G is the centre of gravity of the three points, A, B, and D.

And, in like manner, we can proceed for four or more points.

## Proposition 40.

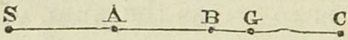
If any number of material points be placed in a line, the sum of the moments of all the bodies referred to any point in that line, is equal to the moment of all the bodies collected in the centre of gravity, and referred to the same point.

This may be easily shown in a particular example, and the same reasoning will apply to any other instance, as is seen in the note below*.

Suppose there are but two bodies, A, 4 ounces, B, 2 ounces; and that AB is 12 inches. (*Fig.* p. 151.)

Therefore, if G be the centre of gravity of A and B, AG will be 4 inches, and GB 8 inches; and the

* Suppose there are three bodies, A, B, C; and that G is their common centre of gravity.




Therefore, by the nature of the centre of gravity, the sums of the moments of all the bodies referred to G being equal on each side,

$$A \cdot GA + B \cdot GB = C \cdot GC.$$

$$\text{or, } A \cdot (SG - SA) + B \cdot (SG - SB) = C \cdot (SC - SG).$$

$$\text{Therefore, } A \cdot SG + B \cdot SG + C \cdot SG = A \cdot SA + B \cdot SB + C \cdot SC.$$

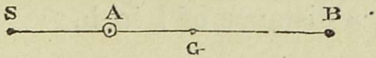
$$\text{Therefore, } (A + B + C)\, SG = A \cdot SA + B \cdot SB + C \cdot SC.$$

$$\text{Therefore, } SG = \frac{A \cdot SA + B \cdot SB + C \cdot SC.}{A + B + C.}$$

The same reason will evidently apply to four or more bodies.

*moment* of A referred to G, or 4 multiplied by 4, is equal to the *moment* of B referred to G, or 2 multiplied by 8.

Now take S at any distance from G, in the line B A: for instance, let S G be 10 inches.




Therefore, S A, the *difference* between S G and A G, will be 6 inches; and the moment of A, referred to S, is 4 multiplied by 6, or 24.

Also, S B, the *sum* of S G and G B, will be 18 inches: and the moment of B, referred to S, is 2 multiplied by 18, or 36.

And the *sum* of these two moments is 60.

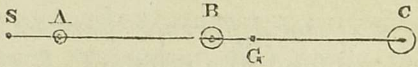
Also, if A and B, were *both* together at the point G, forming a weight of 6 ounces, at a distance of 10 inches from S, the moment of the two together referred to S, would be 6, multiplied by 10, or 60: which is equal to the *sum* of the two other moments.

Hence, to find the distance of the centre of gravity of any number of material points, situated in one line, from a given point in the same line, we have the following rule:

*Multiply together the numbers expressing the weights of each body, and their respective distances from the given point: and add together all the products. Divide this sum by the sum of all the weights, and the quotient will be the distance of the centre of gravity from the given point.*

Ex. 1. Suppose three balls, of 2, 3, and 4 ounces' weight respectively, are placed upon a slender wooden rod, the weight of which may be neglected; at intervals of 3 and 4 inches from one another; and it is required to know at what point the whole must be suspended so as to balance.

Take S A, a distance of 1 inch.




Therefore, S B is 4 inches, and S C 7 inches.

The moment of A referred to S, is 2 multiplied by 1, or 2.

The moment of B referred to S, is 3 multiplied by 4, or 12.

The moment of C referred to S, is 4 multiplied by 7, or 28.

The sum of all these moments is 42; and this sum, divided by 9, the sum of the weights, gives a quotient of $4\frac{6}{9}$ or $4\frac{2}{3}$ inches, for the distance between S and G.

Hence, if B G be taken equal to $\frac{2}{3}$ of an inch, in the direction B C, the three balls will balance one another on the point G.

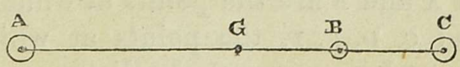
If spherical leaden balls be used, the centres of gravities of each may be considered as coinciding with the centres of the spheres; and the experiment varied at pleasure, by altering the positions of the balls.

Centre of Gravity.

If the point from which the measurement takes place is the point at which one of the bodies, as A, is placed, the moment of A, referred to that point, becomes equal to 0, since its distance from the point is 0.

And if S is so taken that some of the bodies are on one side of it, and some on the other, the moments on one side of S must be subtracted from those on the other side.

Ex. 2. Suppose 3 balls, A of 4 ounces, B of 2 ounces, C of 3 ounces, placed in a line, so that A B is 6 inches, and A C 8 inches, and that it is required to find the centre of gravity.




If we take A as the point from which to measure,
The moment of A referred to the point of A is 0.
. . . . . . B . . . . . . . . . $2 \times 6$ or 12.
. . . . . . C . . . . . . . . . $3 \times 8$ or 24.

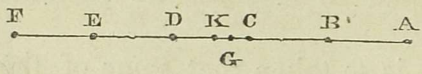
And the sum of those moments is 36.

Also the sum of the three bodies, A, B, C, is 9.

And 36 divided by 9 gives a quotient 4.

Hence the centre of gravity, G, is 4 inches distance from A, or exactly half way between A and C.

Ex. 3. Suppose a man has to carry upon a pole, five feet long, two hares and four rabbits, all hung at the distance of one foot from each other; and that two hares weigh as much as three rabbits, and are hung towards one end of the pole. At what point must he hold the pole, so that he may carry it horizontally; the weight of the pole itself being equal to the weight of a rabbit?




If the weight of each hare be represented by 3, and the weight of each rabbit by 2, the weight of the pole is also 2, and this weight will have the same effect as if it were collected in the centre of gravity of the pole, which must be the *middle* point, K, if the pole be uniform, (Prop. 41); so that A K is $2\frac{1}{2}$ feet.

Also, if A and B are the points at which hares are hung; and C, D, E, F, the points at which rabbits are hung; and we measure the distances from A;

| | | | |
|---|---|---|---|
| The moment of the weight | 3 at A, | referred to the point A, is | 0. |
| . . . . | 3 at B, | . . . . . | $3 \times 1$ or 3. |
| . . . . | 2 at C, | . . . . . | $2 \times 2$ or 4. |
| . . . . | 2 at K, | . . . . . | $2 \times 2\frac{1}{2}$ or 5. |
| . . . . | 2 at D, | . . . . . | $2 \times 3$ or 6. |
| . . . . | 2 at E, | . . . . . | $2 \times 4$ or 8. |
| . . . . | 2 at F, | . . . . . | $2 \times 5$ or 10. |

And the sum of all these moments is 36.

And this sum, divided by the whole weight, 16, gives a quotient $2\frac{4}{16}$ or $2\frac{1}{4}$ feet, or 2 feet 3 inches. Hence the point G, at which the pole must be held, is 3 inches from C, or half way between the middle of the pole and the point C.

## QUESTIONS.

What is the centre of gravity ?

Show that the centre of gravity is a point upon which a body or system, acted on only by gravity, will balance itself in all directions.

Why is gravity different at different parts of the earth's surface.

In what manner does gravity vary in different latitudes, the earth being supposed to be a sphere ?

How can the centre of gravity of any number of material points be found ?

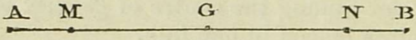
Give a rule for finding the centre of gravity of any number of material points placed in one line.

# LESSON XV.

## THE CENTRE OF GRAVITY.

### Proposition 41.

The centre of gravity of a uniform straight bar, the thickness of which may be neglected, is its middle point.




Let A B be a material *line:* and G its middle point.

Then taking any point M, on one side of G, and another point N, equidistant from G, on the other side, G will be the centre of gravity of these two equal material points, by Prop. 39, 148.

In like manner G is the centre of gravity of any other two corresponding points in the two parts G A, G B, of the material line A B.

Hence the *whole* line will balance itself upon G in all positions; and therefore G is the centre of gravity of the whole line.

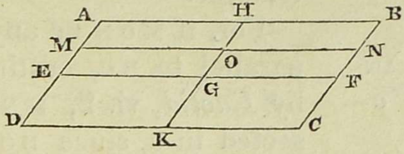
### Proposition 42.

To find the centre of gravity of a parallelogram.

Let A B C D be a thin uniform parallelogram.

Bisect A B, D C, in H and K, and join H K.

Bisect A D, B C, in E and F, and join E F, cutting H K in G.




G is the centre of gravity of the parallelogram.

For, if M O N be one of the material lines, parallel to A B, of which the material parallelogram A C is composed, M N is bisected in O.

Hence M N will balance itself upon O.

In like manner, every such line will balance itself upon some point of H K.

Hence the whole parallelogram will balance itself upon H K.

Therefore the centre of gravity of the whole parallelogram is somewhere in H K.

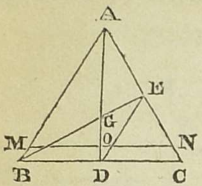
In like manner the centre of gravity of the parallelogram must be somewhere in E F.

Therefore it must be in G, which is the intersection of the two lines.

## Proposition 43.

To find the centre of gravity of a triangle.

Let A B C be a triangle formed of a thin plate of uniform density. Bisect the side B C in D: and join




A D. Bisect another side A C in E: and join B E, cutting H D in G. G is the centre of gravity of the triangle.

For, if M O N be any line drawn parallel to B C, cutting A D in O, by *Euclid*, vi. 2, M N will be bisected in O, since B C is bisected in D.

Hence *every* such material line, of which the triangle is composed, will balance upon A D. And, therefore, the *whole* triangle will balance itself upon A D. And, therefore, the centre of gravity of the triangle lies somewhere in A D.

In like manner, the centre of gravity of the triangle lies somewhere in B E. And it cannot lie in *both* these lines, unless it be their intersection G.

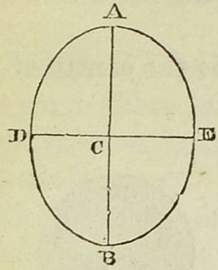
Hence G is the centre of gravity of the triangle. A G can easily be shown to be two-thirds of A D*.

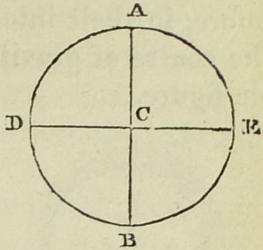
In any figure of uniform density, which is sym-

* Join E D. Then, because A E is equal to E C, and B D equal to D C, D E is parallel to A B, by *Euclid*, vi. 2. And the triangles E D C, A B C are equiangular. Also, the triangles A B G, G D E are equiangular; for the vertical angles at G are equal, and the angle B A D is equal to the alternate angle A D E.

Hence A G is in the same proportion to G D that A B is to E D, or as 2 to 1. And, therefore, A G is two-thirds of the whole A D.

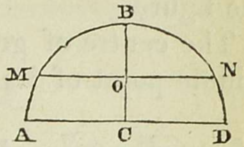
metrical* with respect to any point or line, the centre of gravity can at once be found.



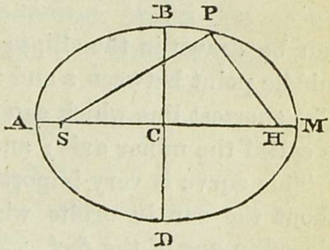

Thus, the centre of gravity of a circle or an ellipse †, is the centre of the figure. For these

* A body is symmetrical with respect to any point or line, when the parts on either side exactly correspond to each other.




Thus, the area of a semicircle A B D is symmetrical with respect to the line B C, which bisects A D, and also bisects the semicircle; but it is not symmetrical with respect to such a line as M O N, even if it be drawn so that the area B N M is equal to the area M A D N.

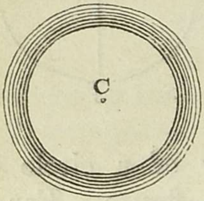
† An ellipse is a figure which may be traced by taking any two points, S, H, and fixing at those points a string S P H. Then, if the string is kept always stretched, the point P will trace the ellipse A P M.

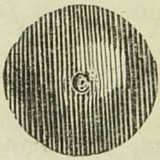



The longest line A M, which

figures are each of them symmetrical with respect to the lines A B, D E at right angles to each other. And the centre of gravity will lie in each; and, therefore, in their intersection C.

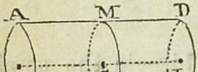
The centre of gravity of a ring is the centre of the whole figure.




The centre of gravity of a sphere is the centre of the figure.

The centre of gravity of a cylinder must be the middle point of its axis.




For every section made by a plane parallel to the circular base of the cylinder A K B must be a circle, the centre of which is its intersection with the axis.

Therefore, the centre of gravity of the whole

can be drawn in the ellipse is that through C, the centre or middle point between S and H, and it is called the major axis. The shortest line which can be drawn through C is B D, which is called the minor axis: and S and H foci.

This curve is very important, since all the planets revolve about the sun in orbits which are ellipses, having the sun placed in one of the *foci*.

cylinder is somewhere in the axis A L. And the whole solid is also symmetrical with respect to the circular section M N which bisects the axis K L in G.

Therefore, the centre of gravity of the cylinder lies somewhere in that circular section. And, therefore, it must be the point G, where the axis K L cuts the circle M G N.

The rules for finding the centre of gravity of any number of bodies not placed in the same line, and generally of bodies of regular forms, are too complicated to be here introduced, and cannot very readily be applied in practice. We will, therefore, proceed to describe a practical method of finding the centre of gravity of any bodies, however irregular.

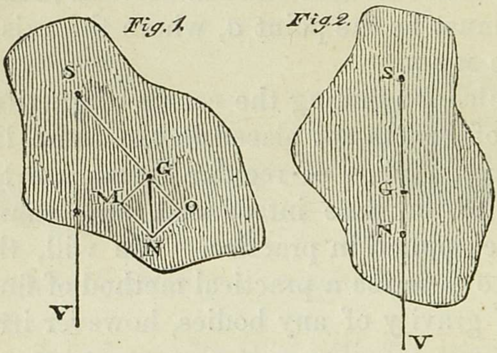
## Proposition 44.

If any body be suspended freely, and acted upon only by gravity, it will not rest until the centre of gravity is in the vertical line passing through the point of suspension.

Let S be the point of suspension, and G the centre of gravity of the body, and S V the vertical line passing through S.

Then, by the definition of the centre of gravity, the effect produced is the same as if the whole mass of the body were collected in the point G. Suppose, then, the body to be in the position represented in *fig.* 1, and the whole mass to be collected in G.

Join S G, and draw G N vertical to represent the force of gravity upon G, and G M, N O, perpendicular to S G, and M N perpendicular to G M.




Then the force G N may be resolved into two forces, G M, G O, of which G O acts in the direction S G O, and is counteracted by the pressure upon the point S, and G M is not counteracted.

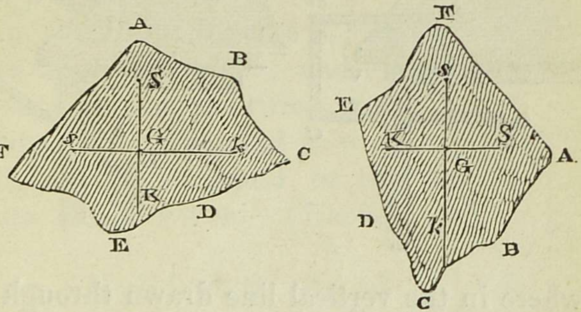
Hence, the point G will move in the direction G M, or towards S V, and a force such as G M will be found to exist in all positions, except in that represented in *fig.* 2, where G is in the vertical line S V.

Therefore the body will rest only in that position.

The knowledge of this property will enable us to find the centre of gravity of many irregular bodies.

Suppose, for instance, A B C D E represents a plane of irregular form, the centre of gravity of which we wish to find.

Suspend the body freely upon some point, S, and through S draw the vertical line, S G K. Then the centre of gravity of the body is somewhere in that line, S G K.



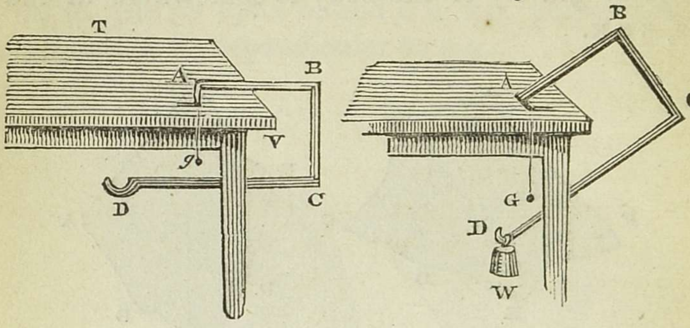

Again, suspend the body freely upon some other point, *s*, and draw the vertical line, *s* G *k*; cutting the line S G K in G.

Then the centre of gravity is also somewhere in the line *s* G *k*, and therefore is the point G the intersection of the two lines.

Several ingenious mechanical puzzles can be explained by this property. Suppose a frame, of the form A B C D, as represented in *fig.* 1, p. 166, rests upon a table, T V, being supported at the point A. Then the centre of gravity of A B C D as *g*, is somewhere in the vertical line passing through A.

Then, if a weight, W, be hung upon the frame at D, it will cause the frame to take another position, as in *fig.* 2, and the whole will then rest supported upon the point A, the centre of gravity of the weight and frame, considered as one system, being

*Fig. 1.* *Fig. 2.*



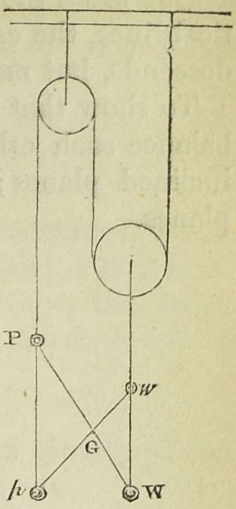

somewhere in the vertical line drawn through A; as at G.

## Proposition 45.

If two weights balance each other upon any machine, and the whole be set in motion, their common centre of gravity neither ascends nor descends.

As a particular instance, we will take the case of two weights, P and W, supporting one another by means of the single moveable pulley, the strings being parallel: and, consequently, W being double the weight of P. (Prop. 23.)

When the weights are in the position P W, join P W, and divide it so that P G is to G W as W to P, or as 2 to 1: and G will be their centre of gravity. (Prop. 39.)




Now suppose P to be moved through the space P $p$, and W to be consequently moved through the space W $w$.

Join $p$ G, G $w$.

Then, (by Prop. 36, case 3,) P $p$ is twice as great as W $w$: and P G was taken equal to twice G W. And, since P $p$ is parallel to W $w$, the angle $p$ P W is equal to the alternate angle P W $w$. (*Euclid*, i. 29.)

Hence, (*Euclid*, vi. 6), the triangles P G $p$, W G $w$, are equiangular: the other sides are proportional, or $p$ G is double of G $w$, and the angles P G $p$, W G $w$ are equal.

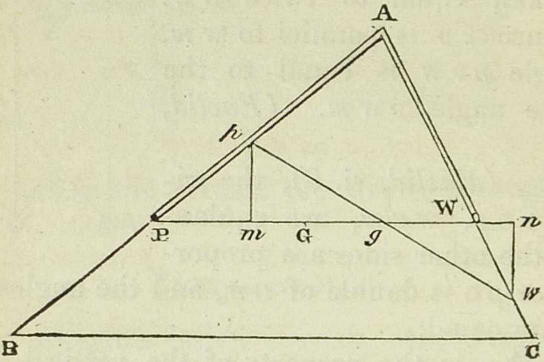
And this is the property of the vertical angles, when two straight lines cut each other. (*Euclid*, i. 15.)

