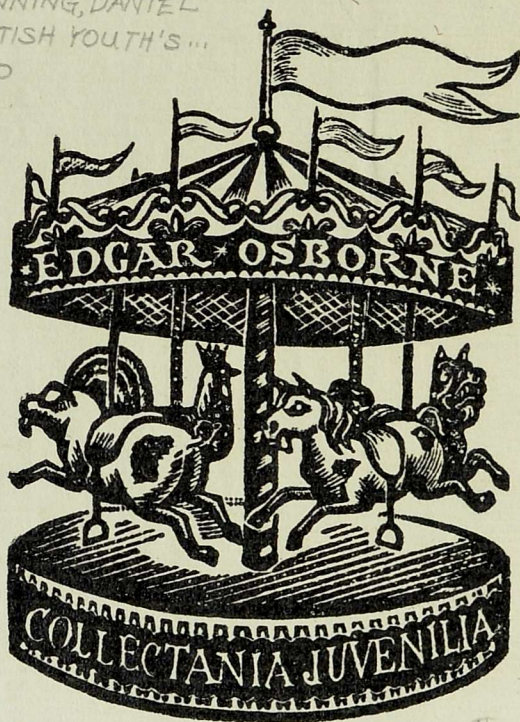


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L O N D O N :

Printed for S. CROWDER, No. 12, in *Pater-Noster-Row*.
M. DCC. LXX.

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DEDICATION

To the SCHOOLMASTERS of

Great-Britain and Ireland.

GENTLEMEN,

THE great Demand for, and
T speedy Sale of the five last Im-
pressions of this small Treatise,
naturally leads me to think,
that many of you have encouraged the
Undertaking.

Permit me, therefore, in this sixth Edi-
tion, to return you hearty Thanks for your
friendly and undeserved Favours.

I am very sensible, (as I told you be-
fore) that many abler Hands have un-
dertaken this Task before me, and have,
in large Volumes, done *that* which can-
not be expected to be found here: But
as you are sensible too many of them
have spent their Time, rather upon Cu-
riositys than Business, (the very Thing
that

iv DEDICATION.

that every Scholar should particularly drive at, and which I am sensible, every one of you would promote the Knowledge of) I have here made it my chief Care to inform the Learner of every Thing that is necessary thereto.

It was not without Reluctance that I appeared upon this Subject; but having published a small Treatise of FRACTIONS and ALGEBRA, by Way of Dialogue, in 1759,* which has met with great Encouragement, I was persuaded by several Schoolmasters, and private Gentlemen, to publish also a Piece of ARITHMETIC, after the same Manner; as they were sensible, they said, (and indeed, I confess, I think the same) that this Way of Writing conveys the Matter better, and communicates Things sooner to the Learner, than the bare setting of Sums, and not working them at all, or, in a dry intricate Manner.

As I do not pretend to recommend the Work by comparing it with other Authors, so, I hope, you will not con-

* The third Edition, with some Quadratic Problems, was published, 1759.

D E D I C A T I O N. v

demn it, till you have perused it through-out.

'Tis true, that the last Method of reducing *C's. qrs. lbs.* into *lbs.* (in the 1st Example of *Tare* and *Tret*) is well worthy your Observations, it being as easy as it is new; and several Persons in Business, to whom I have communicated it, never pretend to use any other Way for Ease and Expedition.

To such of you, indeed, as are perfect Masters of the Sciences, this little Piece may appear insignificant; but to others, I am sensible the Dialogues will be of great Service, as they are intermixed with a Variety of familiar Examples, in natural Conversation: And I persuade myself, they will be of great Help to your Pupils, and, in a great Measure, ease you of that heavy Task, which every diligent Master (with a Number of Scholars) must of Necessity labour under.

This, Gentlemen, is one Reason (as I said) why I built it upon this Plan; and I hope, for the Design's Sake, you will forgive those Errors that have yet escaped my Notice, and which you know are so

vi DEDICATION.

common to a Work of this Sort; tho' I have taken Care to correct all such, as I have at present discovered in former Editions.

I have made no great Alteration in the Work, only have taken off Part of the Double Rule of Three *Direct* and *Inverse*, (as you know the Rule of 5 Numbers will supply that Defect) in Order to make more Room (by the Desire of several Merchants) for a fuller and more clear Explanation, both of the true and customary Way of Discount.

To my Thanks, I add my best Wishes for your Success, jointly and separately, in your several Undertakings; and desire always to subscribe myself,

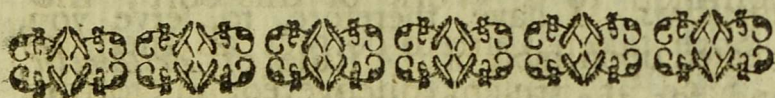
GENTLEMEN,

Your most obliged humble Servant,

Royal-Exchange
Assurance-Office,
London, Sept. 1.
1767.

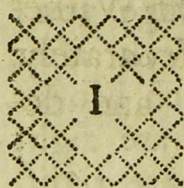
D. Fenning.

THE



THE
P R E F A C E.

KIND READER,



Here present you with the fifth Edition of my small Treatise of ARITHMETIC, which I have improved and endeavoured to render as plain as Time, Room, and Opportunity would admit of.

For such of you, who have too much neglected this Branch of Education; and for others who have neither Time, nor Opportunity, to apply to a proper Master, the following Work was chiefly designed; and was at first carried on no farther than the *Rule of Three Direct*: But I considered with myself, that it might fall into the Hands of many,

who would be glad to have a Notion of *Fractions*. I have, therefore, to serve you, treated upon most of the Rules of *Aritbmetic*, and for the Sake of those that live in the Country in particular, I have given some Instructions to measure Timber, or a common Piece of Ground; as also, to gauge a Cask or Piece of Malt, and several other useful and practical Examples, which, I am persuaded will be of Service to you.

One Thing which is seldom taken Notice of in a PREFACE, I would have every one of you, that are Learners, to observe, and that is, That if, upon the Trial of any Sum, or Question, you do not find it the same as the Answer, do not let that discourage you; for if you examine your own Work after you have done it, and are so much Master of the Rules as to know when you are right or wrong, you may conclude you are right, whatever answer you may find in my Work, or any other Author's: But before you determine this, mind and be sure your own Work is right.

As to such of you that have little or no Notion of Figures, if you have a mind to learn, I am sensible you may very easily do it, with little or no Trouble; for I have taken the more Pains, that you might have the less: And if it does but answer the End of instructing

instructing such of you as are quite ignorant, and improving others that have already some Knowledge of Figures, I shall look back upon the Undertaking with Pleasure, notwithstanding those mean Criticisms and Remarks, that may be made upon such Oversight, which can hardly be avoided in a Work of this Sort.

But, every impartial Reader will consider the Nature and Design of the Work: For 'tis the Plan upon which it is built that is to be minded: If *this* be plain and easy (as I hope it is) there is no fear but the Learner will find a sensible Satisfaction, and the Work be crowned with Success. As for such *Carpers* that are resolved to amuse themselves with the Bone only, they are extremely welcome; but let them be civil, and not snarl at those who would eat the Flesh quietly. I am,

Kind READER,

Your humble Servant,

and Well-wisher,

London, Sept. 1, 1767.

Daniel Fenning.

instructed by the author, and the reader will find that the knowledge of history, I shall look back upon the United States as a nation, notwithstanding those mean criticisms and remarks that may be made upon the work, which can hardly be called a work of this sort.

The author has not been able to find a single copy of the work in the United States, and he has therefore been obliged to send to the publisher, in order to be enabled to send a copy to the author. I have been informed that the work is not in the hands of the publisher, and I have therefore been obliged to send to the publisher, in order to be enabled to send a copy to the author. I have been informed that the work is not in the hands of the publisher, and I have therefore been obliged to send to the publisher, in order to be enabled to send a copy to the author.

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T O T H E
P U B L I C.

 E whose Names are hereunto subscribed, having perused the Plan of this little Treatise of ARITHMETIC, do allow the Dialogues to be very well adapted to the Purpose: And as the Rules and Examples are laid down in a natural, easy, and familiar Manner, We beg Leave to recommend it, as the most useful and easy Book for Learners extant.

Edmund Anguish, *Accomptant*

James Barclay, *Writing-Master*

William Bently, *Surveyor*

George Coles, *Ditto*

Mr. Coulthist, of the Academy, *Prescot-Street,*
Goodman's-Fields

Henry Deacon, *Accomptant*

Randal Evans, *Writing-Master*

Anthony Gilbert, *Surveyor*

Edward Griffiths, *Ditto*

Samuel Hill, *Philom.*

Thomas Humphreys, *Writing-Master*

Thomas Hughes, *Ditto*

Samuel Hornby, *Ditto*

Timothy Langley *Accomptant*

Abraham Longden, *Ditto*

Abraham De Lire, *Philom.*

Thomas Newberry, *Ditto*

John Quant, *Writing-Master*

William Richardson, *Ditto*

David

David Rowland, *Accomptant*
John Smyth, *Writing-Master*
Jonas Smith, *Ditto*
Thomas Smithe, *Ditto*
John Smythe, *Accomptant*
Zachary Snaper, *Ditto*
Erasmus Turner, *Ditto*
Johnson Thompson, *Writing-Master*
James Thurston, *Ditto*
Daniel Trunker, *Ditto*
William Thorley, *Ditto*
James Thorpe, *Ditto*

To the RECOMMENDERS.

GENTLEMEN,

I Return you hearty Thanks for the Honour you have done me, by the Favour of your Names to this little Treatise: Let me crave your further Assistance in noting down those Errors that you may occasionally find, and you will still further oblige,

GENTLEMEN,

Your very humble Servant,

Sept. 24, 1767.

Daniel Fenning.

THE



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THE

INTRODUCTION.

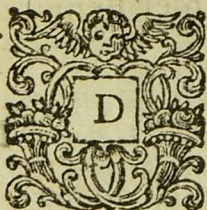
CHAP I.

DILOGUE I.

Between PHILO, a Tutor, or Master, and TYRO, a young Scholar; concerning the Rudiments of ARITHMETIC.

TYRO visits PHILO.

Tyro.



DEAR Sir, I am your very humble Servant—You will pardon me, I hope—I hear you have done instructing *Tyrunculus*, and I am come to lay claim to a former Promise, of your giving me a better Notion of common *Arithmetic*.

Philo. You please me very much, young *Tyro*, I assure you, to see you so willing to learn; but I hear you understand the first four Rules already.

Tyro.

Tyro. When I was at School I had some Knowledge of them, as I thought; but it was not well grounded; and when I left School, instead of practising at Home, and making myself Master of what I learnt there, I bent my Mind to Play and Idleness, like other naughty Boys; and were it not for your kind Offer, I should know the Want of it too late. I choose therefore, Sir, to begin at the very lowest Branch, that I may see the Reason of what I am doing, and not learn by Rote, as too many School-Boys do, to the great Discredit of their Master, Grief of their Parents, and their own future Ruin.

Philo. You say very right; for in beginning again you will be confirmed in what you know already; for I shall proceed with you the same as if you never had began, that others may be the better informed; therefore, for their Sakes do not you be angry, if I should dwell upon some Things longer than you may think there is Occasion for, since I tell you the Reason before hand.

Tyro. Far be it from me, Sir, to take it amiss; for though I do know something of the first four Rules, I am sensible there are many thousands who know Nothing of the Matter, and you do well to consider them also.

Philo. I am glad, *Tyro*, to see you so considerate; it gives me great Hopes of your being Master of what I am about to instruct you in.

DIA

DIALOGUE II.

SECTION I.

Of NUMERATION, ADDITION of Whole Numbers, Money, Weights, and Measures, &c.

Tyro. **W**HAT is *Numeration*, and what does it teach?

Philo. *Numeration* is the true Distinction and Pronunciation of Number; that is, it teaches us to write down, read, and express any Number or Numbers whatsoever. For the better understanding of which, observe the following Table.

N. B. The Letter **C** stands for an Hundred, and **X** for Ten.

TABLE.

Places	C of Millions. X of Millions. Millions.			C of Thousands. X of Thousands. Thousands.			Hundreds. Tens. Units.		
	9 8 7	6 5 4	3 2 1	9 8 7	6 5 4	3 2 1	9 8 7	6 5 4	3 2 1
	9 8 7	6 5 4	3 2 1	9 8 7	6 5 4	3 2 1	9 8 7	6 5 4	3 2 1

NOTE I.

This TABLE you ought to get by Heart, at least, so as to understand the Nature of it.

Cast your Eye now *Tyro* upon it; you see that there are nine Places of Figures from Units

4 Of NUMERATION, ADDITION, &c.

Units to Hundreds of Millions. All the Figures under the first Row are Units; those in the second Row towards the Left-Hand, are under the Place of Tens; all in the third Row are called Hundreds; those in the fourth Row Thousands, &c.

Now, in Order to know the Value of, or how to express any Number in the Table, I begin at the Top Figure towards the Right-Hand, and say Units (1); then Units, Tens, (Twenty-one, 21;) then Units, Tens, Hundreds, (three hundred and twenty-one, 321;) Thus I go on, and find the fourth Figure (viz. 4.) under the Place or Name of Thousands, and accordingly, I call it four Thousand, which joined to the other three Figures, will be four Thousand, three Hundred, and Twenty-one, 4,321. The fifth Figure being in the Place of Tens of Thousands, is thus read: Fifty-four Thousand, three Hundred, and Twenty-one, 54,321. Thus proceed till you come to the last Figure of all towards the left-hand, (which stands in the Place of Hundreds of Millions) and you will easily perceive, that those nine Figures are thus expressed: Nine Hundred eighty-seven Millions, six hundred, fifty-four Thousand, three Hundred, and Twenty-one, 9,87,6,54,3,21. Read this once more, and observe the Commas, or Stops that are put to the Figures, for they answer to or correspond with the Stops in the Words that are written out at Length.

NOTE 2.

You are further to observe, *Tyrol*, that the *Numeration-Table* is not always set with these Figures just in the Form they here stand; for had they been any other nine Figures, they are numbered and expressed after the same Manner: For Instance, suppose they were 1,2,3,4,5,6,7,8,9, this is expressed after the same Manner, only instead of 987 Million, it is now 123 Million; instead of 654 Thousand, it is here 456 Thousand; and instead of 321, it is now 789. So also,

999,999,999

999,999,999 is read thus: Nine hundred, ninety-nine Million, nine hundred, ninety-nine Thousand, nine Hundred, and Ninety-nine, &c. So that you see it is only having a due regard to the Places under which the Figures stand; for you may see that after the first three Places of Figures (*viz.* Units, Tens, and Hundreds) the next three Places have the Name of Thousands, the next three the Place of Millions, as you may observe by the following Table, which, I believe, will be of Service.

T A B L E 2.

Millions Place.			Thousands Place.			Hundreds.	Tens.	Units.
1	2	3.	4	5	6.	7	8	9
	1	2.	3	4	5.	6	7	8
		1.	2	3	4.	5	6	7
			1	2	3.	4	5	6
				1	2.	3	4	5*
					1.	2	3	4
						1	2	3
							1	2
								1

Now here you see they are divided by Stops, or Periods, which is certainly a Help to the expressing the Numbers: Thus, suppose I would express the Number against which this Mark(*) is placed, I find the *Twelve* stands under the Place of Thousands, so that I say it is twelve Thousand, three hundred and Forty-five, &c.

Tyro. I perceive it, Sir, very plainly: But, pray what Use are Cyphers of; or are they of no Signification?

Philo. Cyphers are of no Signification, when they stand alone, (thus 000, or 0000, all stand for Nothing:) Nor are they of any Signification placed before any Figure or Figures, thus, 02 is but 2; and 0005 is but 5 still: But
when

6 Of NUMERATION, ADDITION, &c.

when Cyphers are put after Figures, it makes the Number ten, twenty, thirty, a hundred or a thousand Times more in Value. Thus 1, by adding a Cypher, it is (10) Ten, add two Cyphers, thus, 100, it is an Hundred, &c. as by the following Table.

T A B L E 3.

1	One.
10	Ten.
100	An Hundred.
1000	A Thousand.
10000	Ten Thousand.
100000	An hundred Thousand.
1000000	A Million.
10000000	Ten Million.
100000000	An hundred Million.
1000000000	A thousand Million.
10000000000	Ten thousand Million.
100000000000	An hundred thousand Million.
1000000000000	Million of Millions.

And thus you may go on as far as you please.

Tyro. I perceive it; but who could tell the Value, or express so many Figures out of the Course of the Table?

Philo. You talk like a Learner indeed. You see the twelfth Figure, by the above Table, has the Name of an hundred thousand Millions; therefore, were there 3, 4, or 5 Times, or 5 hundred Times as many Figures, it would be as easy to number them in Order, as well as if there were but 5 in all.

Tyro. That is a little surprising to me, I confess; for some think they do great Things, to number 9. or 10 Figures only.

Philo. Well, *Tyro*, we will not call that Pride altogether; for Pride is good in Learners, so far as it tends to Emulation only; that is, an earnest Desire to excel in Learning: But pray observe, Suppose I had ever so many

many Figures to number; you plainly see, by the last Table, that the seventh Figure is Millions Place, and that the thirteenth Place has the Name of Millions of Millions; so also the nineteenth Figure would be Millions of Millions of Millions: But as the Word Millions would be repeated so often in a very large Number of Figures, as to render it tiresome, there is a shorter and much easier Way of expressing the Number by certain Words, which answer to every seventh Figure, and to any Degree of Millions, as appears by the following Table.

T A B L E. 4.

Dots.

1	The 7th Figure from the Units Place is Millions.
2	13 is Bimillions, or Millions of Millions.
3	19 is Trimillions, or the 3d Deg. of Millions.
4	25 is Quartrillions, or the 4th Deg. of Millions.
5	31 is Quintillions, or the 5th Deg. of Millions.
6	37 is Sexquillions, or the 6th Deg. of Millions.
7	43 is Septillions, or the 7th Deg. of Millions.
8	49 is Octillions, or the 8th Deg. of Millions.
9	55 is Nonmillions, or the 9th Deg. of Millions. or Millions 9 Times repeated, &c.

Suppose it were required to number the 57 following Figures, 321, 987, 654. 321, 987, 654, 321, 987, 654, 321, 987, 654, 321, 987, 654, 321, 987, 654, 321.

Here I make a Period or Dot over every 7th Figure, and find there are 9 Dots in all; Then searching in the above Table, I find the 9th Dot to bear the Name of Nonmillions, the 8th of Octillions, &c. Therefore, I number the above Figures thus; three Hundred 21 Nonmillions, 987 Thousand; 654 Octillions, 321 Thousand; 987 Septillions, 654 Thousand; 321 Sexquillions, 987 Thousand; 654 Quintillions, 321 Thousand; 987 Quartrillions,

8 Of NUMERATION, ADDITION, &c.

Quartrillions, 654 Thousand; 321 Trimillions, 987 Thousand; 654 Bimillions, 321 Thousand; 987 Million, 654 Thousand, 321. And thus you may go on to as many more Places, &c. Or you may number them by the second Part of the Table, and that is, by putting small Figures where the Dots stand, as 1, 2, 3, 4, till you come to 9: Then begin, and say 321^9 of the 9th Degree of Millions, 987 Thousand; 654 of the 8th Degree of Millions, &c. till you come to the End: And thus, by putting small Figures over every seventh Figure, you may number to the 20th, 30th, or any higher Degree of Millions whatsoever.

Tyro. I understand you well; and though it is likely there may never be so many Figures in any Sum; yet, it is good to know how to number them when a Question is asked, though it be for Fancy's Sake only. But now *Philo*, if you please, I will ask you a few Questions, which I am at a Loss to know.

Quest. 1. How is *eleven Thousand, eleven Hundred, and Eleven*, set down in 5 Figures?

Philo. I know this is a common Question, and it is easily done, if you consider; for this is the Answer 12, 111, viz. twelve Thousand, one Hundred, and Eleven.

Proof by ADDITION.

Eleven Thousand is	11,000	} add
Eleven Hundred is	1,100	
Eleven is	11	

Add 12, 111, as above.

Quest. 2. How is *fifteen Thousand, fifteen Hundred, and Fifteen*, set down in five Figures? Answer 16, 515.

But you are to observe, *Tyro*, that though these oddities are easily answered you see; yet, I would not have you be concerned with them; because they are very badly

badly expressed, and are an abuse of language: for, suppose you stood indebted to me, and should ask me what it was, and I should say fifteen pounds, fifteen shillings, and fifteen pence, would you not think it much better to say fifteen pounds, sixteen shillings, and three pence? Take this then for a rule, *Tyro*, that that is the best method of expression, that is the shortest, freest, and most natural. Therefore, tho' the aforesaid numbers run a little smoother in words than what I have now mentioned, (the reason of which is, because they are each but one whole number) yet is the sense of expressing fifteen thousand, fifteen hundred, and fifteen, very little better than fifteen pounds, fifteen shillings, and fifteen pence.

☞ Now there are several things a little dark in expression, relating to numbers, which are actually of use when they are known. I imagine, *Tyro*, you now and then read good books: You remember we read in the book of Kings, and in Isaiah, that an Angel destroyed in the camp of the Assyrians, an hundred, and fourscore and five thousand. How do you set this down properly?

Tyro. Why really, Sir, I am at a loss; and I have seen older persons than myself puzzled at it.

Philo. So have I; but it must be for want of considering. Is not fourscore and five the same as 85. Therefore, the number is one hundred eighty-five thousand, thus, 185,000.

Again, you read in the Revelations, of a multitude of ten thousand times ten thousand; which is 100,000000, viz. one hundred million: and a little farther you read of another, consisting of two hundred thousand thousand, which is, 200,000000 million, viz. two hundred million, which you will see demonstrated, Example the 3d and 4th in compendiums of Multiplication.

I shall finish Numeration with some of the old Roman numbers, as it may be of service.

10 Of NUMERATION, ADDITION, &c.

A TABLE of Old ROMAN Numbers.

L. signifies, or stands for (50) Fifty.	
C. for (100) Hundred.	
CC. (200) Two Hundred.	
CCC. (300) Three Hundred	
CCCC. (400) Four Hundred.	
D. signifies (500) Five Hundred,	or thus I $\overline{D}$.
DC. stands for (600) Six Hundred,	or thus I $\overline{D}$ C.
DCC. (700) Seven Hundred,	or thus I $\overline{D}$ CC.
DCCC. (800) Eight Hundred,	or thus I $\overline{D}$ CCC.
DCCCC. (900) Nine Hundred,	or thus I $\overline{D}$ CCCC.
M. (1000) A Thousand.	or thus C $\overline{I}$.
MM. (2000) Two Thousand, &c.	

ROMAN Numbers explained by FIGURES.

DL. 550.
DCCX. 710.
MDXIV. 1514.
MDCLI. 1651.
MDCCIX. 1709.
MDCCLXX. 1770, &c.

And now *Tyro*, we will proceed to Addition:

SECTION II.

Of ADDITION.

Tyro. WHAT do you mean by Addition?

Philo. Addition signifies the gathering, collecting, and adding together, two, or more numbers into one sum.

Tyro. How many parts are there in Addition

Philo. Two, simple and compound.

Tyro. What is simple Addition?

Philo.

Philo. Simple Addition is that which consists of one simple or single name; that is to say, of whole numbers only, as pounds sterling, tons, yards, ells, or ounces, &c. For ten is ten, and a thousand is a thousand, in any number and quantity; tho' the quality or name be different; and they are all added by one Rule, namely by casting out the tens, and setting down in every line what is over ten, as you will see by and by.

Tyro. What does compound Addition consist of?