Hence the line $p$ G $w$ is a straight line. And $p$ G is double of G $w$; and therefore the point G is still the centre of gravity of the bodies in the position $p$, $w$.

Since this reasoning depends only upon the circumstance that P and W are moved vertically in straight lines, through spaces which are inversely proportional to their weights, the same proof is applicable to all combinations of machinery, where this condition is observed: as in the lever, wheel and axle, pulleys, &c. (See Lesson xiii.)

But when the two weights do not move in vertical lines, the centre of gravity neither ascends nor descends, but moves *in a horizontal line.*

To show that this is the case, when two bodies balance each other, being partially supported upon inclined planes; the strings being parallel to the planes.




Let P, W be supported upon the inclined planes AB, AC, by a string PAW: and let P, W, be their positions when they are in the same horizontal line.

Therefore, their centre of gravity, G, will be somewhere in that horizontal line, and at such a point that PG is to GW as W is to P.

Now suppose P has been moved to *p*, and W to *w*.

Then, if we join *p w*, cutting the horizontal line PW in *g*, *g* will be the centre of gravity of the bodies at *p* and *w*.

For if *p m*, *w n* are drawn perpendicular to PGW;

the angles at *m* and *n* are right angles: and the vertical angles *m g p*, W *g w*, are equal to one another. (*Euclid*, i. 15.)

Hence the triangles *p m g*, *g w n* are equiangular and their sides proportional (*Euclid*, vi. 4): or *p g* is to *g w* as *p m* is to *n w*.

Also, by the principle of virtual velocities, (Lesson xiii.,) *p m*, the space through which P is raised, in the direction of its action, is to *n w*, the space through which W descends, in the direction of its action, as W is to P.

Hence *p g* is to *g w* as W is to P.

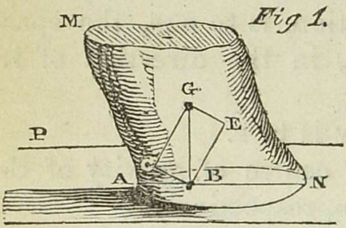
And, therefore, *g* is the centre of gravity of the bodies in the positions *w*, *p*, respectively.

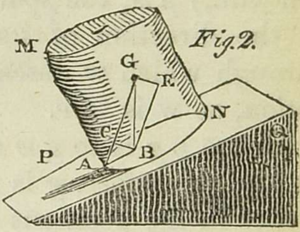
If the weight W were at A, the common centre of gravity of the two bodies would be at the point P; and if W were made to descend gradually towards C, so as to draw the body P up to A, the common centre of gravity of the two bodies would move along the *horizontal line* P G W, from the point P to the point W.

## Proposition 46.

A BODY will rest upon a plane, supposing that it is prevented from sliding, if the vertical line passing through the centre of gravity of the body, *or the line of direction*, falls within the base, and not otherwise.

Let M N represent a body resting on the plane P Q, which is either horizontal, *fig.* 1, or inclined, *fig.* 2. Let G be its centre of gravity. Draw G B vertical, through G; and G A through G to *any* point in the circumference of the base of the body, if the plane P Q is horizontal, or to the lowest point of that circumference, if the plane, P Q, is inclined.

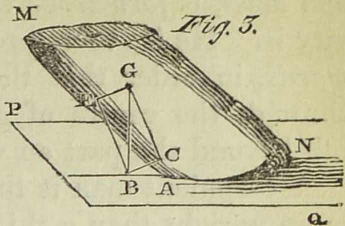



Join G A, and draw B C, G E, perpendicular to G A; and B E parallel to G A.

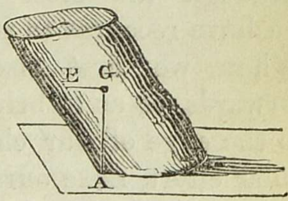
Then, if the point B is within the base of the body, if we take B G to represent the weight of the whole body considered to be collected in G, we may resolve G B into two forces, G C, G E: of which G C acts in the direction G A, and has no tendency to produce motion *about* the point A: and G E acts in such a manner as to *counteract* any force which might be applied to overthrow the body by turning it over at the point A. It is, in fact, a *steadying* force. Hence there is no force tending to overthrow the body, which will consequently remain at rest.

But, if the *line of direction*, G B, falls without the base of the body, as in *fig.* 3, p. 171, taking A, the point of the circumference of the base nearest to B, and joining G A, and drawing B C, G E perpendicular to G A, and B E parallel to G A: we may take G B to represent the effect of the




weight of the whole body applied at G; and resolve it into two forces, G C, G E: of which G C, as before, has no tendency to produce motion about A; but the force G E tends to draw the point G in the direction G E. And this force is not counteracted. Therefore motion will ensue in that direction, or the body will be overturned.

If the body should be of such a form, and in such a position, that the *line of direction* falls exactly upon the circumference of the base, the body will *just* be supported, but will be liable



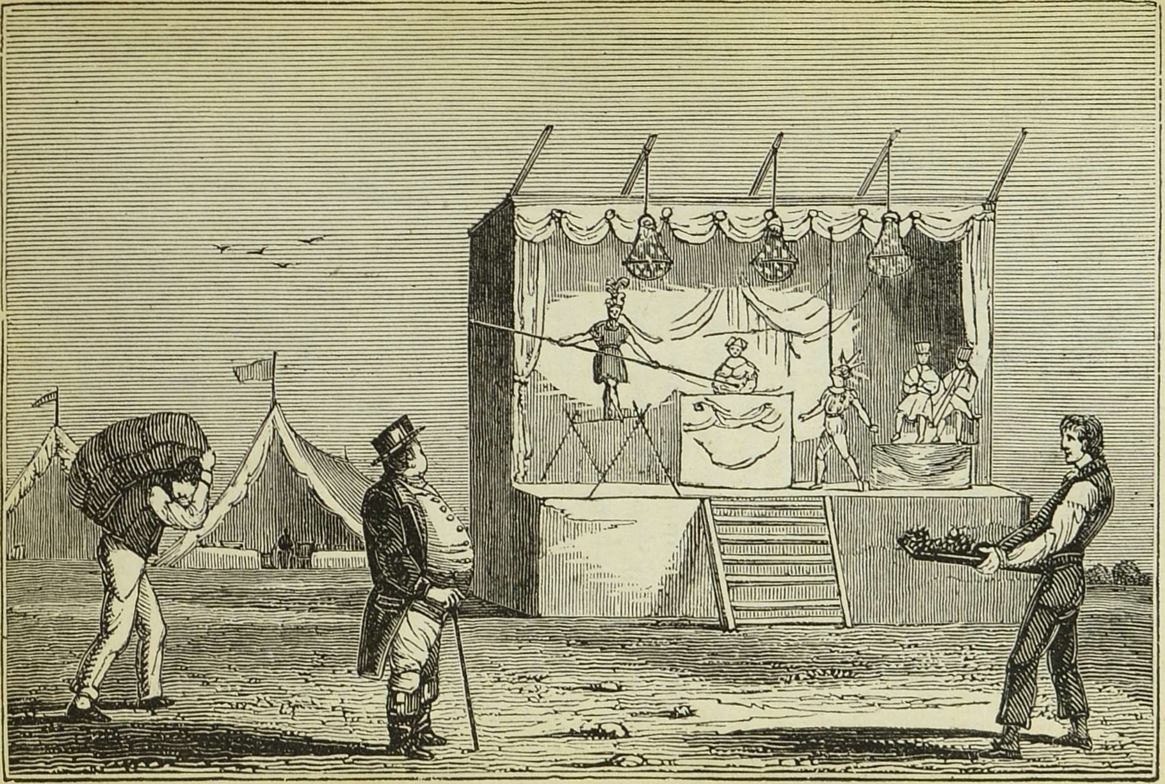

to be overturned by the smallest force tending to move it in the direction G E.

The necessity of having the line of direction fall within the base, in order that the figure may be supported, is the cause of the variety of postures which we are obliged to assume, in order to perform different actions with ease. Thus, if a man carries a weight *before* him, the centre of gravity of *himself* and of the weight which he carries, considered as one mass, is *forwarder* than his own centre of gravity is. He is, therefore, compelled to lean *backwards*, in order that the line of direction, passing through the centre of gravity of the two, may not fall beyond the part on which his feet stand.

A corpulent man is thus obliged to carry himself more upright than a thin man.

If a man carries a load on his *back*, he will fall over *backwards*, unless he leans forwards, so as to bring the line of direction within the base on which he stands.

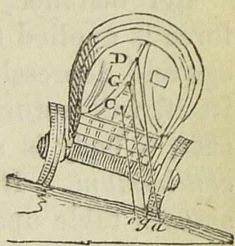
All the postures, assumed by persons who balance weights upon their hands or heads, are regulated by the same principle. But the most remarkable, although the most common, instance, in which we have recourse to it, is in the action of walking. When we first rise from our seats, we lean forwards, so as to bring the line of direction nearly to the edge of our chair; and we then, by a muscular effort, raise ourselves into an upright position. When a person is thus standing easily, the line of direction may fall anywhere between his feet, or on one of them. When we now begin to walk, we rest, for an instant, upon one foot, the line of direction then falling upon it: and we then throw forward the body, in such a manner that we should fall, if we did not, at the same time, bring forward the other foot, upon which the body rests in its turn. Thus, the act of a man's walking is a succession of escapes from falling; and the action is so difficult to



Centre of Gravity.

imitate, that no automaton has been yet constructed so as to perform it. A figure was exhibited, a few years since, which was called a walking figure; but although very ingeniously contrived, its action was more like skaiting than walking.

The property last mentioned shows the importance of not over-loading carriages towards the top.




Suppose a heavy wagon is passing along an inclined road. And, first, suppose the heaviest goods are all packed at the bottom of the wagon, or hung beneath it, and the lighter goods stowed at the top, so as to have the centre of gravity of the whole as low as possible, as, for instance, at the point C.

Then, in the position represented in the figure, the line of direction, C *c*, falls within the base, that is between the wheels of the wagon, and the whole, therefore, will be supported.

But if the goods are differently arranged, or more heavy goods are placed at the top, the centre of gravity of the whole may be higher than C, and may be in such a position, G, that the line of direction, G *g*, falls just within the wheel of the wagon. In this case, the least jerk, tending to throw the wagon over, will overturn it. And if the centre of gravity is still higher, as at D, so that the line of direction, D *d*, falls without the wheel, the wagon will overset by its own weight.

We have seen that a body will rest or not, according as the line of direction, or vertical line through the centre of gravity, is supported or not. But if a body does so rest, and is then acted on by a force tending to overturn it, it may be in such a condition that its own weight either opposes or favours such an impulse, or that it neither opposes nor favours it.

For instance, if an egg is lying *on its side*, and a force is applied to *rock it*, by raising one end of the egg and depressing the other, the egg, when left to itself, will return to its first position. In such a case, a body is said to be *in a condition of stable equilibrium*. If, again, an egg is balanced on a smooth table upon one of its *ends*, and is then acted upon by ever so small a force, it will fall over. In such a case, a body is said to be *in a condition of unstable equilibrium*. But if the egg is again placed upon its side, and is caused to *roll*, by some external force, causing the egg to turn about a line passing from end to end of the egg, it will rest in the position in which it is so placed, without either returning to its first position or falling further than the external force urges it. In such a case, the equilibrium of a body is called *an equilibrium of indifference*.

These three conditions depend upon the position of the *line of direction* of a body, relatively to the base on which the body rests.

Suppose A B C D (*fig.* 1, p. 177) to be a body, of which G is the centre of gravity; and, first, that the body rests upon a horizontal plane, P Q, and that the line of direction G K falls within the base, and, consequently, the body will stand.

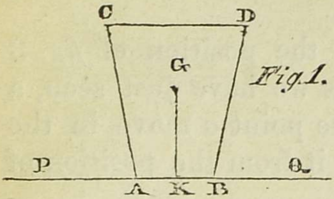


*Fig. 1.*

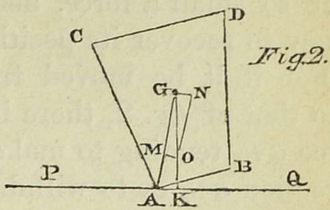


*Fig. 2.*

Now suppose the body to be slightly tilted, into the position represented in *fig.* 2. Then drawing G O K vertical, and taking any portion of that line, as G O, to represent the effect of gravity upon G, and joining G A, and drawing G N, M O perpendicular to G A, the force G O may be resolved, as in the last proposition, into G N, G M: of which G N tends to move the point G *back to its former position*, or to replace the body upon its original base A B.

Hence the position represented in *fig.* 1, is *a position of stable equilibrium.* But if we suppose

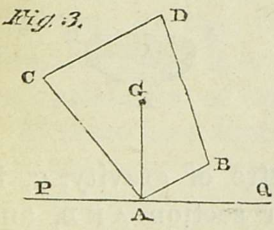


*Fig. 3.*

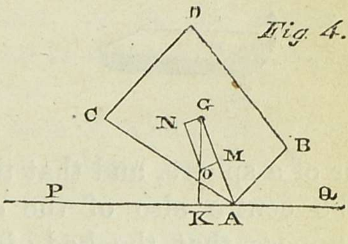


*Fig. 4.*

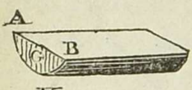
the body to be placed, as in *fig.* 3, so that the line of direction passes through the angular edge A, and that the body can be caused to remain balanced in that position, it will be liable to be disturbed by ever so small a force, and will have then no tendency to recover its position.

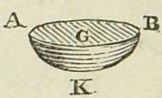
For if it be moved from the position of *fig.* 3 into that of *fig.* 2, there is, as we have just seen, a force G N, tending to make the point G move in the direction G N, or to withdraw it from the position of *fig.* 3.

On the other hand, if it be moved in the opposite direction, as in *fig.* 4, and G O be taken to represent the force of gravity, and resolved into G M in the direction G A and G N perpendicular to G A, the force G N now tends to cause the point G to move in the direction G N, and therefore to overturn the body, instead of replacing it in the position of *fig.* 3.

Hence the position represented in *fig.* 3, is a *position of unstable equilibrium.*

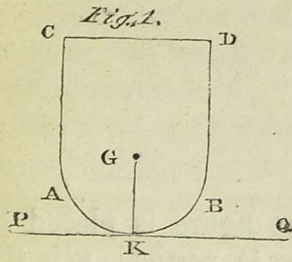
If, again, we take a body of such a form that its lower extremity, A K B, is part either of a cylinder

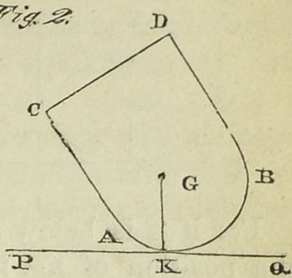



or of a sphere, and that the centre of gravity, G, is the centre also of the circular section A K B, and suppose that the body is placed upon a horizontal plane P Q; then, the body will rest in the position represented in *fig.* 1, p. 179, for the line of direction

G K falls perpendicularly upon the plane P Q, and therefore the body will be supported upon that point.




And if the body be moved into a different position, as in *fig.* 2, so that any *other* point of the cylinder or spherical surface is in contact with the horizontal plane, the line of direction G K still falls perpendicularly upon P Q at the point K; and therefore the body will still rest in the position represented in *fig.* 2.

Hence such a body, when disturbed, has no tendency to move, by its own weight, either in the direction of the disturbance, or in the opposite direction; and it is therefore in the condition called *an equilibrium of indifference.*

If there be a figure, the base of which, as in that last supposed, is a cylinder or sphere, and the figure is symmetrical with respect to some line E G K*, but the centre of gravity of the figure, G, does not coincide with O, the centre of the circular section A K B,

* See p. 160, note *.

the equilibrium upon K will be *stable* or *unstable*, according as G is *below* or *above* O.

*Fig. 1.*

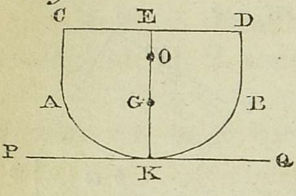
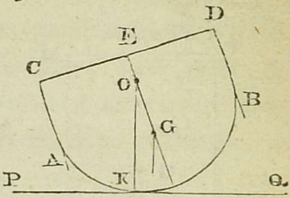



*Fig. 2.*




For if G is below O, and the figure is placed in the position of *fig.* 2, and O K is vertical, K is the point at which the figure is in contact with the horizontal plane. And the effect of the weight of the body is the same as if the whole were collected in the centre of gravity G. Suppose the whole weight so collected. Then the effect of a weight at G would evidently be to turn the body round the point K, towards its former position. Hence the equilibrium in this case is *stable*.

*Fig. 3.*

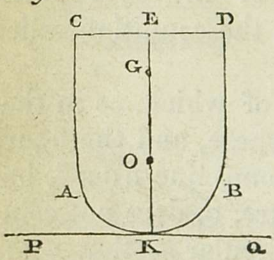
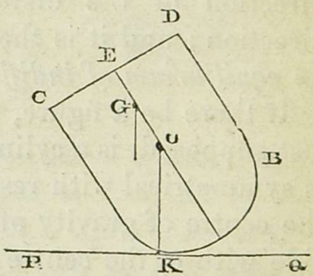
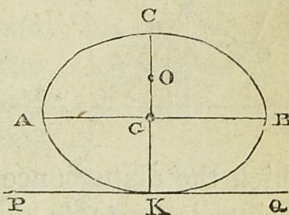



*Fig. 4.*

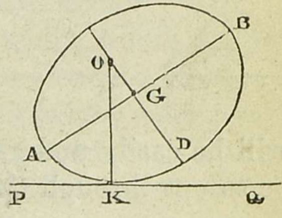



But if G is above O, as in *fig.* 3, and the figure is placed in the position of *fig.* 4, and the whole weight of the body is considered to be collected in G, as the body rests upon the point K, the effect of the weight at G will be to cause motion towards P, or to *increase* the deviation of the body from its first position, in *fig.* 3. Hence the equilibrium in this case is *unstable.*

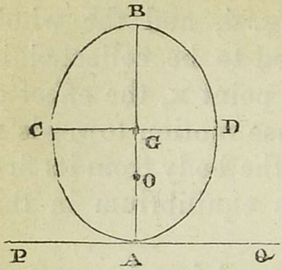
Upon this principle we can explain why an egg-shaped body, or oblong spheroid, of uniform density, resting upon an horizontal plane, has an equilibrium of a different kind, according as it rests on different parts of its surface, and is disturbed by forces acting in different directions, as we supposed in p. 175.




The centre of gravity of the whole body is the centre of the figure, G, and that figure is of such a nature, that the section ACBK is an ellipse, (see p. 161, note †,) of which the part about C and K is *not so much curved* as a circle of which the centre is G, but may be considered as part of a *larger* circle, the centre of which is somewhere above G, as at O.

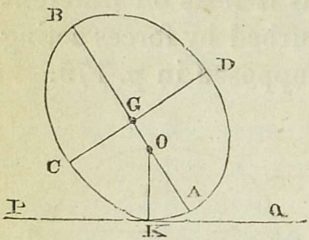



If then the point B be raised, and the point A be lowered, the effect of the whole weight of the body, applied at G, is to bring back the body to its first position,

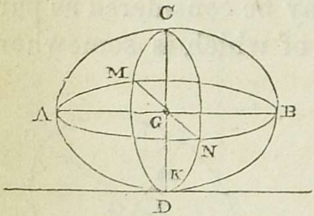



hence the equilibrium is *stable.*

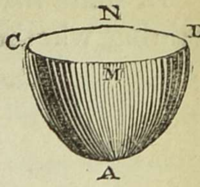
But if the body be placed, so as to rest upon one of its smaller ends, as A; the part of the figure about A is *more curved* than a circle of which the centre is G, and may be considered as part of a *smaller* circle, of which the centre is some point O, lower than G.




If then the point D be raised, and the point C lowered, the body will rest upon some point K, and the effect of the weight of the body collected at G, is evidently to cause the body to move in the direction in which the disturbance has taken place, or the equilibrium is *unstable.*



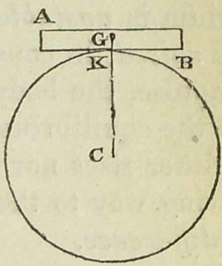

If, again, we consider the body to be resting as in the first instance, upon its side, and suppose a section MDNC to be made through G, by a plane perpendicular to AB, that section will be a circle of which G is the centre: as will be easily understood, if we imagine an egg to be so cut through the middle, as in fig., p. 183.




If, then, the body be moved about the axis A B, by raising the point M, (*fig.*, p. 182,) and lowering N, the body will be brought to rest upon some other point, K, of the circle D N C, *of which* G *is the centre.*

Hence the body will be supported upon K, as well as it was upon D; or the equilibrium, in this case, is *an equilibrium of indifference.*

It is plain that the nature of the equilibrium will depend upon the nature of the surface *on which* a body rests, as well as upon the lower surface of the body itself.

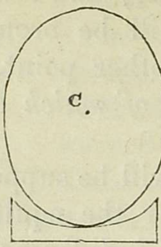



Thus, a body A B, placed upon a sphere C, and resting upon the point K, will, if just disturbed, either settle back to its first position, or fall off, according to the height of its centre of gravity, G, above K, relatively to the radius of the sphere C K.

The equilibrium will be stable, if G K is less than C K: but the investigation of this case is not sufficiently simple to be here introduced.

A body, C, (see *fig.*, p. 184,) which would not rest upon the smaller end upon a *horizontal* plane, may yet have a position of stable equilibrium when placed in a cup of the proper degree of curvature. But the consideration of cases such as this must also be here omitted.

In some instances, it is a convenient way of ascertaining whether the equilibrium is stable, to inquire




whether the centre of gravity can *get any lower than it is*, in consequence of the body obeying a small impulse given to it. If the centre of gravity can get any lower, motion will ensue, and the equilibrium is *unstable.* If the centre of gravity would be *raised*, in consequence of the body obeying the impulse, the body will return to its former position, or the equilibrium is *stable.* If the centre of gravity neither rises nor falls, in consequence of the body giving way to the impulse, the equilibrium is one of *indifference.*

### QUESTIONS.

Find the centre of gravity of a uniform straight bar.

Find the centre of gravity of a parallelogram and of a triangle.

When is a figure said to be *symmetrical* with respect to any point or line?

How may an ellipse be traced?

Show that if a body is suspended freely, it will not rest till the centre of gravity is in the vertical line passing through the point of suspension.

Find the centre of gravity of an irregular body.

Under what circumstances will a body rest upon a plane ?

In what manner does the position of the centre of gravity influence persons in walking, and other actions of the body ?

Whence arises the danger of overloading carriages at the top ?

When is equilibrium said to be *stable ?* when *unstable ?* when *indifferent ?*

Give instances of each.

In what positions will an egg-shaped body, or oblong spheroid, have its equilibrium upon a horizontal plane, *stable* or *unstable ?*

In what manner may it be disturbed so that its equilibrium, when resting on its side, may be an equilibrium of *indifference ?*

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## LESSON XVI.

### ON ABSOLUTE AND RELATIVE MOTION.

---

A body is *absolutely at rest*, when its position in fixed space remains unchanged: and it is *absolutely in motion* when its position in fixed space is changed from time to time. Hence motion may be defined to be *the act of a body changing its place.*

The simplest kind of motion is that which is *uniform*, or that in which equal spaces are constantly passed over in equal successive portions of time; as one mile in one hour; two miles in two hours: and so on.

The motion of a body is said to be *accelerated*, when a *greater* space is passed over in each equal successive portion of time. Thus, if a person walks one mile in the first hour; two miles in the next hour; three miles in the following hour; or a ball rolls one foot in one second of time, three feet in the next, and five feet in the next: the motion in each case is *accelerated;* the acceleration taking place at the end of each interval of time.

The motion of a body is said to be *retarded*, when a *less* space is passed over in each successive portion of time. Thus, if a person walks three miles in the first hour, two miles in the next hour, and one mile in the following hour; or a ball is thrown up an inclined plane, and describes five feet in the first second of time, three feet in the next, and one feet in the next: the motion, in each case, is *retarded;* the retardation taking place at the end of each interval of time.

The *velocity* of a body is *the rate of its motion* at any instant of time.

If the motion of a body is *uniform*, its velocity is measured by the space uniformly described by it in a given time.

If the motion of the body is *not* uniform, its velocity is measured by the space which it *would* describe uniformly in a given time, if the motion became and continued uniform, from that instant of time.

Thus, a stage-coach would be moving with a velocity of eight miles an hour, on passing a given mile-stone, if it had travelled that distance uniformly in the preceding hour, and went on uniformly to complete an equal distance in the next hour.

But it is evident, that it might still be moving at the same rate, that is, with the same velocity, *at the given instant*, however its rate of travelling were altered before or after that instant.

The unit of space and time taken in order to measure velocity may be assumed of any magnitude. In estimating the speed of horses or carriages, the time is usually expressed in hours, and the distance in statute miles. The rate of a ship's sailing is reckoned by the number of *knots* described in half a minute, or the 120th part of an hour. The *knot* is the length between two divisions of the log-line, each of which is the 120th part of a geographical mile. Hence the number of knots which the ship runs past in *half a minute*, is the number of geographical *miles* which she would sail in an *hour*, at the same rate. So that if eight knots run off the reel in *half a minute*, or the ship is going eight knots, she is sailing at the rate of eight *miles an hour*.

In Mechanics, one second ($1^s$) is usually taken as the unit of time; and one foot as the unit of space: so that if a body is said to have a velocity of 25, it is implied that the body is moving at such a rate as would cause it to describe 25 feet uniformly in $1^s$.

We have often occasion to consider whether a body is at rest or in motion *with respect to other bodies*. The terms *relative rest* and *relative motion*, are used to signify these conditions respectively.

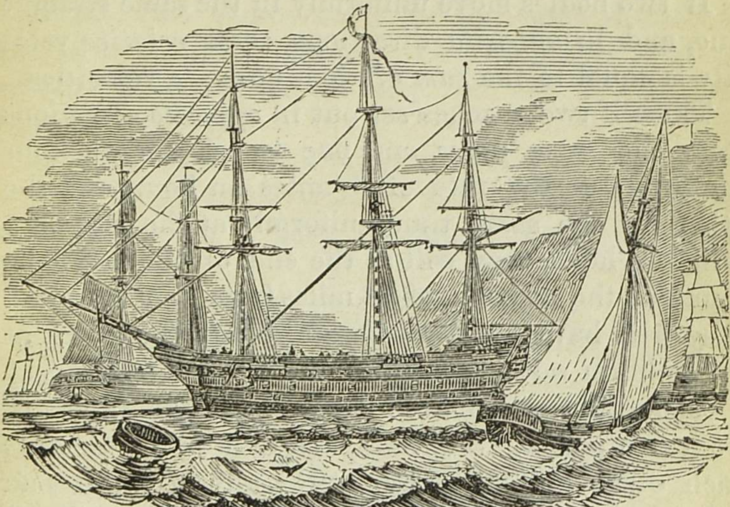
Two bodies are *relatively at rest*, when their position with respect to each other remains unchanged.

This will plainly happen, either where each of the two bodies is absolutely at rest, or where each moves *in the same direction* and *with the same absolute velocity*.

Thus, the passengers in a stage-coach are relatively at rest, with respect to one another, as long as the coach either stands still, or moves smoothly along, however rapid the motion of the whole may be.

Two bodies are *relatively in motion*, when their position with respect to each other is changed from time to time.

Thus, if two vessels are one mile apart, at a given time, and their distance from each other continually increases for an hour, they are relatively in motion during that time.



Absolute and Relative Motion.

If one of these two vessels should be at rest, as lying at anchor, and the other in motion, still *each* of them is in *relative* motion, with respect to the other. For, after a certain interval of time, the sailing vessel has removed one mile further from that at anchor than it was at first, and the *relative distance* of each from the other is equally changed.

*Relative velocity* is the rate of a body's *relative motion;* and is measured either by the space uniformly described by each body, with reference to the other, in a given time, or by the space which *would be* so uniformly described, if their relative motion became, and continued to be, uniform at the given instant of time.

If two bodies move uniformly in the same straight line, and in *opposite* directions, their *relative* velocity is equal to the *sum* of their *absolute* velocities.

Thus, if two coaches set out in *opposite* directions from the *same place*, and one travels for an hour uniformly at the rate of 6 miles an hour, and the other for the same time uniformly at the rate of 7 miles an hour, they will at the end of the hour, be found at the distance of 13 miles from another; and they will have been *separated from one another* at the uniform rate of 13 miles an hour.

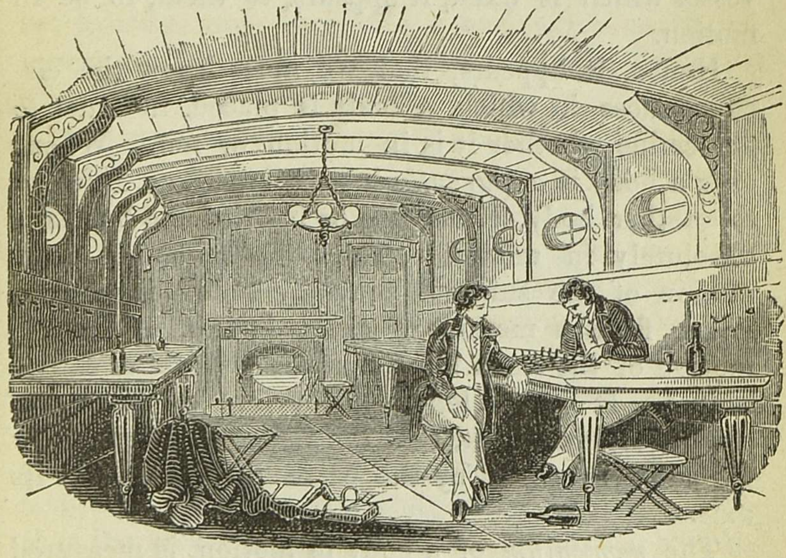
Again, if two coaches set off, at the same instant, from two places, 13 *miles apart*, and move *towards* each other uniformly, one at the rate of 6 miles an hour, and the other at the rate of 7 miles an hour, they will meet at the end of one hour, having *approached one another* uniformly at the rate of 13 miles an hour.

If two bodies move uniformly in the same straight line, and in the *same* direction, their *relative* velocity is equal to the *difference* of their *absolute* velocities.

Thus, if two coaches set out from the same place, and in the same direction, travelling uniformly, one at the rate of 7 miles an hour, and the other at the rate of 6 miles an hour; at the end of an hour the first will be *one mile* in advance of the other; and they will have been separated from one another uniformly at the rate of one mile an hour, which is the difference of their rates of absolute motion.

If the two bodies move in the same direction, and with the *same* absolute velocities, their *relative* velocity is equal to nothing, or the bodies are *relatively at rest*.

It is not so simple a thing as it may at first sight appear, to determine whether a body is in motion or not. Although a body appears to be at rest, with



Absolute and Relative Motion.

reference to any surrounding objects, it may yet be in motion; for all those objects may be moving *with* the body itself, and at the same rate; and in that case they will all be *relatively at rest.*

For instance, if two persons are playing at chess in the cabin of a vessel moving smoothly along, the chessmen will remain *in the places* in which they are set, until the players move them; that is, they will be relatively at rest, with respect to each other, and to the chess-board. But yet they are all the time carried on by the motion of the vessel.

Again, a body may be *at rest*, although it appears to be in motion. Thus, if passengers in such a cabin as we have just supposed, look towards a vessel which is fixed, it appears, to them, to be in motion.

Hence, it appears, that we must know the circumstances in which a body is placed, in order to determine whether it is in motion or not. Whenever a body is *apparently* in motion, when referred to another body, either one or both of the bodies is absolutely in motion. Thus the apparent daily motion of the sun and heavenly bodies must arise either from a motion of those bodies, or from a motion of the earth itself, or from both those causes united. It was a long time before the fact was completely made out, that the rising and setting of the sun are caused by the earth turning round its axis.

The *momentum* of a body in motion, is measured by the product of the numbers expressing its *velocity* and its *quantity of matter*, which is proportional to its weight.

Thus, if one body A, whose weight is 6, moves with a velocity 5, and another body B, whose weight is 8, moves with a velocity 7, their *momenta* are in the proportion of $5 \times 6$ to $7 \times 8$, or of 30 to 56, or of 15 to 28.

When we are considering the condition of bodies *kept at rest* by any forces, it is sufficient to regard the forces as pressures, which may be represented by weights. (See page 5.) But when force is employed *to set bodies in motion*, or to act upon bodies already in motion, several effects are produced, which must be carefully distinguished; such as *velocity* and *momentum;* and some particular *names* must be given, to point out what modification of force we are speaking of.

Suppose two leaden bullets, one weighing 1 ounce, the other 2 ounces, to be let fall freely from the ceiling of a room, by their own weight. They will be found to come to the floor exactly at the same time; and their *velocities* (which can be found in a manner which will hereafter be explained) are found to be the same.

If, therefore, in considering the effect of forces, we regard only the *velocity generated by the action of the forces in a given time*, and we call this modification of force *Accelerating force*, we should say that the *accelerating force* acting upon each of these bullets, was the same.

But if the effect of force, which we wish to consider, is the *momentum* generated in a given time, and if we call this modification of force *Moving force*, since the quantity of matter in each bullet is as 1 and 2, and their velocities equal, the *moving forces* are in the proportion of 1 to 2.

Since the *moving force* is proportional to the product of the *velocity* and *quantity of matter*, and the *accelerating force* is proportional to the *velocity* simply: it follows that the *moving force* is proportional to the *product* of the *accelerating force* and *quantity of matter*, expressed in numbers; and the *accelerating force* is proportional to the *quotient* obtained by dividing the *moving force* by the *quantity of matter*.

For instance, suppose one body A, whose weight is 3, is set in motion by the action of a certain force, and in $1^s$ acquires a velocity 11; and another body B, whose weight is 4, is set in motion by another force, and in $1^s$ acquires a velocity 13.

Then the *Moving force* on A is to the *Moving force* on B, as $3 \times 11$ to $4 \times 13$; or as 33 to 52.

And the *Accelerating force* on A, is to the *Accelerating force* on B, as 11 to 13; or as 33 divided by 3, is to 52 divided by 4.

EXAMPLE.—Suppose a body, whose quantity of matter is represented by 18, acquires in $1^s$ a *momentum* of 108, what is the *accelerating force*?

The *moving force*, measured by the momentum, is 108; and the quantity of matter 18. Hence the *accelerating force* is $\frac{108}{18}$, or 6.

QUESTIONS.

When is a body said to be absolutely at rest, or absolutely in motion?

Define motion.

When is motion uniform?

When is it accelerated or retarded?

What is velocity; and how is it measured?

What units of space and time are usually taken to measure the velocity of a body in mechanics?

How is the rate of a ship's sailing estimated?

When are two bodies said to be *relatively* at rest, or *relatively* in motion?

What is *relative* velocity? How is it measured?

If two bodies move uniformly and in the same straight line, how is their relative velocity found; when they move either in opposite directions or in the same direction?

What is the *momentum* of a body in motion?

Show that two bodies may be *absolutely* in motion, yet relatively at rest.

How is *accelerating force* measured?

How is *moving force* measured?

If the *moving force* and the *quantity of matter* moved are known, how can we find the accelerating force?

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# LESSON XVII.

## ON THE LAWS OF MOTION.

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The simplest principles to which all motions can be reduced, are called the Laws of Motion. They are three in number.

### THE FIRST LAW OF MOTION.

*If a body is set in motion, it will continue to move uniformly in a straight line, until it is acted upon by some external force.*

This law of motion implies two things: first, that a body, once set in motion, has in itself no tendency to stop; and secondly, that its motion is uniform, and in a right line.

With respect to the first part of the law, it must be allowed that it seems at first to contradict our common experience. If we set any body in motion, as by shooting a bullet from a gun, throwing a stone, rolling a hoop, or the like, we find that, when left to itself, it gradually loses its motion, and soon comes to a state of rest. But on further consideration, it will be found that, in all these instances, and any others which may be proposed, there is *an external force* acting upon the body, and destroying its motion.

To take a simple example; suppose a bullet shot along a level road. It leaves the gun with a certain velocity, for instance, with a velocity of 1000 feet in a second. Yet at the end of five seconds, instead of having advanced 5000 feet, and then continuing to move with the same velocity as at first, it may probably not have described half that space, and have already come to a state of rest.

Let us observe, then, what external forces have acted upon the bullet. These are, first, the friction against even the smoothest part of the road itself, which sensibly diminishes the motion of the bullet; secondly, the resistance of *rough* obstacles, such as stones, against which the bullet strikes, losing part of its velocity by every such blow: thirdly, the resistance of the air, which, although small for bodies of inconsiderable bulk moving slowly, becomes very great when bodies move rapidly; as may be readily conceived by any one who has travelled in an open carriage with very great speed, even when the air is still.

If now another bullet be discharged, under more favourable circumstances, that is, with a less velocity, so as to have less resistance of the air, and along a smoother road, so as to be less retarded by obstacles, it will be found to retain the velocity of projection for a longer time. And since, in all experiments which can be made, it is found that the more we can remove the causes which retard the motion of a body, the longer that motion is continued, we conclude that, if all such causes were removed, the motion would continue without diminution.

Again, such a bullet, projected along a rough road, would move very irregularly; it would be found sometimes to leap up into the air, and again to start aside to the right and left. But all such deviations from a right line can be shown to arise from the action of some external force, as that caused by the striking against stones and other obstacles. And as these are removed, the motion is found to become more rectilinear.

From such instances, and from numerous experiments which have been made for the express purpose of determining the fact, we conclude, that if we could remove all the external obstacles which tend to retard the motion of a body, and to cause it to deviate from a right line, the motion would continue to be uniform and rectilinear.