Philo. Of Money, Weights, and Measures, viz. Avoirdupois Weight, Troy Weight, Apothecaries, and Goldsmiths Weight, &c. Likewise, Dry Measure, Liquid Measure, Cloth Measure, Square Measure, Long Measure, Land Measure, and Time. Of these in their order.

SECTION III.

ADDITION of whole Numbers.

THE Rule to be got by heart is,
For every ten in the units place, or first row of figures, you must carry one to the tens place, or second row, and so proceed; because ten units make ten, ten tens an hundred, and ten hundred a thousand.

Add 2	3	2	9
1	2	1	7
1	1	4	8
5	2	3	and 4 together.
—	—	—	—
9 <i>Ans.</i>	8 <i>Ans.</i>	10 <i>Ans.</i>	28 <i>Ans.</i>

Here I say 5 and 1 is 6, and 1 is 7 and 2 is 9; which I place under the numbers, and it is done: but to prove whether

whether it be right, I begin at top, and cast it downwards and find it comes to the same.

Secondly, If it amounts to just ten and no more (as in Example 3) set it down as before; only set the cypher under the row, and set the figure of 1 out towards the left hand. So also, in Example 4, I find it amounts to 28, which I set under the row, viz. the units 8 under the row itself and the two towards the left hand in the place of tens.

Thirdly, When you have two or three rows of figures, then, according to the Rule, add up the first row, or units places, and observe how many tens it contains, and if it comes just to even tens, set a cypher underneath, and carry as many ones to the next row, as there were tens in it; that is, if it be 20, carry 2, if it be 30, carry 3, if 40, carry 4. and for 50, carry 5, &c. And the last row of all set down what it amounts to, as if it were one single row only.

AN EXAMPLE at large.

Add 87625 pounds

94915

62194

76547

81965

92198

and 71964 together

567408 *Ans.*

Now observe once for all. I begin, and say, 4 and 8 is 12, and 5 is 17, and 7 is 24, and 4 is 28, and 5 is 33, and 5 is 38; this is the amount of the first row, or units place, but I must not set down the whole 38, but see how many tens it contains, and find it to be 3 tens and 8 over; this 8 I set under the row, and carry 3 to the next row, or tens place, saying, 3 that I carry and

and 6 is 9, and 9 is 18, and 6 is 24, and 4 is 28, and 9 is 37, and one is 38, and 2 is 40: This being just 4 tens, I set down a cypher, and carry 4 for the 4 tens to the next row, or hundreds place, saying 4 and 9 is 13, &c. and I find the third row to be 44, which is 4 tens and 4 over; therefore I set down 4, and carry 4 to the fourth row, or place of thousands, and find that it amounts to 27, then I set down the odd 7, and carry 2 to the 5th or last row, and find that it amounts to 56, and because this is the last row, I set down the whole 56, that is, the 6 under the row, and the 5 to the left-hand.

- *Fourthly*, When there are several numbers to be added together, consisting of some fewer and some of more figures, they are added after the same manner as before, only observe, in setting down the numbers on your slate, or in a book, that you be careful to set units under units, tens under tens, or else you will be puzzled in casting them up.

Ex. 6. Suppose I were to add 3417, 26, 184, 9, 271, and 3 together: I sent them down as follows, which will be a standing rule for any thing of the like nature.

EXAMPLE 6.

$$\begin{array}{r}
 \text{Loads.} \\
 3417 \\
 26 \\
 184 \\
 9 \\
 271 \\
 3 \\
 \hline
 3910
 \end{array}$$

Here you see I set units under units, and tens under tens, and then I cast up from row to row as before directed.

Tyro. I understand it very well ; but I should have set it down thus, with cyphers, to supply the vacant places :

$$\begin{array}{r}
 3417 \\
 0026 \\
 0184 \\
 0009 \\
 0271 \\
 0003 \\
 \hline
 3910
 \end{array}$$

Philo. It is quite superfluous ; for you have been told already, that cyphers before figures do not at all increase the value ; besides, it is neither so slightly, nor advantageous to cast up, nor are they seldom or ever used by accomptants, even in addition of money, to fill up any line or space whatsoever, as you will see in *example 13, of Addition of money,*

More examples in whole numbers.

<i>Busbels.</i>	<i>Ells.</i>	<i>Yards.</i>
471756	6715	65432
434176	46	2145
621985	2	234
942176	2176	67
354219	200	8
471625	6	76
982196	10	354
853294	1756	3256
	1	76258
5131427		
	10919	147830

And

And now, Tyro, I will set you two or three Questions by way of exercise.

QUESTIONS to exercise ADDITION of whole numbers.

Quest. 1. From London to Rumford is 11 miles, from Rumford to Brentwood 6, from Brentwood to Chelmsford 11, from Chelmsford to Witham 8, from Witham to Kelvedon 4, from Kelvedon to Colchester 10, from Colchester to Manningtree 9, and from Manningtree to Harwich 12: how many miles then is it from London to Harwich? Answer 71.

Tyro. I apprehend the question; it is only setting down the numbers as they stand in their order, as under.

	Miles.
From London to Rumford	11
From Rumford to Brentwood	6
From Brentwood to Chelmsford	11
From Chelmsford to Witham	8
From Witham to Kelvedon	4
From Kelvedon to Colchester	10
From Colchester to Manningtree	9
From Manningtree to Harwich	12
From London to Harwich	71 ans.

Quest 2. A farmer has 7 fields, containing the following acres, viz.

	Acres.	
In one field are	25	} How many acres are there in all? Ans. 192.
In another	57	
In another	18	
In another	25	
In another	15	
In another	43	
In another	9	

Ans 192 acres.

B 2

Quest.

Quest. 3. A draper has 6 pieces of cloth as under.

		Yards.	
* N ^o	1 containing	87	} How many yards are there in all? <i>Ans.</i> 210.
	2 ———	45	
	3 ———	17	
	4 ———	8	
	5 ———	32	
	6 ———	21	

Ans. 210 yards.

Quest. 4. How many days are there in the year, or in the 12 calendar months? First set them down as follows.

		Days.	
1st, January has	———	31	Note, Every fourth year is called <i>leap year</i> , and has 366 days; <i>February</i> having then 29 days.
<i>February</i>	———	28	
<i>March</i>	———	31	
<i>April</i>	———	30	
<i>May</i>	———	31	
<i>June</i>	———	30	
<i>July</i>	———	31	
<i>August</i>	———	31	
<i>September</i>	———	30	
<i>October</i>	———	31	
<i>November</i>	———	30	
<i>December</i>	———	31	

Ans. 365 days.

* Note That in marking any sort of goods N^o stands for Number.

Quest.

Quest. 5. How far is it from London to Carlisle in Cumberland, when,

	Miles.
From London to Newcastle is	149
From Newcastle to Preston	62
From Preston to Lancaster	21
From Lancaster to Penrith	50
From Penrith to Carlisle	19

Ans. 301 miles.

Tyro. I understand what you have shewn me very well.

Philo. Then I shall only leave one question more for you to try at your leisure.

Add 47, 697, 5, 91707, 100000, 26300175, 500, 62, and 987654321 together? *Ans.* 1014147514.

To prove ADDITION.

Tyro. How do you prove addition?

Philo. Two ways. *First*, When you have cast up any sum, as before directed, then begin at the top and cast downwards instead of upwards; and if the figures come the same, no doubt but the work is right: besides in things which require care, you should make a practice of casting every line, first upwards and then downwards.

The *second* way is very well for learners, but to tedious for business; but as it is customary in schools to teach it, I shall shew you the method, which is as follows.

Let us take *Quest. 5*, which amounts to 301 miles; and to prove whether this be right, I cut off the top line of figures by a stroke of the pen, thus——

Of ADDITION.

$$\begin{array}{r}
 149 \\
 \hline
 62 \\
 21 \\
 50 \\
 19 \\
 \hline
 301 \text{ Ans.}
 \end{array}$$

152 Add this to the top 149.

301 *Proof.* See *Addition of money.*

Then I begin to cast the sum up again as I did at first, (except the figures that are cut off, or that stand above the line) and find it amounts to 12, which is 2, and I carry 1; then I proceed to the second row, and find it makes 15, so that this amounts in all to 152, which I place under the 301. *Lastly*, I add this middle line 152 to the top line 149, and find that they make 301, which proves the first work to be right. I shall give you a further reason, when you come to examples of *money*, why this way is not so fit for practice in business as casting the sum upward and downward. And now we will proceed to

SECTION IV.

ADDITION of MONEY.

Tyro. **W**HAT is necessary to the learning of *Addition of money.*

Philo. These three things: *first*, the rule; then the characters, and *thirdly*, pence-tables; all which should

should be perfectly got by heart before you pretend to cast up money.

1. *The* R U L E.

For every 4 farthings carry 1 penny to the pence; for every 12 pence carry 1 shilling to the shillings, and for every 20 in the shillings carry one to the pounds, which are cast up by tens, as in whole numbers.

2. *Of the common* CHARACTERS.

Note 1. £ stands for pounds, s stands for shillings, d stands for pence. Or thus, £. s. d. pounds, shillings, pence.

Note 2. A farthing is one fourth part of a penny, and is thus set down $\frac{1}{4}$. An half-penny is one half, and is thus set down $\frac{1}{2}$. Three farthings being three-fourths, is thus set down $\frac{3}{4}$.

Note 3. 1 quarter, 1 half, and 3 quarters are also set thus, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.

3. *Of* PENCE-TABLES.

TABLE 1.

Pence.		s.	d.
20	} is {	1	- 8
30		2	- 6
40		3	- 4
50		4	- 2
60		5	- 0
70		5	- 10
80		6	- 8
90		7	- 6
100		8	- 4
110		9	- 2
120		10	- 0

TABLE 2.

Pence.		s.
12	} is {	1
24		2
36		3
48		4
60		5
72		6
84		7
96		8
108		9
120		10

Of ADDITION.

By the help of these two tables, the first increasing by tens, or every 10 pence; and the other by even shillings, or every 12 pence, you will soon cast up any small sums, by a due regard to the following examples.

Tyro. But I hope, Sir, you will give me an example in length, by explaining it in words; for that will be of more service to me than an hundred examples without explanation.

EXAMPLE 1.

	£.	s.	d.
Add	9	4	2
	8	5	3
and	7	9	6 together
	24	18	11

EXAMPLE 2.

	£.	s.	d.
add	41	3	8
	62	9	7
and	47	11	6 together
	151	4	9

EXAMPLE 3.

	£.	s.	d.
Add	95	16	2
	68	14	10
and	49	12	11 together.

*

Philo. You seem to be timorous, and doubt your own abilities without occasion. Come pray try at the first sum.

Tyro. I see plainly how you do *that*, because the pence all added together do not exceed 12, nor do the shillings exceed 20; therefore, I set the amount of them under the row to which they belong, and find the total to be twenty four pounds, eighteen shillings, and eleven-pence.

* I leave *example 3* undone for your practice, and will give you another table for the more easily casting up the shillings.

Philo.

Philo The same is to be observed in *Ex. 2.* Thus I say, 6 and 7 is 13, and 8 is 21 pence. Now, by the first table, 20 pence is 1s. 8d. therefore, 21 pence must be 1s. 9d. Or by the 2d table, I ask how many times 12 I can have in 21d. and find it 1, and 9 over; therefore, I set the odd 9 down under the place of pence, and carry 1 to the shillings, saying 1 that I carry and 11 is 12, and 9 is 21, and 3 is 24 shillings. Now, as 20s. make a pound, 24s. is 1l. 4s. therefore, I set the 4 under the shillings, and carry one to the first row of the pounds, casting them up as in *Addition of whole numbers*, and find it 151l. so is the total 151l. 4s. 9d.

T A B L E 3.

Shillings.		Pounds.	s.
20	} is {		1 -
30			1 - 10
40			2 -
50			2 - 10
60			3 -
70			3 - 10
80			4 -
90			4 - 10
100			5 -

Please to add up the following sums.