### THE SECOND LAW OF MOTION.

*Motion, or change of motion, is proportional to the force impressed, and takes place in the direction in which that force acts.*

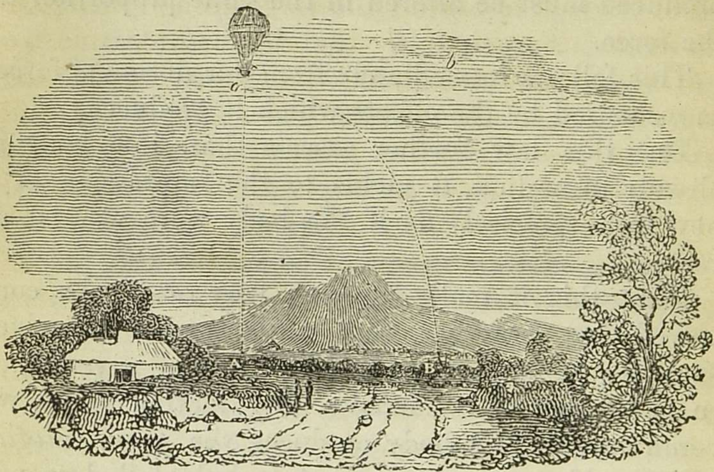
If a force acts upon a body *at rest*, it will set it in motion, unless some other force act upon the body, so as to counteract the effect of the first force. And if a force of a certain magnitude produces a certain quantity of motion, or causes the body to move with a certain velocity, a force twice as great, acting in the same manner, must produce double the quantity of motion, or double the velocity, in the same body. And, in like manner, if the force be altered in any proportion, the quantity of motion produced must be altered in the same proportion as the force.

This follows immediately from our notion of force, as measured by the effects which it produces.

But this law further asserts that, if a body is already in motion, it yields to the impression of a new force precisely as if the body were at first at rest.

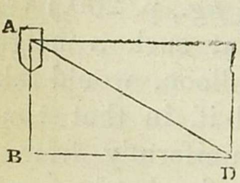
That this is really the case may readily be conceived, by observing, that while we are ourselves smoothly in motion, as in the cabin of a vessel, or in the inside of a carriage, any motion which we communicate to a body produces the same *relative* effect as if we were at rest. A body let fall, descends to the point immediately beneath it. A ball, thrown from one person to another, is caught as readily as if each person were standing still. Thus, also, in feats of horsemanship, a person, riding round a ring with great rapidity, is able to spring from his saddle, and alight upon it again, and that, by leaping, not *forwards*, but *upwards*.

If a person in a balloon, at *a*, (*fig.*, p. 200,) vertically over the point A, were at such a height that a bottle, let fall from the balloon, would take one minute in descending; and if, in that time, the balloon itself would move uniformly from *a* to *b*, a point vertically over B; the bottle would reach the ground, not at A, but at B, having been *directly under the balloon* at every point of its descent.




The effect which the resistance of the air would have upon the motion of the bottle is, of course, here neglected.

If a body have communicated to it at the same instant *two* motions, it will be found, at the end of a given time, at the same point, as if each motion had been communicated to the body in succession.




Thus, suppose a ship, at A, fires a shot at a battery, in the direction A C. If the ship is at rest, the shot, at the end of a given time, will be found at the point C, having described A C uniformly, by the first law of motion. (Neglecting the effect of gravity and the resistance of the air.) But if the ship itself is in motion at the instant when the shot is fired, with a velocity which would cause it to move uniformly through the space A B, in the time in which the shot is fired, and B D is drawn parallel to A C, the shot will describe the diagonal, A D, of the parallelogram A C D B, and will strike the battery at the point D, having described the line A D uniformly in the same time in which it would have described A C, if the ship had been at rest.

The point D is the same at which the shot would have arrived if it had first described A B, in the given time, with the velocity of the ship, and had then described B D, in another equal interval of time, with the velocity communicated to it.

It will be at once seen that this property of the *composition of motion* is analogous to the *composition of force*, as laid down in Lessons III. and IV., and it may be expressed in these terms:

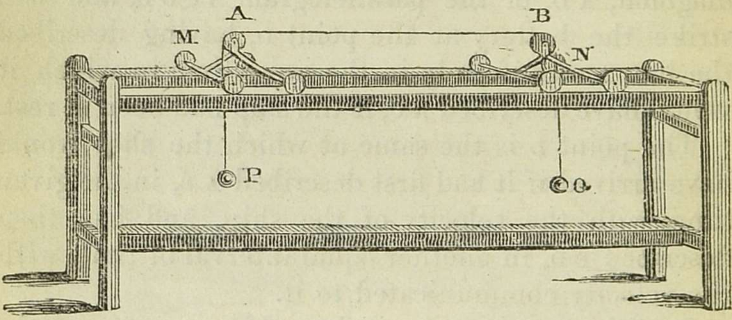
If two motions, which, if communicated separately to a body, would cause it to describe the two sides of a parallelogram uniformly, in a given time, are communicated at the same instant, the body will describe the diagonal of the parallelogram uniformly in the same time.

### THIRD LAW OF MOTION.

*When a body is set in motion by pressure, the moving force is proportional to the pressure.*

This law will be proved, if it can be shown that, when bodies are set in motion by any pressures, their *momentum*, or the product of the velocity multiplied by the quantity of matter moved, in each case, is proportional to the pressure.

Suppose that two pulleys are mounted upon two frames, M, N, resting on wheels, as in the annexed




figure, the frames being freely moveable upon two horizontal bars: and that two equal weights, P, Q, are hung to a string passing over both pulleys, the part of the string, A B being horizontal.

Then, from Prop. 1, 2, (p. 10, 11,) it is evident that the force which stretches every part of the cord, P A B Q, is the same, and is equal to the weight of P. Hence, the pulleys, A and B, are urged towards each other by equal pressures.

1. Let the weights upon the frames, M, N, be equal; and let them be permitted to approach each other. Then at any instant they will be found to be moving towards each other with the same velocities, and they will meet at a point half-way between the points from which they set off.

2. Suppose that one of the frames, M, is loaded

with weights, so that M together with P is twice as heavy as the other, N, together with Q, then M will be found to move only half as fast as N. But the *quantity of matter* moved with M being *double* the quantity of that moved with N, the *momentum* of M will be exactly equal to that of N.

In this case, the bodies will meet at a point which is twice as far from the original position of B as from the orginal position of A.

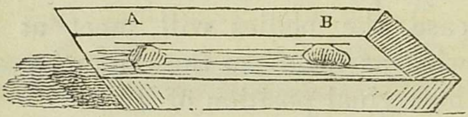
3. Let M be loaded, so that the weight of M, together with P is to that of N, together with Q, as 3 to 2. Then the velocities will be found, at any instant to be as 2 to 3; and consequently the *momentum* of M will be equal to that of N.

In like manner, if the weights of M and N be altered in any proportion, the *momentum* generated *by the same pressure*, will be found to be the same in each body.

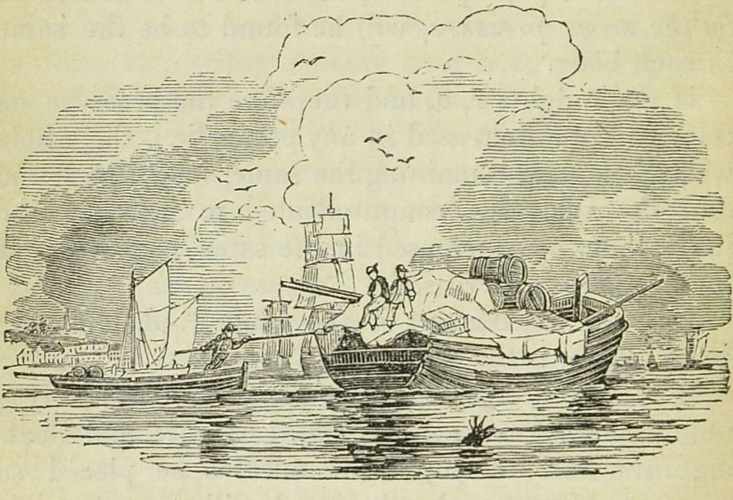
If the weights P, Q, and therefore the pressure on the string, be increased in any proportion, the whole weight on M, N, remaining the same, it will be found that the velocities communicated to M, N, in the same time, are increased in the same proportion as the pressure is increased.

Another experiment of the same kind, but more easily made, is the following.

If A and B represent two pieces of cork, having each a magnetized needle placed upon it, and floating in water. Then, if the needles be placed so near as to attract each other, each will move towards the other. And if one of the bodies, as A, be lighter than the other, B, the lighter body, will move faster than the other, in such a manner, that, neglecting the resistance of the water, the momentum of A is equal to the momentum of B.




If the attraction of one of the bodies, A, upon the other, is called the *action* of A, and the effect produced upon B is called the *reaction*, and each of those is measured by the *momentum* which it generates in the same time, we may say that "*Action and reaction are equal.*"



If a man in a small boat pushes against a larger boat, each boat being freely moveable, the pressure which he exerts upon the boat-hook, will occasion motion in each boat; the smaller boat moving faster than the larger in proportion as its weight is less, so that the *momentum* of each is the same.

If the pressures, which act upon two bodies, and set them in motion, are proportional to the quantities of matter in the bodies, the *velocities* generated in the same time will be equal; or the *accelerating forces*, which are measured by those velocities (See p. 193), will be the same. For the *momentum* of each body, or the product of the velocity multiplied by the quantity of matter in each, is proportional to the *pressure*, and consequently proportional to the quantity of matter itself; which cannot be, unless the *velocities* are *the same*.

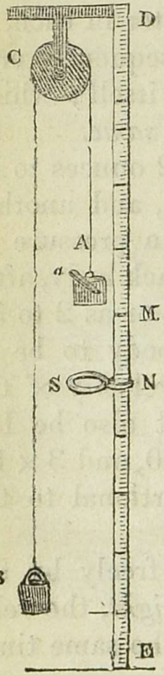
For example, suppose a weight of 2 ounces to be acted on by a pressure of *two ounces*, and another weight of 3 ounces to be acted on by a pressure of *three ounces*, then the momentum of each body, after a given time, will be as the pressures, or as 2 to 3.

Suppose the velocity of the first body to be of any known value, as 10; then the velocity of the second body at the same time must also be 10; and their *momenta* will be $2 \times 10$, or 20, and $3 \times 10$, or 30, respectively, which are proportional to the *pressures* 2 and 3.

Hence, when different bodies fall freely by the action of gravity, or by their *own weight*, the velocities, which they will all acquire in the same time, are the same, the resistance of the air being neglected.

By experiments, made for the purpose, it is ascertained that the force of gravity generates a velocity of 32 feet, nearly, in one second.

The reason why a heavy body, as a leaden bullet, generally falls more quickly than a lighter body, as a feather, is that the surface of the bullet is much less, in proportion to its weight, than the surface of the feather; and if each moves through the air with the same velocity, the resistance to the feather is much greater than that offered to the bullet. If this cause of inequality be removed, by placing both the bullet and the feather under the receiver of an air-pump, from which the air is drawn, they are found each to descend with the same rapidity.




A contrivance such as is represented in the annexed figure affords the means of showing the truth of the third law of motion.

A pulley, C, is made to move very freely upon its axis; and, when great accuracy is required, peculiar contrivances are employed to diminish the friction.

Two boxes, A and B, are suspended over C by a fine cord, and the upright bar, D E, is graduated.

Suppose the two boxes to be equal in weight, and each to weigh 20 ounces; and that a bar, *a*, which weighs one ounce, is laid upon the box, A; and that a moveable ring, S, is placed at N, so that when the top of the box A has descended from M to N, the bar, *a*, may be intercepted by the ring.

Suppose also that M N is found by experiment to be the space through which the box A descends, when *a* is laid upon it, in one second of time.

Then, when the weight *a* is removed, since the two boxes A and B exactly balance each other, A will continue to descend, and B to ascend, *uniformly*, by the first law of motion. And if N O is the space through which A is found to move in one second of time, N O will measure the *velocity* which the whole system had acquired during the action of the pressure of 1 ounce in one second of time.

Also the *quantity of matter* moved will be the sums of the two weights, A and B, and the weight *a*; which, in the case supposed, would be 41 ounces. To this must be added the resistance to motion arising from the inertia of the pulley C, and from the friction, the effect of which is the same as if an additional quantity of matter were to be moved. Hence, the *momentum* generated in the given time, $1^s$, will be known; and, when the experiment is carefully conducted, it is found to be proportional to the pressure.

By varying the weights A, B, and *a*, the pressure and the quantity of matter can be varied at pleasure.

A very perfect machine, upon this principle, was invented by Atwood, and has been used by himself and others to establish the laws by which the motions of bodies are regulated.

The *accelerating force*, in the case supposed, is found by dividing the *moving force*, or the pressure employed, which is proportional to the momentum, by the quantity of matter moved.

Thus, if the *accelerating force* with which the weight *a* alone would descend is called 1, the accelerating force of the system, neglecting the inertia and friction of the pulley, will be the weight of *a* divided by the sums of the weights moved, A, B, and *a;* which, in the case supposed, would be 41.

Hence, the accelerating force would be the fraction $\frac{1}{41}$: and if the velocity acquired by *a* falling freely in $1^s$ would be 32 feet, or 384 inches; the velocity acquired *by the system* in $1^s$ would be $\frac{384}{41}$ inches, or $9\frac{15}{41}$, or $9\frac{3}{8}$ inches nearly.

## QUESTIONS.

What is the first law of motion ?
How is it proved to be true ?
What is the second law of motion ?
By what experiments is it established ?
What is the *composition* of *motion* ?
What is the third law of motion ?
By what experiments is it proved ?
What is meant, when it is said, that action and reaction are equal ?
Give an example.

If the pressures, which set two bodies in motion, are in the same proportion as the weights of the bodies, show that the *accelerating forces* are the same.

Why does a heavy body generally fall more quickly than a lighter body ?

How can it be shown that each would fall with the same rapidity, if all external causes of retardation were removed ?

Explain the construction of a machine, by which the third law of motion may be illustrated.

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## LESSON XVIII.

### ON UNIFORM MOTION.

It will be rembered that the motion of a body is uniform, when it describes equal spaces in all equal intervals of time.

Thus, if a body describes 5 feet in $1^s$, 10 feet in $2^s$, and so on continually, its motion is uniform.

### Proposition 46.

If two bodies move uniformly during different times, and with different velocities, the spaces described will be represented by the product of the numbers expressing the times and velocities in each case.

Suppose a body, A, moves uniformly with a velocity 5, for a time 7; and another body, B with a velocity 6, for a time 8.

Then the space described by A in each interval of time, will be 5; and the whole space, in 7 such intervals, $5 \times 7$, or 35.

In like manner, the space described by B is $6 \times 8$, or 48.

Ex. If one man walks at the rate of 3 miles an hour, for 5 hours; and another at the rate of 4 miles an hour, for 6 hours; the distance which each will travel respectively will be $3 \times 5$, or 15; and $4 \times 6$, or 24 miles.

If the *velocities*, with which two bodies move uniformly, are to one another in the same proportion as the *times* of their motion, the *spaces* described will be proportional to the product of the numbers representing the *times*, multiplied by those numbers themselves; that is, to the *squares* of the times: or again, to the product of the numbers representing the squares of the velocities.

Thus, if the velocities are 3 and 5, and the times 6 and 10, the spaces described will be $3 \times 6$, or 18, and $5 \times 10$, or 50; which are to one another in the proportion of 36 and 100, the squares of the *times*; or in the proportion of 9 and 25, the squares of the *velocities*.

## Proposition 47.

If the time during which a body moves is divided into equal intervals, and the velocity of the body is increased or diminished by the same quantity, at the *end* of each interval of time, but continues uniform *during* each interval, the *space* described is the same as if the body moved *uniformly during the whole time*, with a velocity which is equal to half the sum of the *greatest* and *least* velocities.

Suppose a body moves uniformly for a time 1, Half the sum of 7 and 11 is 9. Hence the whole space described in $12^s$ is $12 \times 9$, or 108 feet.

That this is really the case is easily seen by observing, that in the first $4^s$ the body describes $4 \times 7$, or 28 feet; in the next $4^s$, $4 \times 9$, or 36 feet; and in the last $4^s$, $4 \times 11$, or 44 feet; and the sum of the spaces, 28, 36, and 44, is 108 feet.

Ex. 3. A body moves for $99^s$; it describes 1 foot uniformly in the first second of time, 2 feet in the next second, and so on; describing 99 feet uniformly in the last second; required the whole space described.

Half the sum of 1 and 99 is 50: hence the whole space in $99^s$, is $50 \times 99$, or 4950 feet.

Ex. 4. Suppose a body moves for $100^s$, on the same supposition. Half the sum of 1 and 100 is $50\frac{1}{2}$. Hence the whole space is $50\frac{1}{2} \times 100$, or 5050 feet.

Ex. 5. Suppose that, as in the last example, a body moves for $100^s$; but that its velocity is increased at the end of the *tenth part* of each second of time: describing the tenth part of a foot uniformly in the first tenth part of a second, two-tenths of a foot uniformly in the next tenth of a second, and so on; so as to move with a velocity of 100 feet during the 100th second of time. Required the whole space described.

Half the sum of $\frac{1}{10}$ and 100 is $50\frac{1}{20}$. Hence, the whole space will be $50\frac{1}{20} \times 100$, or 5005 feet.

Ex. 6. Let the same supposition be made, except that the body changes its velocity at the end of the

100*th part* of each second of time; and let the whole space be required.

The velocity for the first interval of time is $\frac{1}{100}$; and, for the last interval of time, is 100.

And half the sum of $\frac{1}{100}$ and 100 is $50\frac{1}{200}$. Hence, the whole space will be $50\frac{1}{200} \times 100$, or $5000\frac{1}{2}$ feet.

By comparing the results of examples 4, 5, 6, it appears that, by dividing the time during which the velocity is continued uniform, into parts less and less, the *whole time* of motion $100^s$, and the *last acquired velocity*, 100 feet, *being the same*, the whole space described approaches nearer and nearer to 5000, which is the product of $50 \times 100$; 50 being half the greatest velocity, and 100 representing the whole time.

## Proposition 48.

If the body moves as before, except that its motion during the first interval of time is o, that is, if the body does not *begin to move* till the beginning of the second interval of time, and then moves uniformly, its velocity being increased by the same quantity at the end of each interval of time, the space described will be the same as if the body had moved during the *whole time*, with half the greatest velocity.

For, in this case, the *least* velocity is o; and therefore, half the sum of the greatest and least velocities is half the *greatest* velocity.

Example 1. Suppose one man, A, sets out at 12 o'clock at noon, and travels uniformly at the rate of 6 miles an hour; another man, B, sets out from the same place at 1 o'clock, and travels till 2 o'clock, at the rate of 2 miles an hour; from 2 o'clock to 3, at the rate of 4 miles an hour; from 3 o'clock to 4, at the rate of 6 miles an hour, and so on; increasing his rate of travelling every hour; how far will he travel before he overtakes the first?