EXAMPLE 4.

£.	s.	d.
45	9	9
94	5	8
76	7	10
51	8	9
25	7	6
75	5	11

368 - 5 - 5

EXAMPLE 5.

£.	s.	d.
249	3	10
176	5	8
649	8	6
148	7	5
219	5	7
154	7	9

EXAMPLE 6.

£.	s.	d.
652	5	8
652	5	8
652	5	8
652	5	8
652	5	8
652	5	8

Tyro. I begin at the row of pence, and find that it amounts to 53. Now, according to my *first pence-table*, 50 pence is 4s. 2d. therefore, 53 pence is 4s. 5d. Or, by *table 2*, I find how many 12 pences I can have in 53 pence, and find that 4 shillings is 48 pence, therefore, 53d. must be 4s. 5d. which odd 5 pence I set down under the row of pence, and carry the 4 shillings to the row of shillings, and find it amounts to 45. Now, by *table 3*, I find 45s. to be 2l. 5s. which odd 5s. I put under the shillings, and carry 2l. to the pounds, which I cast up as in *whole numbers* (by tens) and find the *total*, or whole amount, to be 368l. 5s. 5d.

Philo. Very well; but now you have done it, pray let me hear you say what it comes to; for I have known many school-boys not able to read what they write or cast up.

Tyro. 'Tis very true, sir, but I think it is three hundred and sixty eight pounds, five shillings and five pence.

Philo. Very right; and I make no doubt by your careful proceeding, but you will understand the first four rules of *arithmetic* in a short time. I leave *example 5* and 6 undone for you, to try at your leisure, after the same way and manner: and now I will try you with an example or two with farthings.

EXAMPLE 7.

£.	s.	d.
47	- 8	- $4\frac{1}{2}$
15	- 5	- 10
17	- 4	- $8\frac{1}{2}$
49	- 9	- $6\frac{1}{4}$
15	- 10	- 7
14	- 7	- $9\frac{3}{4}$
97	- 3	- $5\frac{1}{4}$
14	- 9	- 6

Ans. 270 - 19 - $9\frac{3}{4}$

EXAMPLE 8.

£.	s.	d.
271	- 7	- $6\frac{3}{4}$
146	- 5	- $8\frac{1}{4}$
956	- 8	- 3
178	- 7	- $8\frac{3}{4}$
246	- 9	- $7\frac{1}{4}$
101	- 6	- 5
247	- 8	- $3\frac{1}{2}$
604	- 9	- $8\frac{3}{4}$

EXAMPLE 9.

£.	s.	d.
278	- 4	- 3
164	- 8	- $9\frac{1}{2}$
215	- 1	- 10
464	- 5	- $4\frac{1}{4}$
176	- 7	- $8\frac{1}{2}$
291	- 3	- 5
456	- 7	- $6\frac{3}{4}$
278	- 5	- 7

Tyro. I can do these as well as the other. *First*, I begin with the farthings, saying, 3 and 3 is 6, and 1 is 7, and 2 is 9, and 2 is 11 farthings, that is, 2 *d.* and 3 farthings over, which 3 farthings I set under the row of farthings (thus $\frac{3}{4}$) and carry 2 pence to the pence, and find they amount to 57 pence, which is 4 *s.* 9 *d.* This odd 9 I set under the pence, and carry 4 to the shillings, and find them 59, that is, 2 pounds, and 19 over, which 19 I set under the shillings, and carry 2 to the pounds, I find them come to 270: so is the total or answer 270 *l.* 19 *s.* 9 *d.* $\frac{3}{4}$.

Philo. Very well done indeed: proceed in the same manner, and you will find *Ex.* 8. to be 2752 *l.* 3 *s.* 3 *d.* $\frac{1}{4}$.

Tyro. I am obliged to you, sir, and now I should be glad you would give me instructions how you manage *double figures* in the shillings.

Philo. I will shew you three or four different ways, and you may take that which appears most natural and easy.

4. *Of casting up double figures in the shillings, such as 16. 17. 18. or the like.*

EXAMPLE 10.

£.	s.	d.
64	- 17	- 10 $\frac{1}{4}$
47	- 14	- 11
67	- 18	- 10 $\frac{3}{4}$
55	- 17	- 8 $\frac{1}{2}$
49	- 19	- 11
64	- 15	- 5

351 - 4 - 8 $\frac{1}{2}$

EXAMPLE 11.

£.	s.	d.
756	- 17	- 8
175	- 16	- 11 $\frac{1}{4}$
476	- 11	- 9 $\frac{1}{2}$
187	- 17	- 10
356	- 19	- 5 $\frac{3}{4}$
175	- 14	- 11 $\frac{1}{4}$

EXAMPLE 12.

£.	s.	d.
4715	- 11	- 10 $\frac{1}{2}$
1762	- 14	- 5
6471	- 14	- 6 $\frac{3}{4}$
1754	- 15	- 11 $\frac{1}{4}$
6474	- 19	- 8
1762	- 17	- 6

METHOD 1. I cast the farthings up (first upwards and then downwards) and find them 6, I therefore set down $\frac{1}{2}$, and carry 1 to the pence, which are 56, which is

is 4s. 8d. that is, 8 and I carry 4 to the shillings; but as there are two rows of shillings, or *double figures*, I first of all cast up that row, or those figures that stand towards the right-hand, saying thus, 4 that I carry and 5 is 9, and 9 is 18, and 7 is 25, and 8 is 33, and 4 is 37, and 7 is 44, the first row; then I come back upon the second row, from the top to the bottom, calling every figure of 1, ten; saying, 44 and 10 is 54, and 10 is 64, (and so on) 74, 84, 94, and 10 is 104 shillings, which is 5*l.* and 4 shillings over, that is 4, and I carry 5 to the pounds, which I find amount to 35*l.* So that the total is 35*l.* 4s. 8d. $\frac{1}{2}$.

METHOD 2. Some like *this* way best; they cast up the first row of shillings as before, which comes to 44, that is 2*l.* 4s. and set the 4 shillings under the said row, and carry the 2*l.* to the next row of shillings, counting 1*l.* for every two figures in the second row, (because two tens make 20s. or 1*l.*) thus in the above example, there are six figures of 1 in the second row, and counting two of them for 1*l.* then six will be 3*l.* which with the 2*l.* that belonged to the 44 in the first row, make 5*l.* to be carried to the pounds as before.

METHOD 3. Others make use of *dots*: that is, they make a dot at every 4 in the farthings, every 12 in the pence, every 20 in the shillings, and every 10 in the pounds, which method is very easy, and suitable to lads of a dull comprehension, and a bad memory; because they are not able to carry a large number of pence and shillings in their mind, but frequently carry false from figure to figure.

Note, There are different opinions concerning pointing or dotting: Mr. *Dilworth* highly recommends it; but Mr. *Fisher* calls it slovenly and unnecessary. It is not my province to determine between those Gentlemen: But this I will say, that every master should try all methods, and let the scholar use *that* which is most natural and easy to his capacity. He must be very dull who can't tell that 2 tens make 20, or that 13
 twenties.

twenties make 260: and to stop or point at so small numbers, as 3, 4, 5, 6, &c. ought (if possible) to be avoided; because to add up well, and learn to cast out quick, is a great step towards *Multiplication*, and it would look very odd to see a person (after having done a long sum in that rule) dot every 10 in casting up the product. I think therefore, there is no occasion to point or dot at every 4, 10, 11, or at even twenties, when a little practice will make it easy: but indeed when you come to do by 13, 16, 19, 28, or the like, there dotting must unavoidably be used, as you will see in the first example of *Averdupoise Weight*, wrought at large.

N. B. *If you are obliged to dot, mind and make them very small; if you can see them yourself, that is sufficient: but the first method, in my opinion, is best, except it be a very long sum, such as the side of a large book, a bill, or a parish rate, or duplicate, and then the following method is the most certain and expeditious, and is very easy.*

METHOD 4. *Of making and casting up long bills, PARISH RATES, &c.*

Tyro. How is the general way of casting up very long bills, rates, &c.

Philo. The rule is this: *for every 60 in the pence carry 5 to the shillings, because 60 pence make 5 shillings: and for every 60 in the shillings carry 3 to the pounds, because 60 shillings make 3 pounds; and lastly, cast the pounds up by tens as before directed.*

Tyro. Please to give me an example at large.

Philo. I will shew you first, how to make a *parish rate*, which may be of service, and if you can cast it up well, you may also cast up any long bill, by the same rule.

The form of a PARISH RATE, &c.

			£.	s.	d.	
Allen Anthony, <i>Esq;</i>	—	—	3	7	8	$\frac{1}{4}$
Andrews John	—	—	1	14	7	$\frac{1}{2}$
Baker James, <i>Esq;</i>	—	—	15	16	8	
Carter Thomas	—	—		17.	11	$\frac{1}{2}$
Darby Abraham	—	—	5	17	5	
Fernel Abraham	—	—		19	8	$\frac{3}{4}$
Honner Joseph	—	—	7		9.	
Kirton John, <i>Esq;</i>	—	—	21	14.	7	$\frac{1}{4}$
Longman Thomas	—	—	5		11	$\frac{1}{2}$
Lumley, Edward	—	—	3	17	6	
Manning Thomas	—	—		15	10	$\frac{1}{4}$
Martin William, <i>Esq;</i>	—	—	11	11	8	$\frac{1}{2}$
Martin Job	—	—	7	17.	8	
Nicholas Abraham	—	—	2	13	9	
Norton Daniel, <i>Esq;</i>	—	—	17	10	4.	$\frac{1}{2}$
Parker Esther	—	—		8	11	
Powell Judith	—	—		5	10.	
Randall Nicholas	—	—	1	17	6	$\frac{1}{2}$
Robinson Abraham, <i>Esq;</i>	—	—	15	11.	11	$\frac{1}{2}$
Robinson John	—	—	7	10		
Robinson James	—	—	1		10	
Ruggles Nathaniel	—	—	7	11	4	
Rumley Thomas	—	—		15		$\frac{1}{2}$
Solly Thomas, <i>Esq;</i>	—	—	12	11		
Sorsby William	—	—		14.		
Spratly David	—	—	8		11.	
Swallow Thomas	—	—	1	17	4	
Thompson John	—	—		19	11	$\frac{3}{4}$
Twist William, <i>Esq;</i>	—	—	7	7		
Walker John	—	—	1	15.	6	
Wayland Edward, <i>Esq;</i>	—	—	8	14	9	$\frac{1}{2}$
Wingate Samuel	—	—		15	7	
Worley Abraham	—	—	5	4	10	
			£.	189	19	7 $\frac{3}{4}$
						<i>First,</i>

First, I begin with the row of farthings (as in the former questions) and find them amount to 27, which is 6 pence, and 3 farthings over; which 3 I set under the farthings thus ($\frac{3}{4}$) and carry 6 to the pence.

Secondly, for the PENCE.

I say 6 that I carry and 10 is 16, and 7 is 23, and 9 is 32, and 6 is 38, and 11 is 49, and 4 is 53, and 11 is 64; this being 4 above 60, I make a small dot close to the 11, and carry the 4 that is over to the next figure, saying 4 that I carry and 4 is 8, and 10 is 18, and 11 is 29, and 6 is 35, and 10 is 45, and 11 is 56 and 4 is just 60; therefore, as there is nothing over to carry to the next figure, I say 9 and 8 is 17, and 8 is 25, and 10 is 35, and 6 is 41, and 11 is 52, and 7 is 59, and 9 is 68; that is 8 above 60, therefore I make a dot, and carry 8 to the next figure, saying 8 and 8 is 16, and 5 is 21, and 11 is 32, and 8 is 40, and 7 is 47, and 8 is 55: now 50 pence is 4 s. 2 d. therefore 55 pence is 4 s. 7 d. which odd 7 pence I set under the pence, and carry the four shillings to the dots, counting (as I said before) 5 shillings for every dot, which are three in number, that is 15 shillings; and the 4 I carried to them is 19 shillings: this 19 I carry to the shillings, and contrary to the other methods, I now work cross-ways, taking the *double figures* as I go along, *viz.* the right hand figure first, and then the left-hand one belonging to it, counting it for 10, as follows,

Thirdly, for the SHILLINGS.