B will have described the same space as A, who travels *uniformly* at the rate of *six* miles an hour, when B has travelled the *last* hour at the rate of *twelve* miles an hour.

And since his rate of travelling in each hour, reckoning *from noon*, is 0, 2, 4, 6, 8, 10, 12, the number of hours from noon will be 7; or he will overtake the first traveller at 7 o'clock in the afternoon.

This is easily shown to be true, by observing that, in 7 hours, A will have passed over $6 \times 7$, or 42 miles; and B will have passed over a number of miles represented by the sum of 2, 4, 6, 8, 10, 12, which is also 42.

Ex. 2. Suppose one man sets out at 12 at noon, travelling at the rate of 6 miles an hour; and another sets out at *half-past* 12, travelling for *half an hour* at the rate of 1 mile an hour, and increasing his velocity each *half hour*, at the rate of 1 mile an hour; how far will he travel before he overtakes the first?

The second man's rate of travelling, for the first half-hour, is 0; and his rate, for the last half hour, must be 12, as before.

Also his rate of travelling for each *half hour*, reckoned from noon, being 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; the number of half hours, reckoned from noon, is 13.

Hence, he will overtake the first traveller at *half-past* 6 o'clock.

This also may easily be shown to be true, as in the last example. Since the first man travels 3 miles in every *half hour*, in 13 half hours he will have travelled 39 miles.

The distance travelled by the second man, in each successive *half hour*, reckoned from noon, will be as follows:—

| Half-hours. | Miles travelled. |
|---|---|
| 1 | 0 |
| 2 | $0\frac{1}{2}$ |
| 3 | 1 |
| 4 | $1\frac{1}{2}$ |
| 5 | 2 |
| 6 | $2\frac{1}{2}$ |
| 7 | 3 |
| 8 | $3\frac{1}{2}$ |
| 9 | 4 |
| 10 | $4\frac{1}{2}$ |
| 11 | 5 |
| 12 | $5\frac{1}{2}$ |
| 13 | 6 |
| Total distance .. | 39 miles. |

## QUESTIONS.

Determine the space described by two bodies moving uniformly during different times, and with different velocities.

If the time during which a body moves is divided into equal intervals, but the motion is uniform during each interval of time, and the velocity is increased or diminished by the same quantity at the end of each interval of time, how may the whole space described be determined ?

## LESSON XIX.

### ON THE MOTION OF BODIES UNIFORMLY ACCELERATED OR UNIFORMLY RETARDED IN A STRAIGHT LINE.

The motion of a body is said to be *uniformly accelerated*, when the *increase* of its velocity is proportional to the time of its motion; and to be *uniformly retarded* when the *diminution* of its velocity is proportional to the time of its motion.

Thus, if a body moves from rest, and is found to have acquired a velocity of 5 feet in $1^s$, of 10 feet in $2^s$, of 15 feet in $3^s$, and so on for any proportional time, its motion is uniformly accelerated. And if a body begins to move with a velocity of 15 feet in a second, and, after $1^s$, is found to be moving with a velocity of 10 feet; and after $2^s$, to be moving with a velocity of 5 feet; and after $3^s$, to be brought to a state of rest; its motion is uniformly retarded.

### Proposition 49.

If a body moves from a state of rest, by the action of a uniformly accelerating force, the velocity generated is proportional to the time of the body's motion.

Accelerating force is measured by the velocity which it produces in a given time; and if it acts *uniformly*, it must add equal quantities of velocity to the body, in all equal portions of time.

Hence, if in a time 1, a certain velocity be communicated, in a time 2, the velocity will be twice as great; in a time 3, three times as great; and so on for all proportional intervals of time.

Hence, the velocity generated is proportional to the time.

If the time is divided into *equal intervals*, the velocities acquired at the end of each interval of time, will be in *arithmetical progression.*

Thus, if the times are 1, 2, 3, 4, &c., and the velocity at the end of the first interval of time is 32, the velocities at the end of the 2nd, 3rd, &c., intervals of time, will be 64, 96, 128, &c., which are in arithmetical progression.

If the force is represented by the *velocity* which it generates in a body moving from a state of rest, in $1^s$, the velocity generated in any other time, will be represented by multiplying together the numbers expressing the *force* and the *time.*

Thus, if a force generates a velocity 32, in $1^s$, the velocity generated in $12^s$ is $12 \times 32$, or 384.

Hence, generally, the *velocity* generated is the product of the *force* multiplied by the *time.*

Ex. 1. Suppose a body, falling from rest for $1^s$, acquires a velocity of 32 feet. Required the velocity acquired in falling for $10^s$.

Since the velocity is proportional to the time, the velocity acquired in $10^s$ is 10 times as great as the velocity in $1^s$; and therefore is 320 feet.

Ex. 2. On the same supposition, how long must the body fall from rest, to acquire a velocity of 640 feet?

The time, in seconds, will be found by dividing the whole velocity by the velocity acquired in $1^s$; and therefore the time is 640 divided by 32, or $20^s$.

This property may be shown experimentally, by the simple machine described in page 206.

Suppose the weights, A and B, to be equal; and that a weight *a* is added to A, and causes A to descend.

Observe how far A descends from rest in $1^s$; and let the moveable ring N be placed so as to take off the weight *a* at the end of $1^s$.

The system will then continue to move uniformly, according to the first law of motion; and if a stage be placed so that A may just reach it at the end of the next second of time, the distance which A will thus move *uniformly* in $1^s$, will measure the *velocity* which the system had acquired in $1^s$, and this velocity will therefore be known.

Now let the weights be replaced in their first position, and let the space through which A descends from rest, in *two seconds*, be observed; and the moveable ring S, placed so as to intercept the weight *a*, at the end of $2^s$.

Then, the space which A will describe uniformly in the next second of time, will be found to be *twice as great* as in the first instance; or the system, in $2^s$, will have acquired a velocity which is *double* of the velocity acquired in $1^s$.

And, by repeating the experiment for $3^s$, $4^s$, &c., it will be found that the velocity acquired is proportional to the time of falling from rest.

### Proposition 50.

If a body is projected with a given velocity in the direction *opposite* to that of a uniformly accelerating force, the velocity *lost* is proportional to the time.

For the force, acting uniformly, must produce equal effects in equal times; and those effects are the *addition* of velocity, *in the direction in which the force acts*, which is the same as the *destruction* of velocity in the direction in which the body is projected.

Ex. 1. Suppose a body is projected with a velocity of 320 feet, in the direction opposite to that of a uniformly accelerating force, which would generate a velocity of 32 feet in $1^s$.

Required the velocity with which the body is moving at the end of each successive second, until all the velocity is destroyed.

The velocities *destroyed* at the end of 1, 2, 3, &c., seconds, are 32, 64, 96, &c., feet.

And these are taken from the original velocity of projection, 320 feet, with which the body would continue to move uniformly, by the first law of motion, if it were not acted upon by any force.

Hence we may set down the velocities in the following table :—

| Time of Motion. Seconds. | Velocity lost. | Remaining Velocity. |
|---|---|---|
| 0 | 0 | 320 |
| 1 | 32 | 288 |
| 2 | 64 | 256 |
| 3 | 96 | 224 |
| 4 | 128 | 192 |
| 5 | 160 | 160 |
| 6 | 192 | 128 |
| 7 | 224 | 96 |
| 8 | 256 | 64 |
| 9 | 288 | 32 |
| 10 | 320 | 0 |

It appears, then, that at the end of $10^s$, the whole velocity will be destroyed; and, if the body continues to be acted upon by the accelerating force, it will descend again, acquiring a velocity of 32 feet in every successive second of time.

## Proposition 51.

If a body moves from rest by the action of a uniformly accelerating force, the space described from rest is half the space which would be described in the same time, with the last acquired velocity continued uniform.

If the time is divided into *equal intervals*, of any magnitude whatever, the velocities at the ends of those times will be in arithmetical progression, by Prop. 49, p. 220.

And if the motion commences at the beginning of the *second* interval of time, and is afterwards uniform during each successive interval, the whole space described will be equal to that which would be described in the whole time with half the greatest velocity acquired, by Prop. 48. And this conclusion will be true, whatever be the *magnitude* of the equal intervals of time.

And if the intervals of time are taken continually less and less, the time *before* the beginning of the body's motion will be continually less and less. And when those intervals are diminished without limit, the body will begin to move from a state of rest at the beginning of the time, and its motion will be uniformly accelerated.

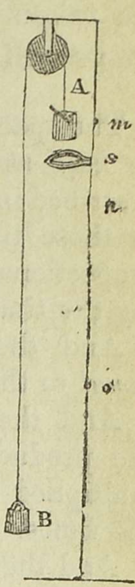
Therefore, in this case, the space described will be equal to that uniformly described in the whole time with *half* the last acquired velocity; and, consequently, will be *half* the space which would be described in the same time with that last acquired velocity continued uniform.

Example. If a body falls from rest for $1^s$, and describes a space of 16 feet, it will acquire a velocity which will carry it uniformly over 32 feet in $1^s$.

And generally the space described from rest is *half* the product of the numbers expressing the time and the velocity.

This proposition may be illustrated by an experiment, with the machine already described in p. 206.

Let the weights A, B, be equal; and the weight *a* be added, as in the experiment described in p. 213.




Let the moveable ring s be placed so as to intercept the weight *a* after any interval of time; and let the distance *m n*, through which the lowest part of A has moved in that time, be measured.

Then, if *n o* be measured equal to *twice m n*, and a stage be placed at *o*, the body A will be found to describe *n o* uniformly, after the weight *a* has been taken off, and to reach *o* in the same time in which it described *m n* from rest.

This experiment may be made with great accuracy, by permitting A to descend just as the pendulum of a clock beats. If *m n* is the space described in $1^s$, A will reach the stage *o* exactly at the second beat of the pendulum.

## Proposition 52.

If a body descends, by the action of a uniformly accelerating force, the spaces described in different times reckoned from the point of rest, are in the same proportion as the squares* of the

* The square of a number is the product of the numbe

time, or as the squares of the velocities acquired; that is if the *times*, and therefore the velocities, are taken as the numbers 1, 2, 3, 4, &c., the *spaces* will be as the numbers 1, 4, 9, 16, &c.

The spaces described from rest, in any time, will be *half* as great as the spaces which would be described in the same time, with the velocity acquired in those times, by Prop. 51, and therefore will be proportional to the products of the numbers expressing the times and velocities, by Prop. 46.

And those *velocities* will be themselves *proportional* to the *times*, by Prop. 49.

And therefore the spaces will be proportional to the product of the numbers expressing the *times* multiplied by *themselves;* that is, to the *squares* of the times.

And the velocities being proportional to the times, the spaces will also be proportional to the *squares* of the velocities. (See Prop. 46, p. 210.)

If the *forces* by which two bodies move are different, the velocities generated are proportional to the product of the *forces* multiplied by the *times.*

Hence generally, when the forces are different, the *spaces* will be proportional to the product of the *forces* multiplied by the *squares* of the times.

For instance, suppose the forces are $\frac{1}{3}$ and $\frac{1}{2}$;

multiplied by itself. Thus the square of 1 is $1 \times 1$, or 1: the square of 2 is $2 \times 2$, or 4: the square of 3 is $3 \times 3$, or 9.

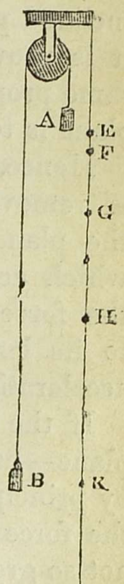
and the times 3 and 2: then the spaces will be as $\frac{1}{3} \times 9$ to $\frac{1}{2} \times 4$ or as 3 to 2.

If the space described in a time 1 is known, the space in any other time is known by multiplying that space by the square of the time.

EXAMPLE. Suppose a body descends from rest 16 feet in $1^s$, and therefore acquires a velocity of 32 feet, by Prop. 51.

Then, in $2^s$ it will acquire a velocity of 64 feet, by Prop. 49; and, if not further accelerated, it would go on to describe 64 feet uniformly in $1^s$, or 128 feet in $2^s$, the whole time of its falling.

And the space described from rest will be *half* this last space, or 64 feet. Again, in $3^s$, the velocity acquired is 96 feet.




The space which would be described uniformly with this velocity in 3 is 288 feet; and the space described from rest is the half of 288 or 144.

In the same manner we may proceed for 4 or more seconds.

Hence, the spaces from rest in the times, $1^s$, $2^s$, $3^s$, are 16, 64, 144, &c., which are in the proportion of 1, 4, 9, &c., or as the squares of the times.

This proposition also may be rendered evident by experiment.

EXPERIMENT 1. Suppose that two equal weights A and B are suspended on a fixed pulley, in the manner described in p. 21, and another weight is added to A, so as to set the whole in motion.

Let the space, E F, through which A descends in $1^s$ be observed.

Then, if E G be taken equal to 4 times E F; E H equal to 9 times E F; E K equal to 16 times E F; and the system be allowed to move till A reaches K, it will be found that A is at F, at the end of $1^s$, at G, at the end of $2^s$; at H, at the end of $3^s$: and at K, at the end of $4^s$.

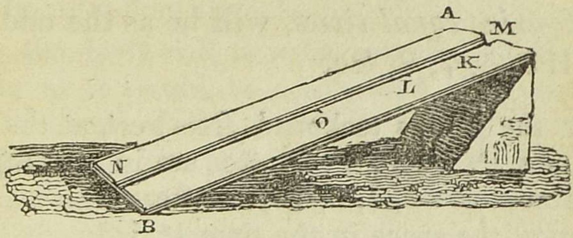
The spaces, therefore, are proportional to the squares of the time from rest.

EXPERIMENT 2. If a body is sustained upon an inclined plane, by a force acting parallel to the plane, it is proved in Prop. 28, that the power is in the same proportion to the weight, as the height of the plane is to its length.

Hence, if the power which sustained the body is removed, the body will begin to slide down the plane, if there is no friction; and the force which accelerates the body, will be constantly to the force of gravity as the height of the plane is to its length; and will therefore be a uniformly accelerating force.

If the body be of such a form as to *roll* down the plane—as a sphere or a cylinder,—it can be proved, by principles which cannot be here introduced, that the force is still a uniformly accelerating force, but not so great as when the body slides; being only $\frac{5}{7}$ of that force for a sphere, and $\frac{2}{3}$ for a cylinder.

The following experiment, however, depends only upon the circumstance that the body is uniformly accelerated, and not upon the actual amount of the accelerating force.




Let a smooth groove, M N, be cut upon a plane A B, of a given length, as 32 inches; and let a spherical ball, of brass or iron, be permitted to roll down the groove from M to N, when the plane is inclined, by raising the end A M.

Let the plane be inclined at such an angle, that the whole time of the sphere's rolling down the plane is a certain number of seconds, for instance, $4^s$.

Then, if M N be so divided that M K, M L, M O, &c., are as the squares of the numbers 1, 2, 3, &c., or 1, 4, 9, 16, &c., the sphere will be observed to be at the points K, L, O, N, at the end of $1^s$, $2^s$, $3^s$, $4^s$, respectively.

If M N is 32 inches, M K will be 2 inches; M L 8 inches; and M O 18 inches.

This was the kind of experiment by which Galileo first established the laws of bodies falling by the action of a uniform force.

### Proposition 53.

If a body moves from rest by the action of a uniformly accelerating force, the spaces described, in *successive equal times*, will be as the odd numbers 1, 3, 5, 7, 9, &c.

For the spaces reckoned *from rest*, at the end of the times $1^s$, $2^s$, $3^s$, $4^s$, $5^s$, &c., are in the proportion of the numbers 1, 4, 9, 16, 25, &c.

Hence, the space in the time $1^s$, is 1.

The space described in the next second is the difference of 4 and 1, or 3.

The space described in the third second, is the difference of 9 and 4, or 5. And so on continually.

Hence the spaces are as 1, 3, 5, 7, &c.

Again, if the intervals of time are taken, each $2^s$, the spaces from rest, at the end of $2^s$, $4^s$, $6^s$, &c., are 4, 16, 36, &c.; and the spaces in each equal interval of $2^s$, are 4, 12, 20, &c.; which are to one another in the proportion of 1, 3, 5, &c.

And the same may be shown, whatever equal intervals of time are taken.

### Proposition 54.

If a body is projected in a direction *opposite* to that of a uniformly accelerating force, and moves till its whole motion is destroyed; the spaces described in successive equal times will be as the odd numbers, 3, 1: 5, 3, 1: 7, 5, 3, 1: &c.: the numbers of intervals varying according to the time which is requisite to destroy the whole motion.

For the body will now be retarded, in the same manner as it would be accelerated, if it moved in the direction of the uniform force; the spaces described from the beginning of one motion, being the same as those described in equal times from the end of the other motion.

## Proposition 55.

If a body is projected in the direction of a uniformly accelerating force, the space which it describes in a given time is equal to the space which it would describe in the given time uniformly with the velocity of projection, together with the space through which it would fall in the same time by the action of the uniformly accelerating force.

For, by the third law of motion, the effect produced by the constant force, is the same as if the body had no motion given to it by the force of projection. And the motions arising from the uniform force, and from the force of projection are in the same direction; and the spaces described will be the *sum* of the spaces, which the body would describe by the action of each force separately.

EXAMPLE. Suppose a body projected downwards with a velocity of 80 feet in $1^s$, into a pit 800 feet deep; the force of gravity being represented by 32 feet: required the place of the body at the end of $1^s$, $2^s$, $3^s$, &c.

The spaces in successive equal times, when a body falls by gravity, will be 16, 48, 80, &c., by Prop. 53.

In $1^s$ the body would describe 80 feet with the velocity of projection; and 16 feet by the force of gravity.

Hence, in the first second, it will describe the sum of those spaces, or 96 feet.

In the next second, the sum

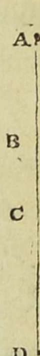
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | of the spaces is | . | 80 | and | 48, | or | 128 |
| „ | third, | . | 80 | „ | 80, | „ | 160 |
| „ | fourth, | . | 80 | „ | 112, | „ | 192 |
| „ | fifth, | . | 80 | „ | 144, | „ | 224 |

And the sum of all these spaces is 800 feet.

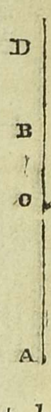
Hence, the body will reach the bottom of the pit in $5^s$; whereas, if it had fallen by the action of gravity alone, the space which it would have described would have been $16 \times 25$, or 400, only half as great as the space it actually describes. The other 400 feet is the space which it would have described uniformly in $5^s$, with the velocity of projection 80.

If a body is projected *downwards*, or in the direction in which a uniformly accelerating force acts upon it, it will continually descend, and it will be found at a given distance below the point of projection, only at one instant of time.

For instance, if a body is projected from A towards D, and acted on by a uniform force in the same direction; and if A B is the space through which the body would move in a given time, with the velocity of projection, and B C is the space through which it would move in the same time, from rest, by the action of the uniform force, the body, at the end of the same time, will be found at C, having described the space A C, equal to the *sum* of the two spaces, A B, B C.