I say 19 that I carry a 4 is 23, and 5 is 28, and the 1 on the left hand of it, which is always counted for 10, is 38, and 4 is 42, and 10 on the left-hand belonging to it, is 52, and 5 is 57, and the 10 belonging is 67: here according to the same rule, I make a dot, and carry the odd 7 to the next figure, as I did in the pence; saying, 7 that I carry and 7 is 14, and 9 is 23,
and

and 10 is 33, and 7 is 40, and 10 is 50, and 4 is 54, and 10 is 64; that is, dot, and I carry 4, to 11 is 15, and 5 is 20, and 10 joining it is 30, and 11 is 41, and 10 is 51, and 11 is 62, which is dot, and I carry 2 to the next figure, proceeding in the same manner, as before directed, till I come to the top of all, and find there are 39 shillings, besides the last dot, that is 1 £. 19 s. therefore, I set the 19 under the shillings, and carry the odd 1 £. to the dots (counting 3 £. for every dot, because 60 shillings is 3 £.) and find them 6, which is 18 £. and 1 I carried to them is 19 £. which I carry to the first row of the pounds, casting them up by tens, as in *whole numbers*, and find them 189 £. So that the total of the rate, or bill, is 189 £. 19 s. 7 d. $\frac{3}{4}$. Pray, Tyro, run it over once more, and the method also, and you will not lose your labour.

Tyro. Sir, I see the nature of it very well; but why do you leave those vacancies in the shillings and pence, for most people fill them up with cyphers.

Philo. I know it is the common custom of schools, but it is a very idle one, nor is it so slightly or convenient; for the cyphers hinder sight, and prevent expedition in casting up. See example 6, in *whole numbers*.

5. Of CYPHERS, where necessary, and where not.

EXAMPLE 13, without cyphers, according to the true Order of BOOK-KEEPING.

£.	s.	d.
5478	- 9	- 6
1	- 19	- 5
	- 6	- 8 $\frac{3}{4}$
179	-	-
	- 17	- $\frac{1}{2}$
7	- 5	- 9
1	- 17	- 7
	4	- $\frac{1}{2}$
5670	-	$\frac{3}{4}$

EXAMPLE 13, with cyphers, according to the common custom of schools.

£.	s.	d.
5478	- 09	- 06
0001	- 19	- 05
0000	- 06	- 08 $\frac{3}{4}$
0179	- 00	- 00
0000	- 17	- 00 $\frac{1}{2}$
0007	- 05	- 09
0001	- 17	- 07
0000	- 04	- 00 $\frac{1}{2}$
5670	- 00	- 00 $\frac{3}{4}$

Now,

Now, *Tyro*, I would ask you which is most sightly, *that* which stands clear, or the other confused with cyphers. And it is not only this, but every line of the sum, and the total also, reads much better: for it is only five thousand, six hundred, and seventy pounds, and three farthings; whereas the other, according to the manner of cyphers (and the too common custom of school-boys) must be thus read, *six thousand, six hundred, and seventy pounds, no shillings, and no pence, three farthings*. And tho' custom some years ago prevailed upon one of the most ingenious authors *, to use the *expression* after this sort; yet it does not at all justify the correctors of the last edition, since it is now quite superfluous, and out of date.

Note, Though I say it be not customary to use cyphers in *addition of money*, yet, in *weights and measures* they are often made use of, and the learner may take his own fancy, that teaches himself; but it is my opinion that masters in general would find it much easier and better, both to themselves and scholars, to set their addition sums quite clear, and use no cyphers at all before figures.

Tyro. I am obliged to you, sir; and now, if you please, I would know how you prove *addition*?

Philo. That is quite easy, *Tyro*.

6. To prove ADDITION of MONEY.

There are two ways, one by cutting off the top line (of which see under the proof of *addition*,) but it is not practicable in business, the best way is this: begin at the bottom and cast up to the top, noting down what it comes to; then begin at the top, and cast the same row downwards, and if it amounts to the same as before, there is no doubt but the work is right, provided you observe carefully to set it down right.

* *Wingate*, Page 8.

SECTION V.

Containing some farther explanation of things necessary to be known, with the manner of drawing out bills, writing notes and receipts, &c. being very proper to exercise the young beginner.

Tyro. I Shall be at a loss under this section.

Philo. You are mistaken; for I shall not leave you to yourself wholly, but will explain every thing I am capable of, to your understanding; because I am sensible many school boys can add up sums that are set them very well; but when they are called upon out of school, to write out, and cast up a small bill, or the like, how awkward do they go about it, and will set puzzling over it so long, that one would think they knew nothing at all of the matter: and when it is done, it is often set in such confusion, that it will puzzle much better scholars than themselves to cast it up; because they do not observe to place their figures right under one another, which remember, *Tyro*, you be always careful of, for it is a great advantage, and the work will look neat, and you will have the praise.

Tyro. 'Tis very true, sir, I have known several that could cast up a bill if others would figure it down, but have not had any notion of drawing it out themselves: nor can I say I have, and therefore should be obliged to you to shew me.

Philo. That I will; but first of all, it will be proper to shew you some *signs* and *contractions* made use of both in arithmetic and business.

I. *Of contracted signs and characters used in common arithmetic.*

Note 1. This character (+) signifies addition, and when placed betwixt numbers shews that they are all to be added together. — Thus $5 + 7 + 11 + 9$, is thus read 5 more 7, more 11, more 9, &c.

2. This

2. This character (—) signifies less, and is the sign of subtraction, and shews the number after it, is to be taken out of the number before it: thus $11 - 7$ is 11 less 7 which is 4; and $148 - 93$ shews that 93 is to be subtracted from 48, &c.

3. This ($\times$) is the sign of multiplication and signifies into, and shews that all the numbers betwixt which it is placed are to be multiplied continually into each other.—Thus, $4 \times 5 \times 3$, shews that 4 is to be multiplied into or by 5 and that product by 3.

4. This ($\div$) is the sign of division, and shews the number before it is to be divided by the number after it.—Thus $48 \div 6$ shews that 48 is to be divided by 6.

5. This (:) signifies to.

6. This (::) signifies so is.

7. The 2 last characters put together thus ($:: ::$) are the sign of the rule of 3 or proportion. Suppose you see $2 : 4 :: 8 : 16$; it is thus read, as 2 to 4, so is 8 to 16, &c. As $a : b :: c : d$. As a to b so is c to d , &c.

8. This ($=$) is the sign of equality, and signifies that the figures or quantities placed before it are equal to those after it: thus $4 + 8 = 16$, is read 4 more equal to 16. So also $25 - 16 = 9$, is 25 less 16, or 16 subtracted from 25 equal to 9. So also $5 + 9 + 12 - 2 \div 5 = 11$, is thus read; 5 into 9, more 12, less 2, divided by 5, is equal to 11.

9. This ($\sqrt{}$) is the sign of the square root.

10. This $\sqrt[3]{}$ the sign of the cube root.

2. Of contracted words, and what they signify.

Note 1. *Bt.* stands for, or signifies bought.

2. *Dr.* is debtor. *Cr.* is creditor.

3. *Do.* stands for ditto, and signifies the same thing, place or sort of goods, as was wrote in the line before; it is used by merchants, tradesmen and accountants, both in books and bills, to avoid writing the same thing over and over again.

4. *Co.*

4. *Co.* signifies *company*; that is, when two or more are in trade and in partnership.

5. *Messrs.* signifies *Messieurs*, or gentlemen concerned together in some trade or business, and is used with the foregoing contraction, (*Co.*) on the head or front of the bills, instead of writing all their names at length: thus, suppose *John Sharp* to have a bill upon *Aaron Nelson*, *John Long* and *Joseph Truman*, it would be ridiculous to write thus:

Mr. *Aaron Nelson*, Mr. *John Long* and Mr. *Joseph Truman*, bought of *John Sharp*.—But we write thus:

Messrs. *Aaron Nelson* and *Co.* bt. of *John Sharp*, as you will see in the following example.

A Linen-Draper's BILL.

EXAMPLE I.

Messrs. *Aaron Nelson*, and *Co.* bt. of *John Sharp*.

1769.

		£.	s.	d.
Jan. 4	To 11 pieces of <i>Irish</i> , at 3 <i>l.</i> per piece.	33	-	
12	To 2 ditto, at do. ———	6	-	
Feb. 8	To long lawn as by agreement	3	- 15	- 6
Apr. 12	To 2 pieces of check, as by do.	1	- 7	- 10
28	To four do. as by do. ———	2	- 2	-
May 24	To 2 yards of dowlas at 1 <i>l.</i> d.		- 1	- 10
Aug. 9	To a parcel, as per bill ———	1	- 15	- 6

This bill I cast up as a common sum in }
addition of money, and find the total } 48 - 2 - 8

EXAMPLE 2.

A Stationer's BILL.

Mr. Samuel Long bt. of John Page.

1769.

			£.	s.	d.
May 19	2 ream of fool's cap superfine	}	£.	1	10
	paper				
June 12	4 ditto			3	-
	20 ream of coarse brown			1	10
Aug. 14	To sundry goods, as per bill			3	19
				£.	10

Note, Some persons never use the word *To* at the beginning of the bill. *Note* farther, the word *To* is used in books of accompts on the *Dr.* or left hand side. The word *By* is used on the *Cr.* or right-hand side.

And now, *Tyro*, I shall draw you out a few more bills, and leave you to set them on a slate, and cast them up yourself which will be of service, by way of exercise.

EXAMPLE 3.

A Goldsmith's BILL.

Mr. James Proud bt. of Paul Fineshew:

		£.	s.	d.
May 9	To 1 Diamond ring	15	-	-
	2 Pair of silver salts	2	2	6
	1 Quart tankard	10	10	-
	1 Pint do.	4	7	6
	1 Dozen of knives and forks	3	2	6
	1 Silver tea kettle	5	5	-
	12 Tea spoons, tongs, and cream-pot	1	15	-

Total £.

Ex-

EXAMPLE 4.

A Taylor's BILL.

Mr. Robert Patience Dr. to John Trimmer.

1769

Mar. 15	To 2 yards of cloth, at 18s. per	yard	_____	_____	_____	£. s. d.
	Making your coat	_____	_____	_____	_____	1 - 16 -
	4 yards and a half of shalloon, at 2s.	_____	_____	_____	_____	- 8 -
	Buckram, staytape, and canvas	_____	_____	_____	_____	- 9 -
	Silk, twist, and mohair	_____	_____	_____	_____	- 3 - 6
	Making a suit for your son	_____	_____	_____	_____	- 2 - 8
	Buckram, silk, twist, mohair, &c.	_____	_____	_____	_____	- 15 - 9
	Buttons to the same	_____	_____	_____	_____	- 4 - 3
						- 3 - 6

£.

Now, from these four examples, *Tyro*, you may form any other tradesman's bill in due order; and as for making out or balancing any reckoning between one person and another, to see how much is due to either, is not the work of *addition*; but I shall explain it very fully to you in *subtraction*.

2. *The manner or common form of receipts, and notes of hand.**Tyro.* How am I to give, or write a receipt?*Philo.* According to what money you receive, and the persons you receive it of, or for. Let us take the stationer's bill in example 2, which is 10*l.* Now, if he receives it in full, the receipt will run thus:

August 6, 1769. Received of Mr. Samuel Long ten pounds, in full.

£. 10.

John Page.

If

If the receipt be wrote upon the bill itself then this is better :

Received at the same time the contents in full.

John Page,

EXAMPLE 2. If only part be received, then thus :

August 16, 1769. *Received of Mr. Samuel Long five pounds, on account.*

John Page.