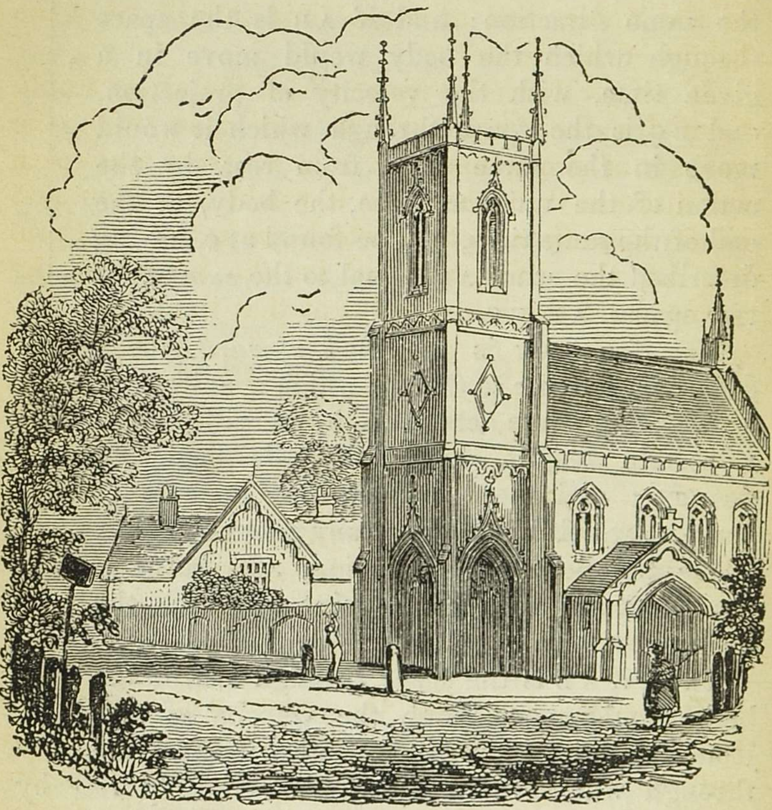

But if a body is projected *upwards*, or in the direction *opposite* to that in which a uniformly accelerating force acts upon it, it will rise to a certain height, and afterwards begin to descend; and there will be *two* instants of time, at which it will be at any given distance, above the point of projection, and between that point and the highest point to which the body rises.




Thus, if A B is the space through which the body would move, with the velocity of projection continued uniform, and B C the space through which the body would fall from rest by the action of the constant force, in the same time;

at the end of the time, the body will be found at the point C, at the distance A C from A, A C being the *difference* of the two spaces A B, B C.

And if D is the highest point to which the body rises, it will be at the height C, at two different times, first as it rises, and again as it descends.



Motion of a Body uniformly accelerated or retarded.

For instance, suppose a boy stands at the foot of a tower 144 feet high, that there is a window 128 feet from the bottom of the steeple, and that he shoots up an arrow with a velocity of 96 feet, gravity being represented by 32 feet (that being the velocity which it generates in $1^s$); and that it is required to find whether the arrow will rise as high as the top of the tower, and at what time it will be at the height of the window.

If we set down the space which the arrow would have described uniformly in any time, with the velocity of projection, and subtract from that space the distance through which it would fall from rest in the same time, both of which are known by Prop. 46, 52, the difference will be the height of the arrow above the lowest point at that time.

| Time. Seconds. | Space with uniform Velocity. | Space described from Rest, by Gravity. | Differences of the first spaces, or Height above the lowest point. |
|---|---|---|---|
| 1 | 96 | $1 \times 16$ or 16 | 80 |
| 2 | 192 | $4 \times 16$ or 64 | 128 |
| 3 | 288 | $9 \times 16$ or 144 | 144 |
| 4 | 384 | $16 \times 16$ or 356 | 128 |
| 5 | 480 | $25 \times 16$ or 400 | 80 |
| 6 | 576 | $36 \times 16$ or 576 | 0 |

It appears, then, that the arrow will have come to the ground again in 6 seconds, three of which will be occupied in its ascent, and three in its descent: that it will *just reach* the top of the tower, 144 feet, at the end of $3^s$; and that it will pass the window, 128 feet from the ground, at the end of $2^s$ in its ascent, and again at the end of $4^s$, in its descent.

## Note.

The properties of motion, when a body is acted on by a uniformly accelerating force, can be best understood, when expressed in general terms. Thus, if $v$ represents the velocity generated by a body falling from rest, during a time $t$, and $f$ represents the *force*, or the *velocity* which the force will generate in a time $t$;

By Prop. 49, $v = ft$. (1.)

That is, to find the velocity, multiply the force by the time.

By Prop. 51, 52, $s = \frac{1}{2} ft^2$. (2.)

That is, to find the space, multiply *half* the force by the square of the time.

And, since $t = \dfrac{v}{f}$, $s = \frac{1}{2} f. \dfrac{v^2}{f'}$

$$= \tfrac{1}{2} \quad \frac{v^2}{f},$$

Therefore $v^2 = 2fs$. (3.)

That is, to find the square of the velocity, multiply *double* the force by the space described from rest.

Also from (1) $t = \dfrac{v}{f}$, or the time is found by dividing the velocity by the force.

Also, from (2) $t^2 = \dfrac{2s}{f}$, or the square of the time is found by dividing double the space described from rest by the force.

Example 1. Find the velocity generated by gravity (32 feet), in $12^s$.

By (1) $v = 32 \times 12 = 384$.

Example 2. Find the space described from rest in the same time.

By (2) $s = 16 \times 12^2 = 16 \times 144 = 2304$ feet.

Example 3. Find the velocity acquired in falling by gravity through 2304 feet.

By (3) $\quad v^2 = 64 \times 2304$

$= 64 \times 16 \times 144$

Therefore $v = 8 \times 4 \times 12$

$= 384.$

---

QUESTIONS.

When is the motion of a body said to be uniformly accelerated, or uniformly retarded?

Show that if a body is uniformly accelerated, and moves from a state of rest, the velocity generated is proportional to the time; and is represented by the product of the numbers expressing the force and the time.

Show that if a body is projected in a direction opposite to that in which a uniformly accelerating force acts, the velocity lost is proportional to the time.

Prove that, if a body moves from rest by the action of a uniformly accelerating force, the *space* described from rest is half the space which would be described in the same time with the last acquired velocity continued uniform.

Prove that, on the same supposition, the spaces from the beginning of the motion are proportional to the *squares* of the numbers expressing the *times* of motion, or to the *squares* of the numbers expressing the velocities: and that the spaces described in equal *successive times* are as the odd numbers, 1, 3, 5, 7, &c.

By what experiments can the last four questions be shown to be true?

How can the space be determined, which a body describes when it is projected either in the *same* direction, or in the *opposite* direction to that in which a uniformly accelerating force acts?

On the same supposition, how can we find the instant of *time*, at which the body so projected is at a given distance from the point of projection?

# LESSON XX.

## ON THE MOTION OF BODIES DOWN INCLINED PLANES, AND UPON CURVED SURFACES, AND ON OSCILLATING BODIES.

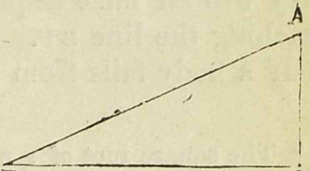
### Proposition 56.

The velocity acquired in falling down an inclined plane, is the same as that acquired in falling freely by gravity down the perpendicular height.

By Prop. 52, and by the note at the end of Lesson xix, it appears that, if a body moves by the action of a uniformly accelerating force, the *square* of the velocity is found by multiplying *double* the *force* by the *space* described.

Hence, if the *force* is *diminished* in the same proportion as the *space* is *increased*, the velocity acquired is the same.

Now, if one body falls freely by gravity from A to C, it describes the *space* A C.




And if another body falls down the plane A B, it describes A B, which is *greater* than A C; but the *force* which accelerates the body, is *less* than the force of gravity, in the proportion of A C to A B, by Prop. 28.

Hence, the velocity acquired by the body in falling down the plane from A to B, is the same as that acquired in falling freely from A to C.

For example: suppose A C is 16 feet, and A B 64 feet, or 4 times as great as A C; and therefore the force on the plane is $\frac{1}{4}$ the force of gravity, which is represented by 32 feet.

Now, the *square* of the velocity acquired in falling through A C, or 16 feet, is $2 \times 32 \times 16$, or 1024; and therefore the velocity itself is the *square root** of 1024, or 32 feet.

And the *square* of the velocity acquired in falling through A B, or 64 feet, on the plane, the force being $\frac{32}{4}$, or 8, is $2 \times 8 \times 64$, which is also 1024; and the velocity itself 32 feet, as before.

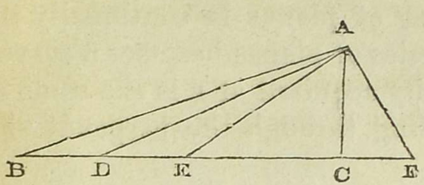
Hence, if any number of bodies fall down different inclined planes, all having *the same altitude*, the bodies will all acquire the *same velocity* at the lowest point of their several descents.

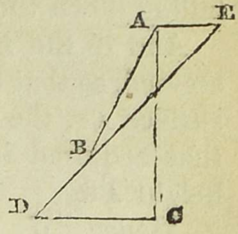
Thus, if A C is 16 feet, and B C F horizontal, and bodies fall from A down the planes A B, A D, A E, A F, they will all have acquired a velocity of 32 feet, on reaching the line B F.

If a body falls from rest down the plane A B, and

* The square root of a number given is such a number as, when multiplied by itself, produces the number given. Thus, since $1 \times 1$ is 1, 1 is the square root of 1; since $2 \times 2$ produces 4, 2 is the square root of 4; so 3 is the square root of 9; 4 the square root of 16.

then goes on to describe B D, *without any loss of velocity*, and continues to fall down the plane B D, it will have acquired at D a velocity equal to that acquired in falling from A to C.

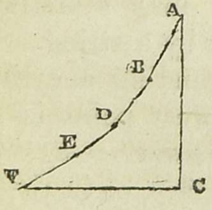


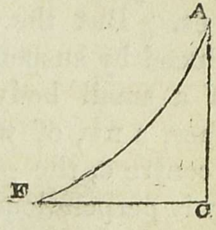




Let D B be produced to meet the horizontal line A E in E. Then, the velocity acquired in falling through A B is equal to that acquired in falling through E B, which is of the same altitude.

And no velocity being lost at B, the body goes on to describe B D, in the same manner as if it had fallen through E B.

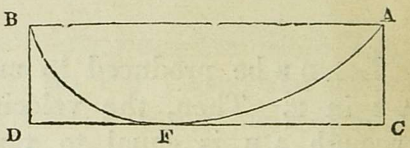
Hence, the velocity acquired at D, is that which would be acquired in falling through E D; which is equal to that acquired in falling through A C, the perpendicular height.



In the same manner, if a body descends down any number of planes A B, B D, D E, E F, if no velocity is lost in passing from one plane to the other, the velocity acquired at F is equal to that acquired in falling down the perpendicular height A C.

And if the number of planes is continually increased, so that the series of planes becomes a curved surface A F, the velocity acquired at F is the same as that acquired in falling through the perpendicular height A C.

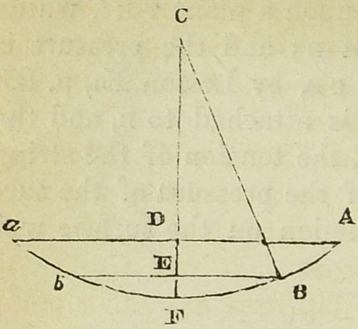



Hence, if two curves A F, B F, are united at their lowest point F, and have the same perpendicular altitude A C, B E, and a body falls down the curve A F, and loses no velocity in passing from one curve to the other, it will rise up the curve F B to the point B, which has the same perpendicular altitude as that from which the body fell.

It is not very easy to make experiments upon bodies sliding, or even rolling, upon curved surfaces, in consequence of the friction and other disturbing causes. But the same effect may sometimes be produced by suspending a body by a string.

If a small body, B, is descending a cylindrical surface, A B F, of which the lowest point is F, and the centre C, the *pressure* of the surface against the body is perpendicular to the surface, or in the direction of the radius B C. For it will be supported for

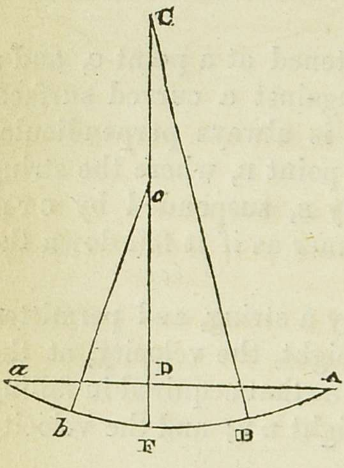
Motion on a Curve.




with which the body is moving at any point B is the same as that acquired in falling through the perpendicular height D E.

Also, if the body begins to fall from the point A, and is permitted to ascend again, on the other side of the lowest point F, it will rise to a point *a*, which is at the same perpendicular height above F as A is: and the velocity of the body at any point, *b*, of its ascent, will be the same as that which it had at the point B, at the same perpendicular height above F, in its descent: the effects of the resistance of the air being neglected.

This kind of motion is called *oscillation*.

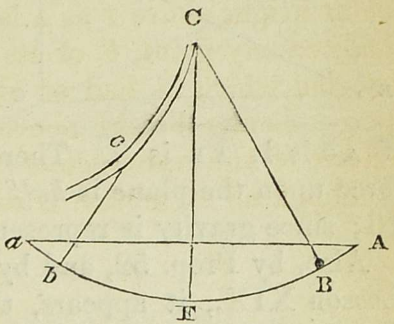



An easy experiment, first made by Galileo, shows that the velocity at the lowest point is the same, in different circles, provided the perpendicular height from which the body falls is the same.

Let a body, B, be suspended from the point C, and descend from A to F in the circular arc A F, of which the centre is C.

In the vertical line, C F, let a nail, *c*, be placed, against which the string C *c* F will rest, so that the body, after passing F, will rise up the circular arc F *b a*, of which the centre is *c*.

Then it will be found, that, if the body falls from A, it will rise in the arc F *a*, to the point *a*, which is at the same perpendicular altitude as A, above F. Or, if it begins to fall from *a*, it will rise to A. And if the body begins to fall from any other point B, it will rise to *b*, a point at the same perpendicular height.



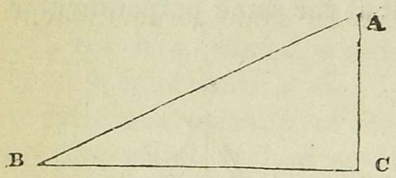

This experiment may be varied, by placing any smooth curved surface C *c* against which the string may be wrapped, after the body has reached the lowest point F, so that in rising again, on the side F *a*, it may describe some curve F *b a*, the nature of which will depend upon the curve C *c*.

And it will be found, that the point *a*, to which it rises, is always at the same perpendicular height above F as A.

In these experiments, the resistance of the air will cause some little variation; for which allowance must be made.

## PROPOSITION 57.

IF two bodies fall from rest, one down an inclined plane and the other down its perpendicular height, the time of falling down the plane s to the time of falling down the height in the same proportion as the length of the plane is to its height.



To show that this is true, we will take a particular case, and suppose the height of the plane, A C, to be half its length, A B: so that, if A C is 1, A B is 2. Therefore, by Prop. 28, the force upon the plane is *half* the force of gravity, or 16; since gravity is represented at 32.

Also, by Prop. 52, and by the note at the end of Lesson XIX., it appears, that when a body falls from rest, the *square* of the time is found by dividing *double the space* described, by the *force*.

Hence, the *square of the time* down the plane A B, or 2, is represented by the fraction $\frac{4}{16}$.

And the *square of the time* down the height A C, or 1, is represented by the fraction $\frac{2}{32}$, or $\frac{1}{16}$.

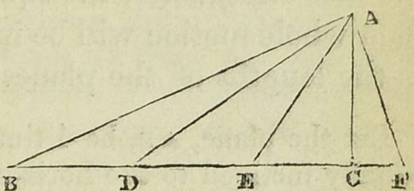
Hence, the *square* of the time of falling down the plane, is to the *square* of the time down the height as $\frac{4}{16}$ to $\frac{1}{16}$, or as 4 to 1.

And therefore the time down the plane is to the time down the height, as the *square root* of 4 to the *square root* of 1; or as 2 to 1; which is the proportion of the length of the plane to the height.

By using general terms, the same may be proved for any inclined plane*.

Hence, if different bodies fall down different planes, having the same perpendicular height, the times of falling down the planes will be in the same proportion as the *lengths* of the planes.

Thus, if A B is 5 feet, A D 4 feet, A E 3 feet, and A F 2 feet, the times of falling from A, down these planes, will be in the proportion of 5, 4, 3, and 2.




* Since $s=\frac{1}{2}ft^2$, $t^2=\frac{2s}{f}$.

And if $h$ is the height of a plane, and $l$ its length, and $g$ the force of gravity, $f=\frac{h}{l}.g$. and $s=l$.

Therefore $t^2=\dfrac{2l}{\frac{h}{l}.g}=\dfrac{2l^2}{gh}$. and $t=\sqrt{\dfrac{2}{gh}}.l$.

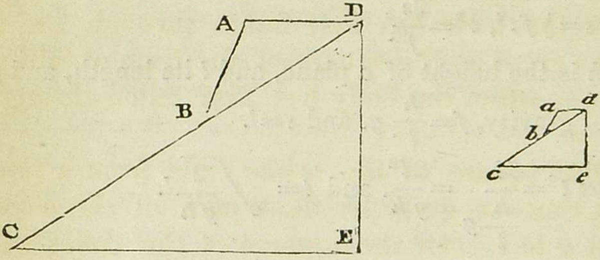
And if a body falls through $h$ by gravity, and $t'$ is the time, $t'^2=\frac{2h}{g}$. and $t'=\sqrt{\frac{2h}{g}}$.

Therefore $t:t'::\sqrt{\dfrac{2}{gh}}.l:\sqrt{\dfrac{2h}{g}}::l:h$.

## Proposition 58.

If two bodies descend from rest down two planes equally inclined to the horizon, and then, without any loss of velocity, proceed to descend down two other inclined planes, also equally inclined to the horizon, the lengths of which are to each other in the same proportion as the lengths of the first two planes, the squares of the times of their whole motion will be in the same proportion as the lengths of the planes.

Let the plane, A B, be 4 times as long as *a b*, and equally inclined to the horizon; and the plane, B C, be also 4 times as long as *b c*, and equally inclined to the horizon.




Then, the proportion of the height to the length being the same, in the planes A C, and *a c*, the accelerating force will be the same; and therefore, by Prop. 52, the *squares* of the time falling through

A B and *a b*, will be in the proportion of A B to *a b*, or as 4 to 1.

Also, if C B be produced to D, and *c b* to *d*, by Prop. 56, the velocities which the bodies have at B and *b*, will be the same as if the bodies had fallen through D B, *d b* respectively, which have the same perpendicular height as A B, *a b*.

And the bodies will describe B C, *b c* respectively in the same time, whether they have first fallen through A B and *a b*, or D B and *d b* respectively.

But if they fall through D B and *d b*, the square of the time of falling through D B from rest, will be 4 times as great as the square of the time of falling through *d b;* and also the square of the time of falling through D C *from* rest, will be 4 times as great as the square of the time of falling through *d c from* rest.

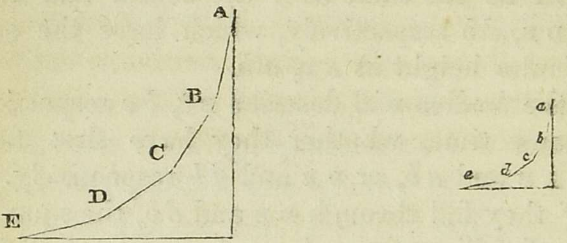
Hence, the square of the time of moving through B C, when the body has first fallen through A B, is 4 times as great as the square of the time of falling through *b c*, when the body has first fallen through *a b*.

And the square of the *whole time* from A to C, will be 4 times as great as the square of the *whole time* from *a* to *c*. Or the squares of the times will be to each other in the same proportion as the lengths of the planes.

Hence, the times themselves will be to each other as the *square roots* of the lengths of the planes.

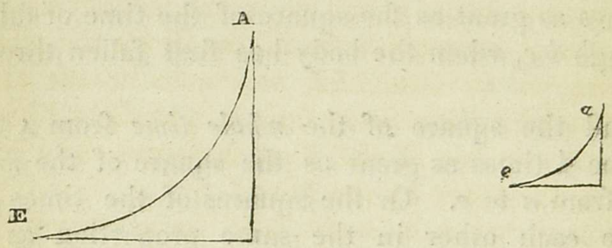
The same reasoning will apply to any number of planes similarly inclined.

Thus, if A B C D E, *a b c d e*, are two systems of planes, of which each part, as A B, is to the corresponding part, *a b*, as 9 to 4; and two bodies descend from rest, through each, no velocity being




lost in passing from one plane, as A B, to another, as B C, the time from A to E will be to the time from *a* to *e*, as the square root of 9 to the square root of 4, or as 3 to 2.

If the number of planes between A E and *a e*, are increased without limit, we shall have two similar




*curves;* and the times of falling down those curves will be in the proportion of the *square roots of their lengths.*

If a body, on reaching the lowest point of a series of inclined planes, or of a curve, were projected back with the velocity acquired, it would rise to the same point, A, in the same time as that in which it descended, if there were no friction or other extraneous cause, to prevent it so doing. And if a similar system of planes, or a similar curve, were placed on the other side of E, it would rise up the system of planes, or curve, after reaching E.

## Proposition 59.

If the force of gravity should be diminished or increased, the time of falling down a series of planes, or down a curve, will be *increased*, or *diminished*, in the same proportion as the *square root of the force is diminished*, or *increased*.

It is plain, that if the force of gravity is *diminished*, the body is not so much accelerated, and will take a longer time in falling down the planes.

Also, as in Prop. 52, and note at the end of Lesson XIX., if a body falls in a straight line from rest, the square of the time is found, by dividing double the space by the force.

Hence, when a body falls in a straight line, through any space, the *square* of the time is *increased* or *diminished* in the same proportion as the *force* is *diminished* or *increased*.

Hence, the *time itself* is *increased* or *diminished*, in the same proportion as the *square root* of the force is *diminished* or *increased.*

And, by proceeding as in the last proposition, the same relation can be shown to subsist, when bodies move down a series of planes, or down a curve.

For example:—Suppose two bodies descend from rest, down two planes of equal length, and similarly inclined to the horizon; the length of each plane being 16, and the height 8. Therefore, the force accelerating the body on the plane is $\frac{8}{16}$, or *half* the force of gravity, by Prop. 28, p. 86.

And if gravity, acting upon one of the bodies, is represented by 32, the force on the plane is 16. Hence, the *square* of the time of falling down the plane, is found by dividing twice the length of the plane, or 16 by 16; or, the *square* of the time is $1^s$, and therefore the time itself is $1^s$.

Again, suppose that the force of gravity acting upon the other body, is by some means reduced so as to be one-fourth of 32, its former amount, or 8; that is, that the force is diminished in the proportion of 1 to 4, or the *square root* of the force diminished in the proportion of 1 to 2.

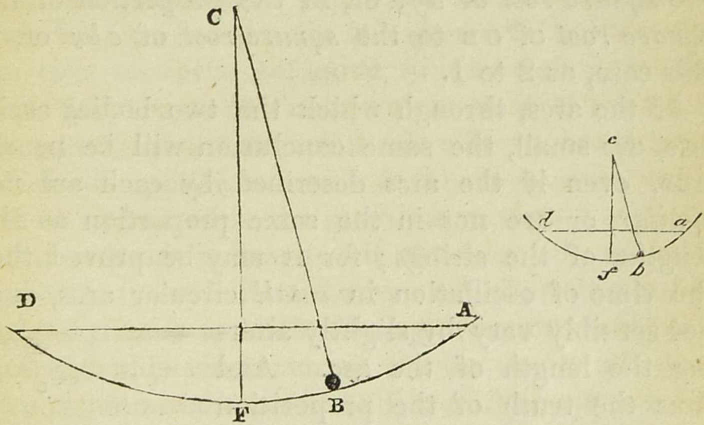
Then the accelerating force of the body on the plane would be one-half of 8, or 4.

And the *square* of the time of descending down the whole length of the plane, 8, will be 16 divided by 4, or $4^s$.

Hence, the time itself is $2^s$; or the time of descent is *increased* in the same proportion as the square root of the force is diminished.

## Proposition 60.

If two bodies, considered as points, *oscillate* in similar circular arcs, the times of their oscillation are in the proportion of the *square roots* of the lengths of the strings by which they hang.



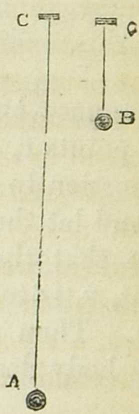

Suppose two bodies B, *b*, to be suspended from two points C, *c*, by strings C B, *c b*, which are in any proportion to each other; for instance, as 4 to 1.

And let the body, B, be drawn up to A, and *b* to *a*, so that the arc, A F, measured from the lowest point, F, is to *a f*, as C B to *c b*, or as 4 to 1, in this case. Then A F, *a f*, are *similar arcs*. Now let each body descend, describing the circular arcs, A F, *af*, and rising to D, *d*, respectively, in the same time in which they descended through A F, *af*.

Then the times of the bodies moving from A to D, and from *a* to *d*, are called the *times of their oscillation*, and will be twice as great as the times of falling from A to F, and from *a* to *f*, respectively.

Now, by the last proposition, and Prop. 55, the times of falling from A to F, and from *a* to *f*, will be in the proportion of the *square root* of A F, to the *square root* of *af*, or, in the proportion of the *square root* of C B to the *square root* of *c b*; or, in this case, as 2 to 1.

If the arcs, through which the two bodies oscillate, are small, the same conclusion will be nearly true, even if the arcs described by each are not *similar*, or are not in the some proportion as the lengths of the strings; for it may be proved that the time of oscillation in *small* circular arcs, does not sensibly vary by slightly altering the length of the arc. And thus the truth of the proposition is capable of being easily verified by experiment.




Let C A be 4 times as long as *c* B; and let A and B each be made to oscillate.

Then it will be found, by counting the number of oscillations made by each in a given time, that B makes *two* oscillations, while A makes one oscillation; or that the *times* of each oscillation of A and B, are to each other as 2 to 1.

If C A is 9 times as long as *c* B, the time of A's oscillation will be three times as great as that of B.

If C A is 16 times *c* B, A will oscillate only once, while B oscillates four times.

And, by altering the lengths of the strings at pleasure, the same relation may be shown to subsist for any lengths of the strings.

If the *length* of a pendulum remains *the same*, and the force of gravity is *increased*, by Prop. 59, the *time* of each oscillation is *diminished*, in the same proportion as the square root of the force is *increased;* and therefore the *number* of oscillations which the pendulum will make in a given time, will be *increased* in the same proportion.

The reverse is the case, if the force of gravity is *diminished;* and since the number of oscillations in a given time is capable of being observed with great accuracy, this circumstance enables us to discover very small variations in the force of gravity, at different parts of the earth's surface.

For instance, if the force of gravity should be increased in the proportion of $(10{,}001)^2$ to $(10{,}000)^2$, or of 100,020,001 to 100,000,000, or of 10,002 to 10,000 nearly; the number of oscillations which a pendulum would make in a given time, would be increased in the proportion of 10,001 to 10,000. Hence, a clock, which kept true time, would gain one second in every ten thousand seconds; which would amount to $8{\cdot}64^s$, or more than $8\frac{1}{2}^s$ in $86{,}400^s$, or 24 hours.

The length of a pendulum which vibrates seconds, in the latitude of London, is found by experiment to be 39·1386 inches. The length at the equator is only 39·0117 inches; and that at the poles 39·2193 inches.

It has been shown, in p. 146, that the sensible gravity of a body increases, in passing from the equator towards the poles, in consequence of the centrifugal force arising from the rotation of the earth upon its axis. This effect is rendered very sensible by means of a pendulum. For, although the *time* of one oscillation cannot be accurately observed, the *number* of oscillations in a given time, as in 24 hours, can be readily ascertained, by comparing the number of oscillations made by the pendulum in a certain time, with the number made, in the same time, by a pendulum which vibrates seconds.

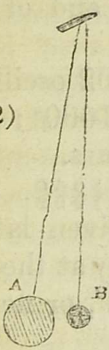
(1)

The best method of observing this, is by causing the two pendulums to be suspended, one before the other, so that when they both hang vertically, one may be seen exactly in the same line with the other, as in *fig.* 1.

Suppose that the larger pendulum, A, oscillates once in a second; and that the smaller, B, oscillates nearly, but not exactly, in the same time; and let each of the pendulums be set in motion.

At first, they will appear to keep nearly together; but after a little time they will separate, as in *fig.* 2, till that which vibrates the fastest, as B, has *gained* one whole oscillation on the other, A. They will then cross each other, at the lowest point, moving in *opposite* directions; and when at the highest points, will be as far apart as they can be, as in *fig.* 3.

(2)

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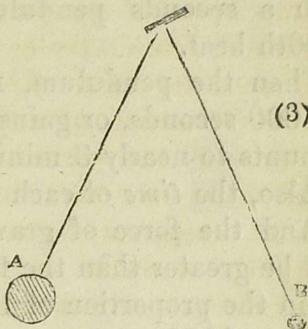



After that time, they will approach more and more nearly to each other, when at their lowest points, until the fastest pendulum, B, has gained *two whole* oscillations upon A, when they will again be together at the lowest point, moving in the *same direction.*

At that instant, the pendulum B will be observed at the lowest point, *exactly before* the pendulum A, as in *fig.* 1; and, although each is in motion, B will appear, as it were, to *hang* to A, for an instant, since each is moving in the same direction, and, sensibly, with the same velocity.

And the number of oscillations made by A, in the given time, being known, the number made by B in the same time, which will be two more than the number made by A, will be known.

For instance, suppose a pendulum, B, which vibrates seconds under the equator, is carried to another latitude, and there is found to be coincident with a seconds' pendulum, A, at the end of every 1000th beat.

Then the pendulum, B, makes 1002 oscillations in 1000 seconds, or gains $2^s$ in every $1000^s$; which amounts to nearly 3 minutes in 24 hours.

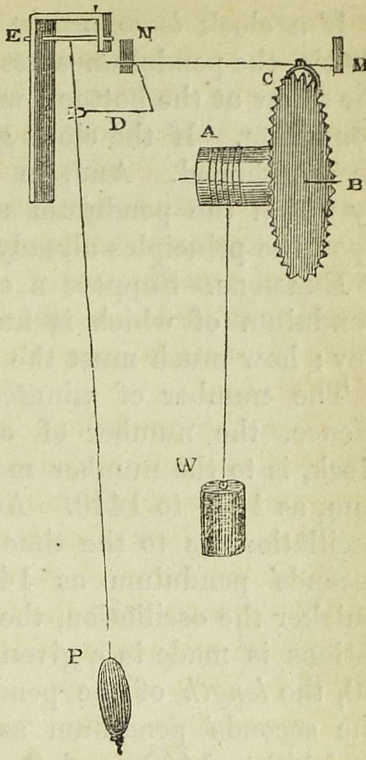
Also, the *time* of each oscillation is $\frac{1000}{1002}$.

And the force of gravity in the given latitude, will be greater than the force of gravity at the equator in the proportion of $1002^2$ to $1000^2$, or nearly as 1004 to 1000.

Experiments thus made are capable of great accuracy; and afford the easiest means of detecting small variations in the sensible force of gravity in different latitudes.

Since a pendulum of given length vibrates in a certain time, if we can connect a pendulum with machinery, as in a clock, it will regulate the rate at which the machinery moves.

Thus, if a weight, W, is attached to a string wound round a cylinder A, upon the same axis as a toothed wheel B, if the cylinder and wheel can revolve




freely, W will descend with a uniformly-accelerated motion.

But if a ratchet, C, be so placed as to be alternately elevated and depressed on each side by the action of the pendulum, P, suspended at E, and communicating its motion to C by means of the bent wire D attached to the axis M N on which C turns, *only one tooth* of the wheel B can escape, at each beat of the pendulum. And if the pendulum is sufficiently heavy, the descent of W will be checked at each beat of the pendulum, and the motion of the machinery will be rendered uniform.

Such a ratchet as C, is called a *scapement*. That represented above is one of the most common. But, among other defects, it interferes with the free motion of the pendulum itself.

In clocks which are required to keep time with great accuracy, a scapement of a different construction is employed.

If a clock *loses*, or the pendulum oscillates too slowly, the pendulum must be shortened, by turning the screw at the bottom, which raises the bob of the pendulum. If the clock *gains*, the pendulum must be lengthened. And, in either case, the quantity by which the pendulum must be altered depends upon the principles already laid down.

EXAMPLE. Suppose a clock, the length of the pendulum of which is known, gains a minute in a day; how much must the pendulum be lengthened.

The number of minutes in 24 hours is 1440. Hence, the number of oscillations made by the clock, is to the number made by a seconds' pendulum, as 1441 to 1440. And the time of *each* of its oscillations is to the time of each oscillation of a seconds' pendulum as 1440 to 1441; since, the quicker the oscillation, the greater number of oscillations is made in a given time. Hence, by Prop. 60, the *length* of the pendulum is to the length of the seconds' pendulum as $(1440)^2$ to $(1441)^2$, or, as 1440 to 1442 nearly*, or as 720 to 721.

Hence, the pendulum will require to be lengthened by a quantity equal to $\frac{1}{720}$ of its own length.

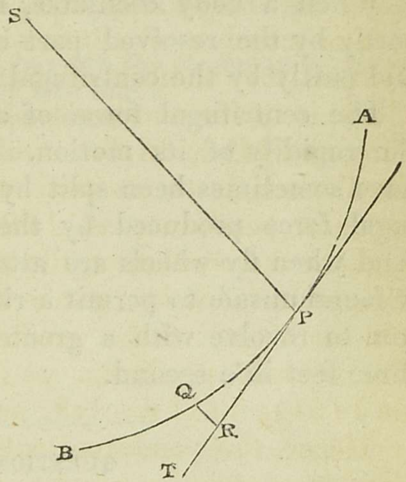
The principles upon which depend the motion

$$\begin{aligned} * \ (1440)^2 : (1441)^2 &:: (1440)^2 : (1440+1)^2 \\ &:: 1440^2 : 1440^2+2\times1440+1 \\ &:: 1440 : 1440+2+\tfrac{1}{1440} \\ &:: 1440 : 1442 \quad \text{nearly.} \end{aligned}$$

The principle of reduction here introduced is one of frequent use.

of a compound pendulum, that is, of one in which the oscillating body cannot be considered as a point, are too complicated to be here introduced.

When a body describes a straight line, it has no tendency to recede from that line. But if a body is caused to describe a curve, some force is necessary to retain it in that curve. For, suppose a body to be moving in the curve, A P Q B, from A to B, and that P T is a straight line which *touches* the curve at the point P. Then, at the instant when the body is at P, it is moving in the direction P T; and, by the first law of motion, it would go on to describe the line P R T, unless acted upon by some external force.




Suppose P R to be the space through which the body *would* move uniformly in a very small time, and P Q to be the space through which it *does* move in that time, then the space, R Q, is that through which it must be drawn, or turned aside from the straight line, by some force, in order that it may describe the curve, P Q.

If S P is a string to which P is suspended, and perpendicular to P R, then, if the string were cut at the instant when the body was at P, in the next instant the body would be found, not at Q, but at R, having *receded* or *fled*, as it were, from the centre S, through the space Q R. This tendency of a revolving body to recede from the centre, is called a *centrifugal force*. It must be remembered, however, that the body does not *exert* any force, matter being quite passive, but that it cannot be kept in motion in a curved line without constant constraint.

When a body oscillates, the string is stretched partly by the resolved part of the force of gravity, and partly by the centrifugal force.

The centrifugal force of a body increases with the rapidity of its motion. Ponderous mill-stones have sometimes been split by the enormous centrifugal force produced by their revolving too fast. And when fly-wheels are attached to machinery, it is found unsafe to permit a rim of the best malleable iron to revolve with a greater velocity than thirty-three feet in a second.

## QUESTIONS.

How is it proved that the *velocity* acquired by a body in falling down an inclined plane is equal to the velocity acquired in falling down its perpendicular height?

Is the same thing true of a curve?

By what experiments can this fact be established?

Prove that the *time* of falling from rest down an inclined plane is to the time of falling down its perpendicular height, as the length of the plane is to its height.

Show that if two bodies descend down two systems of planes similarly situated with respect to the horizon, the times of their falling to the lowest point are proportional to the square roots of the lengths of the planes.

If the force of gravity is diminished or increased, in what proportion is the time of falling down a given plane increased or diminished ?

Show that, if two bodies oscillate in similar circular arcs, the times of their oscillation are in the proportion of the square roots of the length of the strings by which they hang.

How can this be verified by experiment ?

If the force of gravity slightly varies, how may the variation be rendered sensible by means of a pendulum ?

How may a pendulum be employed to regulate the rate of a clock's motion ?

If a clock gains or loses, how can the pendulum be altered so as to make the clock keep true time ?

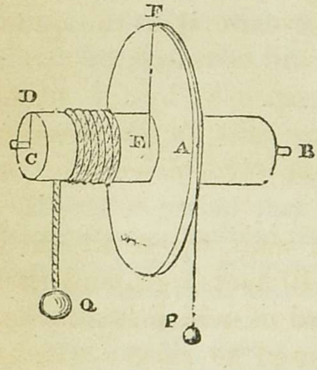
How is centrifugal force produced ?

## LESSON XXI.

### ON THE WORK DONE BY MACHINES, AND BY ANIMAL FORCE.

We have already investigated the general principles, upon which depends the proportion between weights, or forces of any kind, which *sustain* one another upon any machine. But, in the case of equilibrium, the machine, by which such forces act, is supposed to be kept at rest; or, if set in motion, as in Lesson XIII., to be moved by some force different from that of the forces sustaining each other.