£. 5

EXAMPLE 3. If the Person that receives the money be a son or servant, he must write thus :

August 16, 1769. *Received of Mr. Samuel Long ten pounds in full, for my father or (master.)*

John Page, jun.

EXAMPLE 4. For rent.

June 14, 1769. *Received of Mr. John Lumley twelve pounds, ten shillings, for half a year's rent, due at Lady-Day last.*

Abraham Gripe.

£. 12 - 10

EXAMPLE 5. When there has been an account of long standing, and at last the two parties agree to have a reckoning, but still he that owes money upon the balance has none at that time to pay, then the following form is counted better than a common note of hand, because it shews the reason of such an acknowledgment: but then this writing should be drawn in the book of the person to whom the balance is due, and if signed in the presence of witnesses the better. The form is thus :

August

August 16, 1769. *Reckoned and balanced all accompts, and I Samuel Long acknowledge myself to be indebted to John Page three pounds, ten shillings, which I promise to pay on demand, for value received. Witness my hand,*

Test.

Abraham Justice.

Samuel Long.

EXAMPLE 6. When a Person has no money about him, or has his money in other persons hands, and gives you a note or draught upon them for the payment of any sum, it is wrote after this manner :

Sir, please to pay to Mr. John Page, or bearer, three pounds, ten shillings, and place it to the account of

Your humble servant,

To Mr. Jonathan Trusty.

Samuel Long.

3. *Of the Value of the common coins used in England, how they are expressed, and how set down.*

A *Port, or Portugal* piece, is set down 1*l.* 16*s.* but is expressed, or commonly called a six and thirty.

A double *Port* is 3*l.* 12*s.*

A *Moidore* is set down 1*l.* 7*s.* but is called a seven and twenty.

A *Guinea* is 1*l.* 1*s.* and expressed a guinea.

A *Crown* is expressed a crown ; but set down 5*s.*

Half a crown is 2*s.* 6*d.*

A *Tester* is six-pence.

A *Groat* is four pence.

An useful EXAMPLE.

A servant laid out cash as follows : for *coals* a guinea and a half. *Cloth* three and twenty and six-pence. *Meat* seven groats. *Butter, eggs, and bacon* nineteen-pence half-penny, and *thread* seven farthings. What was laid out in all ?

Tyre.

Tyro. I fet them down as follows.

	£.	s.	d.
For Coals	1	11	6
Cloth	1	3	6
Meat	-	2	4
Butter, eggs, and bacon	-	1	7 $\frac{1}{2}$
Thread	-	-	1 $\frac{3}{4}$
In all	2	19	$\frac{1}{4}$

Philo. Very well done indeed. *Obferve farther then,* that from *one* to *two* fhillings, and from *one* or *two* pounds, the *exprefion* is different from the *fetting-down*. Thus *1s. 10d. $\frac{1}{2}$* is expreffed two and twenty-pence halfpenny; and *1l. 19s. 6d.* is expreffed nine and thirty and fix-pence.

So alfo, though thefe numbers, 1300, 1753, and 2500, are properly, one thoufand three hundred, one thoufand, feven hundred, and fifty-three, and two thoufand, five hundred; yet they are thus expreffed; thirteen hundred, feventeen hundred and fifty-three, and five and twenty hundred.

Tyro. I heartily thank you, kind fir, and if it were not too troublefome, I could wifh you would fet me a few queftions by way of exercife.

Philo. You do very well to ask me, *Tyro*; but it will be better at the end of *Addition*, where I fhall give you fome ufeful examples.

SECTION V.

AVOIRDUPOIS WEIGHT.

Tyro. **P**RAY what is the ufe of this rule, or what is weighed by it?

Philo. Moft Things that are commonly dealt in, fuch as *grocery-wares*, and alfo, *cheefe*, *butter*, *foap*,
C
candles;

candles, allum, brass, iron, copper, salt, hemp, and all such sort of goods.

Tyro. What are the different names or denominations of the weights used in this rule.

Philo. The greatest denomination is a *ton*, and the least a *dram*. They run thus in order: *tons*, *hundreds*, *quarters*, *pounds*, *ounces*, and *drams*; of which is composed the following table, with the characters that stand for each denomination after them.

The T A B L E.

16 Drams	} make	1 Ounce marked thus,	oz.
16 Ounces		1 Pound	lb.
28 Pounds		1 Quarter of an hundred	qr.
4 Quarters		1 Hundred weight	Ct.
20 Hundreds		1 Ton	T.

Tyro. Should I get this table by heart?

Philo. You may do as you please; I know some masters call it unnecessary; but if you take my advice, learn every table perfectly. 'Tis true, you may do the sums without it, if you have the table before you, but it would be a very bad thing to make an excuse, by saying, you could do such a sum if you knew the rule by heart; and I have known many a lad lament the omission.

Tyro. I don't doubt it at all, and I will take your advice: please to tell me how you cast up those sums.

Philo. The same as in *addition of money*, only you stop (as is plain by the table) by different figures; and for your further information, I shall put over every row and denomination, the quantity you are to stop at, or do by, and shall give you one example at large as a standing rule for all that follows.

EXAMPLE 1.

(10)	(20)	(4)	(28)
<i>Tons</i>	<i>C.</i>	<i>qrs.</i>	<i>lb.</i>
25	- 14	- 1	- 15.
18	- 11	- 2	- 16
27	- 17	- 3	- 21.
46	- 14	- 1	- 17.
16	- 17	- 2	- 15
45	- 15	- 1	- 17.
16	- 17	- 3	- 14

198 - 9 - 1 - 3

EXAMPLE 2.

(10)	(4)	(28)	(16)
<i>C.</i>	<i>qrs.</i>	<i>lb.</i>	<i>oz.</i>
42	- 1	- 17	- 10
17	- 2	- 19	- 14
21	- 3	- 22	- 11
17	- 2	- 17	- 12
24	- 1	- 22	- 15
65	- 3	- 17	- 13
24	- 1	- 21	- 10

I begin at the *lbs.* and say, 14 and 7 is 21 and the 1 on the left hand (which is always called 10) is 31; that is, 3 above 28; therefore, I make a dot against the 7, and carry the odd 3 forward; saying, 3 that I carry and 5 is 8, and 10 on the left of it is 18, and 7 is 25, and 10 on the left is 35, that is, 7 above 28; then I dot again, and carry 7 to the next figure, saying 7 and 1 is 8, and 2 on the left-hand, which stands for 20, is 28; therefore, as there is nothing over, I only say 16 and 5 is 21, and 10 is 31, that is, 3 above 28, which 3 I place under the row of pounds, and then telling my dots, I find them 4 (that is 4 qrs.) which I carry to the next row of qrs. saying, 4 and 3 is 7, and 1 is 8, and 2 is 10, and 1 is 11, and 3 is 14, and 2 is 16 and 1 is 17 quarters. Now as 4 quarters make an hundred, I ask how many *fours* I can have in 17, and find 4 *fours*, and 1 over, that is 4 hundred, 1 quarter, which 1 I place under the row of quarters, and carry the 4 to the hundreds, which I cast up by *twenty*, the same as in *addition of money*, saying, 4 that I carry and 7 is 11, and 5 is 16, and 7 is 23, and 4 is 27, and 7 is 34, and 1 is 35, and 4 is 39; then I come back with the tens, saying, 39, and 10 is 49, and 10 is 59, and so on, 69, 79, 89, 99, and 10 is 109, which is 5 twenties, and

9 over; that is, 5 tons, and 9 hundred over, which 9 I place under the hundreds, and carry 5 to the first row of the tons, and casting them up as whole numbers by tens, I find the first row 48, that is, 8, and I carry 4 to the next row, which amounts to 19. Therefore the total is 198 tons, 9 c. 1 qr. 3 lb.

☞ This is a standing rule, *Tyro*, for all your other sums in *Addition*, which are cast up after the very same manner: those that you find done are for your information and satisfaction; and such as are left undone are for your practice. Are you satisfied with what I have told you?

Tyro. Sir, I am; but in *example 2*, I perceive you set 10 over the *hundreds* place, contrary to the table; whereas in *example 1*, you have set 20 over the *hundreds* place, which I own puzzles me at present.

Philo. That is for want of a little consideration, *Tyro*; for whatever *name* or *denomination* stands first (that is, whatever you add up last) is always added up like *whole numbers*, by *tens*; be they *tons*, *hundreds*, *pounds*, *shillings*, *yards*, *ells*, or any thing else, as you will see hereafter.

Tyro. I am obliged to you, sir, and desire no further instruction in *Addition*, but only the rules and examples to go by.

Philo. You shall not want for either, and they are all done after the same manner as this example before you, though by different figures: and pray take notice, all those examples that you find ready done, are not to indulge you in idleness; but the answers are inserted for your information and satisfaction; and those that are left undone, are for your exercise, practice, and improvement; and though I told you in *addition of money* it is not customary to use cyphers; yet, if you chuse it, take your own way; but you will find it better to leave them out, when there is no occasion to use them.

AVOIRDUPOIS *small Weight.*

Tyro. What is the use of this rule?

Philo. It is chiefly used by such as deal in filk, worsted, thread, &c. by *retail*, or in small quantities only; that is, from a dram to pounds, 16 drams make an ounce, and 16 ounces 1 pound.

EXAMPLE 1.

(10)	(16)	(16)
lb.	oz.	d.
2	- 10.	- 5.
4	- 7	- 3
1	- 6	- 11.
6	- 9.	- 0
3	- 7	- 9
18	- 8	- 12

EXAMPLE 2.

(10)	(16)	(4)
lb.	oz.	grs
3	- 14	- 2.
1	- 11	- 1
3	- 9	- 0
2	- 01	- 2
2	- 00	- 3

Note, Worsted is weighed by ounces and quarters, as in *example 2*, and no drams are used here, nor in retailing many other commodities.

Of W O O L.

Wool is weighed by the *clove, stone, tod, wey, &c.* as follows.

7 Pounds make 1 clove
 2 Cloves, or 14 lb. 1 stone
 2 Stone, or 28 lb. 1 tod
 6 $\frac{1}{2}$ Tod 1 wey
 2 Weys 1 sack
 12 Sacks 1 last.

Note, In some places 7 tod are allowed to 1 wey, and 12 score, or 240 lb. is called a *pack of wool*.

Note farther.

A *Firkin* of soap is 64 lb.

A *Firkin* of butter 56 lb.

A *Clove* of cheese 8 lb.

A *Wey* of cheese in *Essex* is 32 cloves, or 256 lb.

A *Wey* in some parts of *Suffolk* is the same; but in other parts of it, 42 cloves, or 336 lb. make a wey.

☞ See another table of weights and measures, section 6.

TROY-WEIGHT.

Tyro. Of what use is *troy-weight*?

Philo. By this is weighed gold, silver, jewels, electuaries, &c. and liquors in general.

T A B L E.

Note,	{	24 Grains 20 Penny wts. 12 Ounces	}	make	{	1 Penny-wt. 1 Ounce. 1 lb. troy.	}	<i>dwt.</i> thus <i>oz.</i> <i>lb.</i>

Note 1. You need not *point* or *dot* at any row except the first, where you do by 24; the others are easy and common, as before, the penny-weights being cast up by 20, are done the same as shillings; and the ounces being done by 12, are cast up like pence, in addition of money.

(10)	(12)	(20)	(24)	(10)	(12)	(20)	(24)
lb.	oz.	d-w.	gr.	lb.	oz.	d-w.	gr.
42	- 11	- 14	- 18.	647	- 11	- 17	- 16.
26	- 9	- 10	- 21	197	- 10	- 15	- 23
49	- 10	- 11	- 11	494	- 11	- 19	- 14
65	- 8	- 14	- 12.	165	- 9	- 15	- 21
73	- 9	- 15	- 16	219	- 1	- 11	- 19
87	- 11	- 18	- 11	648	- 8	- 15	- 21
347	- 2	- 5	- 17				

Note 2. That 1 lb. *avoirdupois*-weight is equal to about 14 ounces 12 penny weights *troy*.

Note 3. Custom only introduced *avoirdupois*-weight, and beer-measure: for, according to the statute laws, there should be but one weight and one measure throughout the whole realm, as you may see under *dry-measure*, Note 1. Therefore, it is evident, that from these two different weights, came the different sorts of measures, as you will more plainly see under *dry-measure* and *liquid-measure*.

Note farther the Value of gold and silver.

Gold.		£	s.	d.
That 1 Pound weight of gold is worth		48	- 0	- 0
1 Ounce	— — — — —	4	- 0	- 0
1 Penny-weight	— — — — —	0	- 4	- 0
1 Grain	— — — — —	0	- 0	- 2
Silver.		£	s.	d.
That 1 Pound weight of silver is worth		3	- 0	- 0
1 Ounce	— — — — —	0	- 5	- 0
1 Penny-weight	— — — — —	0	- 0	- 3
1 Grain about $\frac{1}{2}$ a farthing.				

APOTHECARIES WEIGHT.

Tyro. What is the use of this Weight?

Philo. By it *apothecaries* mix and compound their medicines, their pound being the same as the pound *troy*,

only differently divided, as you see in the following Table.

Note, Though apothecaries mix their medicines by this rule, they buy and sell their drugs by avoirdupois-weight.

The TABLE.

20 Grains	}	make	{	1 Scruple	}	thus marked.
3 Scruples				1 Dram		
8 Drams				1 Ounce		
12 Ounces				1 Pound		

(10)	(12)	(8)	(3)	(20)	(10)	(3)	(20)
℥	3	3	℥	gr.	3	℥	gr.
42 -	10 -	7 -	2 -	19	18 -	1 -	14
17 -	8 -	5 -	2 -	11	12 -	2 -	17
43 -	7 -	6 -	1 -	10	41 -	1 -	10
15 -	4 -	5 -	1 -	9	24 -	2 -	15
47 -	11 -	3 -	1 -	8	16 -	1 -	11
64 -	9 -	6 -	2 -	15	17 -	2 -	15
232 -	5 -	4 -	0 -	12			

D R Y M E A S U R E.

Tyro. What is the use of *dry measure*?

Philo. By this rule is measured all dry goods, such as coals, sand, salt, fruit, oysters, wheat, barley, peas, and other grain, as appears by the two following tables.

T A B L E 1.

2 Pints	}	make	{	1 Quart
2 Quarts				1 Pottle
2 Pottles, or 8 pints				1 Gallon
2 Gallons, or 16 Pints				1 Peck
4 Pecks				1 Bushel
4 Bushels				1 Coomb
2 Coomb, or 8 bushels				1 Seam, or quarter
5 Quarters, or 40 bushels				1 Load

TABLE

TABLE 2. For Coals.

4 Pecks	}	make	{	1 Bushel
9 Bushels				1 Quarter of a Chaldron
4 Quarters, or 36 bushels				1 Chaldron
Note, 5 Pecks				1 Bushel, water-measure

Note 1. That 33 cubic inches, and 3 fifths, make a corn pint, 268 inches, and 4 fifths, a corn gallon, and 2150 inches, 2 fifths, a true *Winchester* bushel, according to act of parliament, made in 1697, which says, that every round bushel with an even bottom, that is, 18 inches wide, and 8 inches deep, shall be deemed a true legal *Winchester* bushel.

(10)	(5)	(8)	(4)	(10)	(4)	(9)	(10)	(36)	(4)		
Loads	qrs.	B.	Pecks	Chal.	qrs.	Bush.	Chal.	B.	Pecks		
24	-	4	-	7	-	3	64	-	2	-	8
19	-	3	-	5	-	2	17	-	1	-	6
49	-	2	-	3	-	1	48	-	2	-	3
47	-	3	-	1	-	2	17	-	1	-	4
56	-	1	-	5	-	3	96	-	2	-	5
17	-	3	-	4	-	2	17	-	3	-	7
215	-	4	-	4	-	1					

Note 2. The common received contents of a corn gallon is 272 inches. For dry measure is a mean, as it were, between wine and beer measure. For as 12 oz. troy is to 224 inches; so is 14 $\frac{1}{2}$ oz. to 272 inches nearly. See note 3, in wine measure.

Note 3. That salt, sea-coal, and many other commodities, are heaped in the measuring in general; and where they are not, it is customary to allow 5 struck pecks to the bushel. Bran is double measure; that is, 2 pecks struck are allowed for one peck.

Observe farther.

A Score of coals	—	is	21 chaldron, that
is, those that buy 20 chaldron have			21 for 20
A sack of coals	—		3 Bushels
A sack of corn in common	—		4 Bushels
A sack of flour	—		5 Bushels
A load, called a market load	—		5 Bushels
A load in general means	—		40 Bushels
A wey is 10 quarters, or 2 loads			80 Bushels
A last in some places is 12 wey, viz.			960 Bushels
See section 6, in Addition.			

LIQUID-MEASURE.

Tyro. What is the use of this measure?

Philo. All sorts of wine, spirituous liquors, and beer and ale are measured by it, under the names of *Wine measure* and *Winchester-measure*.

I. Of WINE-MEASURE.

T A B L E.

2 Pints	} make	1 Quart
4 Quarts		1 Gallon
10 Gallons		1 Anker of brandy, or rum
18 Gallons		1 Runlet
31 $\frac{1}{2}$ Gallons		1 Barrel of wine or vinegar
42 Gallons		1 Tierce
63 Gallons		1 Hogshead
2 Hhds. or 126 gallons		1 Butt, or pipe
2 Pipes, or 252 gallons		1 Tun

Note 1. A *puncheon* is 8 ankers, or 80 gallons; but any cask between a *bogshead* and a *pipe*, is called a *puncheon*.

(10) (2) (2) (63)	(10) (63) (8)
Tuns Pipes. Hhds. Gall.	Hhds. Gall. Pints.
47 - 1 - 1 - 21.	47 - 19 - 7
49 - 0 - 1 - 57	26 - 14 - 1
64 - 1 - 0 - 16.	94 - 17 - 5
45 - 1 - 1 - 18	57 - 47 - 6
27 - 0 - 0 - 15	19 - 14 - 4
56 - 1 - 1 - 17	45 - 49 - 5
991 - 1 - 0 - 18	

Note 2. Cyder, perry, oil, vinegar, &c. are bought and sold by this measure; and milk is also sold in the city of London by it; there being no standard to the contrary, corrupt custom has reduced the largest of liquid pints to one half its proper quantity.

Note 3. The wine pint (according to custom) is reckoned to contain 28 cubic inches, and 7 eighths; and the gallon 231 inches. But by an experiment made at Guild-Hall, in London, (1688) a vessel containing but 224 cubic inches, was filled with water, and carefully emptied into the wine gallon kept there, which did exactly fill it. But notwithstanding this, it was thought proper to continue 231 inches to the gallon, which remains to this day. And from this increase of the wine gallon (answering to troy-weight) came the increase of the beer gallon, which answers to avoirdupois-weight. See note 1. in Winchester measure

2. WINCHESTER-MEASURE.

Tyro. What are the different measures and denominations for beer and ale?

Philo. They are as under

T A B L E.

2 Pints	} make {	1 Quart
4 Quarts		1 Gallon
9 Gallons		1 Firkin
2 Firkins, or 18 gallons		1 Kilderkin
2 Kilderkins, or 36 gallons		1 Barrel
1 $\frac{1}{2}$ Barrel, or 54 gallons		1 Hoghead
2 Hhds, or 3 bar. or 108 gall.		1 Butt
2 Butts, or 216 gallons		1 Tun

Note

Note 1. That 35 cubic inches 1 quarter make a beer pint, and 282 (nearly) 1 gallon, which answers to *avoirdupois weight*. For as 12, the ounces in 1*lb. troy*, is 231, the inches in a wine gallon; so is 14 $\frac{1}{2}$ ₀ ounces to 282, the inches in a customary beer gallon. See *note 2, Troy weight*.

(10)	(2)	(1 $\frac{1}{2}$)	(2)	(2)	(9)	(10)	(3)	(36)
Butts	Hbds.	Bar.	Kild.	Firk.	Gall.	Butts	Bar.	Gall.
15	-	1	-	0	-	1	-	5
23	-	0	-	1	-	0	-	2
41	-	1	-	1	-	1	-	3
22	-	1	-	0	-	1	-	0
7	-	1	-	1	-	0	-	5
1	-	0	-	1	-	1	-	0
112	-	1	-	1	-	1	-	0
						3		

Note 2. That in gauging beer and ale in London, 32 gallons is a barrel of ale, and 36 a gallon of beer; but in other places 34 gallons is a barrel, one with another, and common brewers in the country allow victuallers 36 gallons of both sorts to the barrel.

CLOTH-MEASURE.

Tyro. What are the denominations of this measure?

Philo. Yards, quarters, and nails: also, ells *English*, ells *Flemish*, and French ells, as appears by the table.

TABLE.

2 inches and a quarter	}	make	1 nail
4 nails			1 quarter of a yard
4 quarters			1 yard
3 quarters of a yard			1 ell <i>Flemish measure</i>
5 quarters, or 1 yd. 1 qr.			1 ell <i>English</i>
6 quarters			1 <i>French ell</i>

Note 1. That things in common, such as woollen, linen, silk, tape, cord, &c. are measured by the yard: but *hollands* are in general measured by the ell *English*, and *tapestry* by the ell *Flemish*.

Note

Note 2. That 16 nails make a yard, 20 nails an ell *English*, and 12 nails an ell *Flemish*.

(10) (4) (4)	(10) (5) (4)	(10) (3) (4)
<i>Yds. qrs. Nails</i>	<i>Ells Eng. qrs. N.</i>	<i>Ells Fl. qrs. N.</i>
47 - 3 - 2	64 - 4 - 3	17 - 2 - 3
19 - 2 - 1	17 - 2 - 1	41 - 1 - 2
16 - 3 - 2	19 - 3 - 2	19 - 2 - 1
14 - 1 - 1	15 - 4 - 3	64 - 1 - 2
17 - 1 - 2	16 - 2 - 1	14 - 2 - 1
21 - 2 - 1	17 - 2 - 3	25 - 1 - 3

137 - 2 - 1

LONG-MEASURE.

Tyro. What does *long measure* teach?

Philo. To know the length or breadth of any thing, and the distance of one thing, or place from another, as by the following table.

T A B L E.

3 barley corns	}	make	1 inch
12 inches			1 foot
3 feet			1 yard
2 yards, or 6 feet			1 fathom
5½ yards			1 rod, pole, or perch
40 rods			1 furlong
8 furlongs			1 mile
3 miles			1 league
20 leagues, or 60 miles			1 degree
360 degrees			the circumference of the earth and sea.

Note 1. An hand, or hand's breath in horsemanship is 4 inches.
A common pace is 2 feet 6 inches.
A geometrical pace is 5 feet.

(10)	(20)	(3)	(8)	(40)	(10)	(5 $\frac{1}{2}$)	(3)	(12)
Deg.	Leag.	Miles	Fur.	Rods	Rods	Yds.	Feet	Inches
25	- 19	- 2	- 7	- 21.	42	- 4	- 1	- 11
17	- 14	- 1	- 5	- 19	61	- 3	- 0	- 10
16	- 8	- 2	- 2	- 15.	17	- 1	- 1	- 9
21	- 5	- 1	- 4	- 27	24	- 0	- 2	- 8
47	- 7	- 2	- 1	- 14.	16	- 2	- 1	- 7
15	- 10	- 1	- 5	- 37	72	- 1	- 0	- 6

144 - 7 - 0 - 3 - 13

Note 2 According to the table 60 miles make one degree, therefore, the earth is 21600 miles round: but $69\frac{1}{2}$ miles (very nearly) make but one degree, and therefore the circumference of the earth is about 25000 miles; as you will see in *Reduction*.