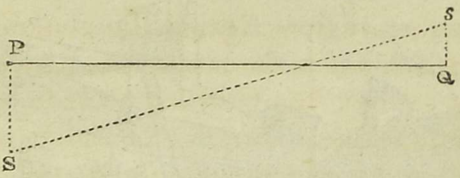
But the principal use of machinery is to *do work;* and this will be effected by causing one of the forces acting upon a machine to become greater than that which is sufficient simply to *sustain* another force.




For instance, in the wheel and axle, a weight, acting at P, will *sustain* the weight Q, if the weight bears the same proportion to Q, as the radius of the axle to the radius of the wheel. (Prop. 21, p. 62.) If a weight, P, greater than this, be suspended to the wheel, it will more than counterbalance the weight Q, and Q will be raised.

If P descends through any space, Q will be raised through any space which is to that described by P, as the radius of the axle to the radius of the wheel.

In this case, the pressure exerted by P, remains the same during the whole descent of P.

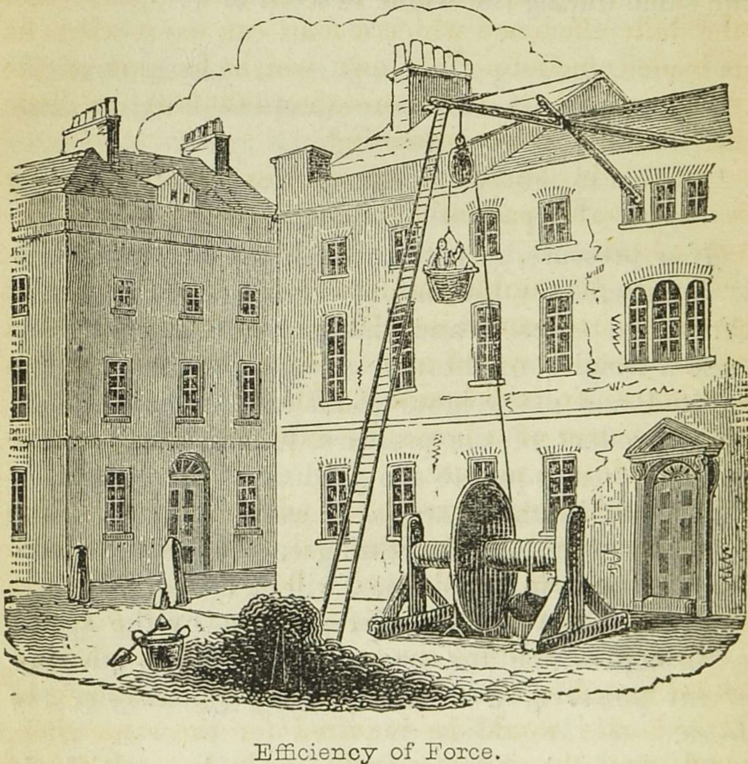



If a pressure, P, acts in such a manner, that it is not altered by the rate of working, and descends through a space, S, and this pressure is caused to raise a weight, Q, through a space, *s*, the pressure exerted, and the weight, would *balance* each other, if the product of the pressure, P, multiplied by the space, S, were equal to the product of the weight, Q, multiplied by the space, *s*. (Lesson XIII., p. 120.) And if the pressure be at all greater than the quantity sufficient to sustain Q, Q will be raised.

The product of such a pressure, P, by the space, S, through which the pressure acts, is the *measure of the power of a machine*, or *the efficiency of the force.*

For example:—Suppose a man, whose weight is 150 lbs., mounts a ladder 40 feet high, and then descends in a bucket, the cord of which is attached to any machinery, and is caused to raise a weight. The efficiency of the force employed, measured in pounds and feet, is $150 \times 40$, or 6000. And this force may be caused to raise a weight of 6 lbs.

through a space of 1000 feet; or a weight of 1000 lbs. through a space of 6 feet, or a weight of 6000 lbs. through a space of 1 foot.



Efficiency of Force.

It has been found that a man can ascend 10,000 feet in the course of a day. If we suppose that, the time employed in his descent is not taken from that employed in working, and that his weight is 150 lbs., his daily efficiency is $150 \times 10,000$, or 1,500,000.

Hence, a man so employed, would raise a million and a half of lbs., to the height of one foot, in a day.

By experiments made of the weights which porters can carry up a flight of stairs, it appears that the daily efficiency which a man can exert when he is loaded, including his own weight as part of the weight raised, is not more than 800,000; or little more than half the preceding.

In this instance, too, the man exerts ineffectually a considerable part of his force in descending without a load.

Under favourable circumstances, it is said that a good labourer can raise 370 lbs., 10 feet high, in a minute, which would give 3700 for the efficiency in a minute; or, in 8 hours, 1,776,000.

The power of a horse, by experiment, appears to be 22,000 in a minute.

The *estimated horse-power* adopted by engineers is greater than this by one-half, being such a force as will raise 33,000 through a height of one foot in a minute; or through 15,840,000 feet in 8 hours.

Hence, to produce the power of every *two* horses, as estimated in the effect of an engine, the labour of *three* horses would be required for the same time. And, since the engine may be made to work for 24 hours in the day, which would require three relays of horses, the power of every *two* horses as estimated, would be nearly equivalent to that of *nine* horses.

A small deduction must be made for the time of stoppage, and the necessary oiling and stuffing of the machine.

It must be observed, however, that, owing to the different form of a man and horse, the work which they can perform is very different, as their force is differently applied. The most advantageous way in which the power of a horse can be employed, is in draught; and perhaps the least advantageous is in carrying weights up a steep ascent.

A man's force, on the other hand, is advantageously employed in climbing; provided he has but a small weight to carry. A man can exert his force to the greatest advantage in the action of *rowing;* the muscles of his arms, his legs, and many of those of his body, being then all put in action.

The following appears to be nearly the relative force which a man of ordinary strength can exert in different methods of employing his muscular power.

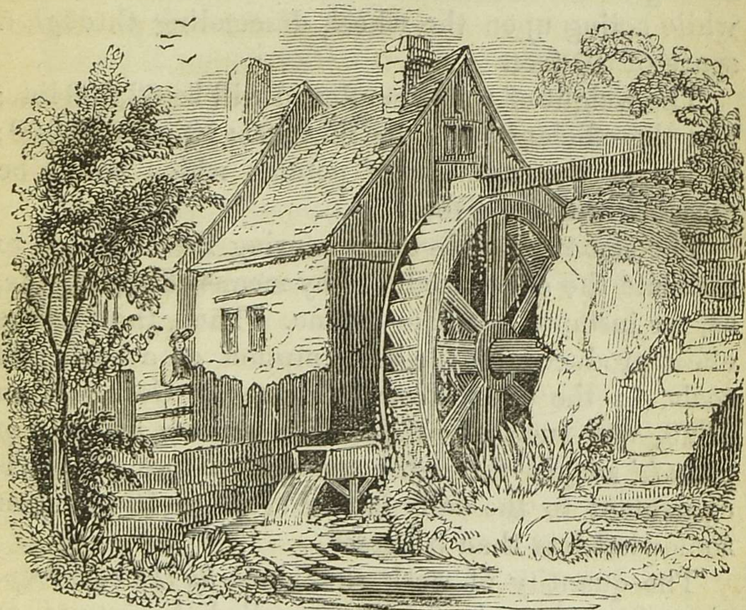
| | |
|---|---|
| Digging . . . | 5. |
| Turning a winch . | 6. |
| Ascending stairs . | 10. |
| Ringing . . . | 13. |
| Rowing . . . | 14. |

There is great difference in the estimate which different writers make of animal force.

One horse will not be able to draw up a steep hill so much as three men can carry up the same ascent. Whereas, a horse, drawing on level ground, will move a weight which seven men could not draw. In the first of these cases, the force of the horse is applied to the least advantage; and that of the men very advantageously. In the second, the force of the horse is applied advantageously, and that of the men, to the least advantage.

A man of ordinary power, who could *carry* a burden of 100 lbs. up a steep hill, could not exert a greater force than the pressure of 27 lbs., when *drawing* horizontally.

This shows the difficulty of comparing the force of men and other animals; as well as the importance of employing animal force in the most efficacious way.



Overshot Wheel.

If water falls upon a mill-wheel, so as to set it in motion by the *weight* of the water, the wheel is called an *overshot* wheel; and if the water descends through any space, the efficiency of the force may be computed in the same manner as above.

Hence, it is convenient to estimate any force, by the weight which it would raise through a space of *one foot:* since that weight would represent the efficiency of the force employed.

Thus, suppose a reservoir of water contains 1000 cubic feet, each foot weighing 62 lbs.; and that all the water is employed, without loss of force, in setting an overshot mill-wheel in motion, the water, while acting upon the wheel, descending through a space of 10 feet.

The weight of water employed will be 62,000 lbs.; and the efficiency will be $62{,}000 \times 10$, or 620,000; which is the number of pounds which could be raised through one foot.

The same principle of measuring force is used to estimate the effect produced by means of any engine, as, for instance, a steam-engine. Thus, the average effect produced by the consumption of a bushel of coals, in the Hual Towan engine in Cornwall, in 1829, was such as would raise *seventy millions of pounds' weight* through a space of a foot. This is equivalent to the daily labour of about 40 men; or somewhat less than 5 horses.

The quantity of work thus done *by the consumption of one bushel of coals,* is called *the duty of an engine.*

The duty of an engine can be computed, if we know the weight which it raises through a given space, by the consumption of a given number of bushels of coals.

EXAMPLE. Suppose an engine is found to raise 40,320 cubit feet of water from a depth of 120 fathoms, or 720 feet, by the consumption of 36 bushels of coals, and that one cubit foot of water weighs 62 lbs.

The weight, in pounds, raised through 720 feet is $40{,}320 \times 62$, or 2,499,840. This work being done by 36 bushels of coals, *one* bushel of coals would raise that weight through a 36th part of the same height, or through 20 feet.

Hence it would raise through *one foot* a weight 20 times as great, or $20 \times 2{,}499{,}840$, or 49,996,800 lbs.

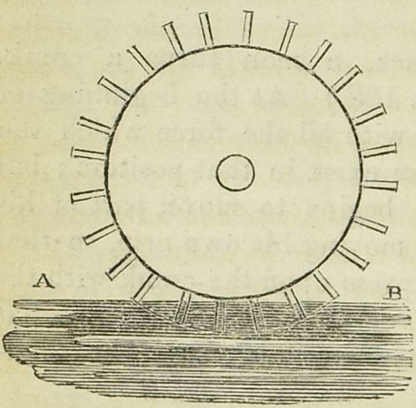
And the *duty* of the engine would be about 50 millions.

If a *weight* is employed to raise another weight, the pressure is not altered in consequence of the motion of the system. But in many kinds of force the pressure is very different in a state of rest, and in a state of motion.

Suppose, for instance, a man turns a crank. (See plate opposite p. 108.) At the beginning of the motion, he presses with all the force which the muscles of his arms can exert in that position; but as soon as the crank begins to move, part of his force is employed in moving his own arm, so that he can now no longer press upon the crank with the same force as at first. This is seen, in the familiar instance of turning a heavy grindstone. The grindstone itself, when once set in motion, continues to move with the velocity acquired, and at length it is caused to move round so fast, that the hand can only just keep up with the crank, and ceases to press upon it at all.

When horses draw a carriage, the force with which they draw, is diminished as they move with greater rapidity. Thus, if the utmost speed of a horse unloaded is 12 miles an hour, and he were placed in a carriage which was moving at that rate, all he could do would be to keep out of the way of the coach. Whereas, if the carriage moved at the rate of only 6 miles an hour, he would be able to exert some part of his force in drawing the coach, but not so much as he could exert *at a dead pull.*

Another familiar instance of the same kind occurs, when a stream of water is made to move an *undershot* wheel, upon which the water acts by its *impulse*, not by its *weight*, as in an *overshot* wheel, or in a *breast wheel*, in which the water falls upon *part* of the wheel, but is not brought *over* the wheel.




Suppose a stream of water to be flowing from A towards B: and that a wheel, with floats attached to its circumference, has its lower end immersed in the fluid. Suppose also that the stream flows at the rate of 3 miles an hour, and that, while the wheel is at rest, the pressure upon the floats immersed, tending to set the wheel in motion, is equal to some given weight, as 60 lbs., and is attached to machinery of such a nature that a weight of 900 lbs. exactly balances the pressure of the stream upon the wheel at rest.

Now suppose the machinery loaded with some less weight, and the wheel to be set in motion by the stream, and observe what change takes place.

As soon as the wheel begins to move in the direction of the stream, it is no longer acted upon by so great a force as when it was at rest. This is plain; for if the wheel moved at the same rate as the stream, or 3 miles an hour, it would move on *with the water*, and be neither accelerated nor retarded by the action of the water moving at the same rate. If the wheel moved with a greater velocity than the water, as the paddle of a steam-vessel does, for instance, it would *drive the water* in the direction of its own motion, and be retarded by the water instead of being urged forward by it.

But if the wheel moves in the same direction as the water does, but with a velocity less than that of the water itself, the pressure of the water on the floats of the wheel will become less and less, as the velocity of the wheel becomes more nearly equal to the velocity of the stream. Thus, if the velocity of the stream, as we have supposed, is 3 miles an hour, and the pressure on the floats, at rest, 60 lbs.; if the wheel have a motion of one mile an hour, the pressure of the water upon the wheel will be diminished by one-third, or will now be 40 lbs. If the wheel moves at the rate of two miles an hour, the pressure will again be diminished by one-third of its first amount, and will now amount to only 20 lbs.

In order to find the quantity of work done in such a case, it would be necessary to introduce principles different from any which we have hitherto established, and too complicated for our present purpose.

In the instance supposed above, the greatest quantity of work would be performed, if the wheel moved with *one-third* the velocity of the stream, or at the rate of one mile an hour; and the weight with which the machine was loaded was 400 lbs., being four-ninths of the weight (900 lbs.,) which would balance the pressure of the stream upon the wheel at rest.

When the pressure is diminished, or the resistance increased, in consequence of the motion impressed upon a system, the velocity of the motion increases to a certain point, after which it becomes uniform.

Thus if a windmill is set in motion by the action of the wind blowing with a velocity of 15 miles an hour, the rate at which the sails revolve increases for a time; but as the velocity of the sails increases, the effect of the wind to increase the motion diminishes; and the motion becomes uniform as soon as the force of the wind upon the sails is just sufficient to balance the resistance occasioned by the friction of the various parts of the machinery, and other obstacles to the motion. If the force of the wind should now increase to 20 miles an hour, the velocity of the sails would also increase, until another uniform rate of motion was attained greater than the first.

If a body falls by the action of gravity in perfectly free space, it will continue to move more and more quickly, according to the laws of uniformly accelerated motion. But if a body falls by the action of gravity *in a fluid*, as, for instance, if a stone bullet descends in water, its velocity soon becomes uniform; for in that case, the resistance increases with the velocity; and as soon as that resistance is equal to the force with which the body is acted upon in the fluid, no further increase of velocity takes place; but the body moves on uniformly with the velocity acquired, according to the first law of motion.

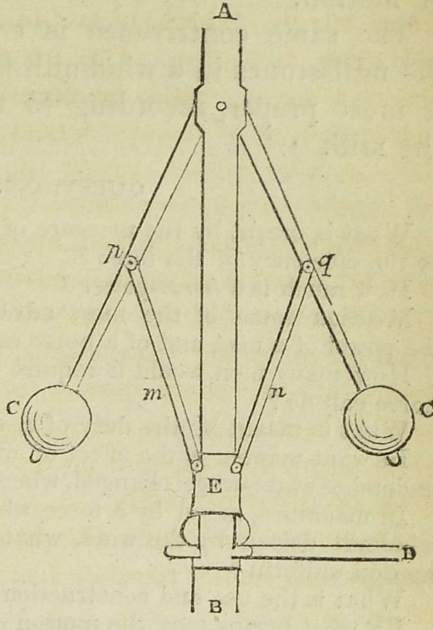
Various contrivances are employed to render the work done by machinery and by animal force uniform, although the intensity of the force itself, or the resistance to be overcome, may vary from time to time.

If the machinery itself is ponderous, as in mill-work, when once it is set in motion, and has attained a uniform rate, it will continue to move with a nearly uniform velocity, although the force or the resistance should be considerably altered for a short time. In many machines a heavy wheel, called *a* *fly-wheel*, is introduced, for the purpose of equalizing the motion. Thus, in driving piles, the force of many men or animals is employed to lift a very heavy weight, which is suddenly detached and let fall upon the pile. The instantaneous removal of the resistance would be accompanied with serious consequences, if a large wheel were not attached to the machine, which keeps on uniformly revolving, when the weight is removed, and prevents any inconvenience from the sudden jerk. A fly-wheel is also very necessary to *accumulate force*, so as to overcome a sudden resistance. In the operations of coining, drawing iron plates, and many others, a comparatively small power is employed for some time in setting in motion a large mass, and the whole momentum so collected is expended in producing a force of great intensity.

In many machines the moving power itself is capable of being regulated. Thus, in some windmills, notwithstanding the variable nature of the force of the wind, considerable uniformity of motion is produced by having the sails so constructed as to *yield* to the pressure of the wind. The sails are formed of a succession of flat plates, moveable upon an axis, and kept in their position by the pressure of a weight. If the pressure of the wind upon those plates becomes greater than that of the weight, the plates open, and part of the air passes between them. When the force of the wind again decreases, the plates close, and the wind produces its full effect.

In the steam-engine, and in watermills and other machines, a very simple but efficacious method is used, to regulate the force of the original moving power.

The *governor*, as it is called, is thus constructed. Two heavy balls, C C, are suspended to some part of a vertical axis, A B, which revolves by the action of the machine. Two bars, moveable upon joints at each end, are attached, at *p*, *q*, to the arms on which C C are hung; and at their other extremity are fastened to a ring, E, moveable upwards and downwards upon the axis, A B, and carrying a lever, D.




If now the balls, C C, were in the position represented in the figure, and the velocity of the machine were increased, by increasing the force of the moving power, the balls would revolve more rapidly; they would therefore *recede further* from the axis, A B, by the increased effect of the centrifugal force arising from rotation, and consequently would *elevate* E, and with it the lever D. That lever is made to communicate with the valve of the boiler in a steam-engine, and with the door which supplies water, in a watermill; and thus at once reduces the moving force to the degree requisite for securing uniformity of motion.

The same contrivance is employed to separate the mill-stones in a windmill, to the distance which is most proper, according to the rate of motion of the mill.

### QUESTIONS.

What is meant by the measure of the power of a machine, or the efficiency of the force?

How much is *a horse-power*?

Mention some of the most advantageous ways in which the power of a man and of a horse can be employed?

How many men would it require to do as much work as a horse can do?

What is meant by the duty of a steam-engine?

In what manner is the effect of water upon the floats of an undershot water-wheel changed, when the wheel is set in motion?

In machines, urged by a force which continually acts, as a windmill driven by the wind, what causes tend to make the motion uniform?

What is the use and construction of a fly-wheel?

By what means may the motion of a machine be rendered uniform?

THE END.

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