LAND-MEASURE.

Tyro. What are the denominations of *land measure*?

Philo. Almost the same as in *long-measure*; but as they never regard the inches and barley-corns, this table is sufficient.

T A B L E.

5 $\frac{1}{2}$ Yards, or 16 feet $\frac{1}{2}$	} make {	1 rod, pole or perch
40 rods or poles		1 furlong in length
40 rods in Length, and 1 in breadth		1 rood or quarter of an acre
4 roods, or quarters		1 acre

Note 1. That though $16\frac{1}{2}$ feet make a statute pole or rod, yet it is customary in some low, fenny countries, and barren lands, to allow 18 feet, and in measuring forests 21 feet to the pole.

(10)	(4)	(40)	(5 $\frac{1}{2}$)	(10)	(4)	(40)
Acres	Roods	Poles	Yds.	Acres	Roods	Poles
7	- 1	- 14.	- 1	15	- 1	- 19
9	- 3	- 27	- 2	26	- 2	- 24
4	- 1	- 29.	- 3	19	- 3	- 27
15	- 2	- 27	- 4	47	- 1	- 15
37	- 1	- 18	- 4 $\frac{1}{2}$			

Note

Note 2. The common instrument used in measuring land is an iron chain, containing 100 links, which is 4 rods, or 22 yards in length; therefore 10 chains in length, and 1 in breadth, make an acre, and 8 chains in length only make a mile, or 1760 yards.

S Q U A R E - M E A S U R E .

Tyro. What do you mean by *square measure*?

Philo. You are not to expect, *Tyro*, as yet to know the nature of it; it is sufficient at present, that you only know this, that *long-measure* shews you only the length or breadth of any thing; but *square-measure* tells you the content of any thing, which you can have no notion of till you have learnt *Division*; but it will not be amiss to learn the following table by heart (at your leisure) that you may be the better able to understand it by and by.

16 square quarters	}	make	{	1 square inch
144 square inches				1 square foot
9 square feet				1 square yard
30 square yards, 1 qr. or				1 square rod, or
272 square feet 1 quarter				1 pole
160 square rods	}		{	1 acre of ground

Note, An example or two will be sufficient.

(10) (9)

Yards Feet

45 - 6

29 - 3

47 - 8

25 - 5

148 - 4

(10) (9) (144) (16)

Yards Feet Inches Qrs.

17 - 5 - 71 - 3

64 - 2 - 14 - 6

16 - 4 - 17 - 12

78 - 5 - 94 - 11

O f T I M E .

Tyro. What is *time*, and how is it divided?

Philo.

Philo. Time shews us the beginning, mutation, (or changing) continuation, and ending of all mutable things. It is measured by *years, months, days, hours, minutes, and seconds*, and divided as follows, which will serve all common purposes.

T A B L E.

60" seconds	}	make	{ 1 minute of time
60' minutes			{ 1 hour
24 hours			{ 1 natural, or real day
7 days			{ 1 week
4 weeks			{ 1 month
13 months, or			{ 1 year
365 days			

Note 1. Though the table says 13 months, or 365 days make a year, yet it is not truly so; for 13 months (allowing 4 weeks to the month) is but 364 days; whereas 13 months, 1 day, 6 hours, make a year; for these odd 6 hours make 24 hours, or 1 day, every fourth year, which is added to *February*, which has then 29 days, and is called *leap year*: but a true year is 365 days, 5 hours, 48 minutes, and is called a *solar year*, being the time that the sun performs its apparent revolution through the *ecliptic*; but you must not busy yourself, and lose your time about these things, which are too hard for you to understand at present.

Note 2. Though 13 months are said to make a year, and servants commonly reckon a month 28 days; yet you are to observe, that in trade, and transacting business, by a month is meant a *calendar-month*; that is from any day of the month to the same day of the next month; thus, from the 5th of *February* to the 5th of *March*, or from the 18th of *April* to the 18th of *May*, is a month.

Ex-

EXAMPLE 1.

(10) (13) (4) (7)
Yrs Mths. Wks. Days

27 - 11 - 3 - 6

43 - 9 - 2 - 5

27 - 5 - 1 - 4

36 - 1 - 2 - 3

45 - 10 - 2 - 1

28 - 12 - 1 - 4

209 - 12 - 2 - 2

EXAMPLE 2.

(10) (24) (60) (60)
Days Hrs. Min. Sec.

41 - 22 - 50 - 27

17 - 17 - 17 - 15

24 - 15 - 27 - 14

19 - 21 - 19 - 24

27 - 11 - 18 - 19

35 - 14 - 25 - 25

Note 3. In example 2, you must point or dot at every row but the last.

And now, *Tyro*, I think you may by this time be perfect in *Addition*, I shall only see you a few questions, and proceed to *Subtraction*.

SECTION VI.

Containing some other useful things necessary to be known in number, weight, and measure.

Note 1. Of things bought and sold by the dozen, score, or gross.

A Dozen is 12. A score is 20. A common hundred is 100. A long hundred is 120. A gross is 12 dozen, or 144, and a great gross is 12 times as many, or 1728, oranges and lemons, corks, bungs, and many other things, are bought and sold by the dozen or gross. Herrings, and several other sorts of fish; and all sorts of nails, and many such small things, have six score, or 120, to the hundred; but a hundred of ling cod is 124 in number; and a hundred books in printing is 104.

Note

Note 2. Of PARCHMENT and PAPER.

1 dozen is 12 skins; 5 dozen 1 roll of *parchment*; 24, and sometimes 25 sheets make a quire of *paper*, 20 quires 1 ream, and 10 ream 1 bale.

Note 3. Of the different sizes of Books.

Folio is the largest of all books, and has but 2 leaves to the sheet. *Quarto* (marked 4to) has 4 leaves to a sheet. *Octavo* (or 8vo) is a sheet doubled into 8 parts and *duodecimo* (commonly called *twelves*, and marked 12mo.) has 12 leaves to the sheet.

Note 4. Of WEIGHT, MEASURE, &c.

A faggot of steel 6 score, or 120 lb. A burthen or gad of steel 9 score. A barrel of anchovies 30 lb. A barrel of figs from 98 to 300 lb. A barrel of gun-powder 1 Cwt. A puncheon of prunes from 10 to 12 Cwt. A ton, or fother of lead 19 Cwt. 2 qrs. A quintal of fish 100 in tale. A stone of iron, shot, or horseman's weight 14 lb. A stone of meal 8 lb. A stone of hemp 32 lb. A stone of glass 5 lb. A seam of glass 24 stone, or 120 lb. A keg of herrings, &c. 60 in number, an hundred is 120. A cade of sprats 1000. A cade of herrings 500. A barrel 1000. A last 12 barrels, or 12,000. A last of corn or rape-seed 10 qrs. A last of gun-powder 24 barrels. A last of leather 20 dickers. A dicker 10 skins. A last of hides 12 dozen; of tar 14 barrels; of wool 12 sacks; of flax or feather 1700 lb. A wey in some places is 5 chaldron. A wey of meal 6 qrs. A gallon of train oil $7\frac{1}{2}$ lb. A tun 252 gallons. A tun of sweet oil 236 gallons. A load of hay in some places is 25, in others 30 cwt. In London it is sold in trusses, containing 56, or 60 lb. a truss, and 36 trusses to the load. A load of Scotch coals 1 cwt. A load of tiles 1000.

Of

Of bricks 500. Bricks in general are 9 inches long, $4\frac{1}{2}$ inches broad and $2\frac{1}{2}$ thick. A square of tiling or thatching contains 10 feet every way, that is 100 feet, and a rod of brick-work 272 feet, 1 quarter; but 272 is reckoned for common work. A stack of wood is three feet high, 3 feet wide, and 12 feet long; but this is according to the agreement of the master and the workmen.

Tyro. I am extremely obliged to you, sir; but pray am I bound to get these things by heart before I learn *subtraction*?

Philo. Not at all, *Tyro*: learn the rules and the common tables is sufficient. The others I have only added for your further instruction: They are not set you for a task, but for your diversion; and if you now and then read them over, you will soon find the benefit of thus improving your mind; since it will naturally gain you the good will of your parents, your master, and mankind in general, rather than squandering away your time in idleness and mischief, besides the disgrace of living and dying a dunce.

Rice Price 56 Reg

SECTION VII.

Contains some useful and diverting questions to exercise the learner in Addition only.

Quest. 1 **A** Man borrowed of his friend a certain sum of money, and paid him in part 15*l.* 10*s.* and left unpaid 24*l.* 10*s.* What did he borrow? *Ans.* 40*l.*

Rule. Add the two sums together gives you the answer.

2. Suppose a person was born in 1709, when will he be fourscore years old. *Ans.* in 1789.

Rule. Add as many to 1709 as will make it 1789 is the answer.

3. *A.*

3. *A, B, and C agreed to purchase an Estate, A laid out, or paid for his part, 140*l.* 10*s.* B. paid 217*l.* 10*s.* C paid 500*l.* 10*s.* What did the estate cost? *Ans.* 851*l.* 10*s.* Rule. *Add all the sums together.**

4. A factor bought 4 bags of hops, the first, (No. 1.) weighed 2 C. 1 *qr.* 14 *lb.* No. 2. 3 *qrs.* 17 *lb.* No. 3, 2 C. 3 *qrs.* 13 *lb.* and No. 4. weighed 1 *qr.* 27 *lb.* What is the weight of all? *Ans.* 6 C. 2 *qrs.* 15 *lb.* *Add all together is the answer.*

5. A shop-keeper having opened a shop, sold the first day as many goods as came to 15*l.* 13*s.* 7*d.* $\frac{3}{4}$, and thus he went on for one week, (*viz.* 6 days.) How much did he take in all? *Ans.* 94*l.* 1*s.* 10*d.* $\frac{1}{2}$.

Rule. *Set down 15. 13. 7. $\frac{3}{4}$, six times, and add them together gives the answer.*

6. A farmer carried out 100*l.* in order to buy cattle, and brought home but 34*l.* 15*s.* 6*d.* What did he lay out? *Ans.* 65*l.* 4*s.* 6*d.* For these two added together make just 100*l.* and so on for any other sum.

DIALOGUE III.

SECTION I.

SUBTRACTION in whole numbers.

Tyro. **WHAT** does subtraction teach?

Philo. Subtraction teaches, to take a less number from a greater to discover the difference. To prove the work, add the difference to the less number, and if that sum be the same as the greater, the work is right, otherwise false.

Tyro. How is subtraction performed?

Philo.

Philo. Quite contrary to *addition*; for in *addition* you use the word *and*, but *here* you use the word *from*. Thus 4 *from* 7, that is, 4 *taken out of* 7, there remains 3, for the *difference*. Again, 5 *from* 10, there remains 5; and 6 *from* 14 there remains 8, and so for any other numbers, as you may see by the following examples.

EXAMPLE 1. Ex. 2. Ex. 3. Ex. 4.

	£.	Yards	Ells	lbs.
Greater number	27	95	468	441756
Lesser number	14	61	162	240256
Difference	13	34	306	201500
Proof	27	95		

The WORK in words at length.

In *example 1*, I say, 4 *from* 7 there remains 3; and 1 *from* 2 there remains 1. To prove it, I add the *difference* to *less numbers*, saying, 3 *and* 4 is 7, and 1 *and* 1 makes 2. In *Ex. 2*, I say 1 *from* 5 there remains 4; and 6 *from* 9 there remains 3. *Proof*, I say 4 *and* 1 is 5; 3 *and* 6 is 9: That is, 34 *and* 61 added together make 95, and so for the other examples.

Tyro. This is quite plain indeed.

Philo. I shall work *example 3* and 4 in the same manner, and leave you to prove them as I have directed. Observe then, *Ex. 3*, I say, 2 *from* 8 there remains 6; then 6 *from* 6, or 6 *taken out of* 6, there remains 0, and 1 *from* 4 there remains 3. Lastly, in *Ex. 4*, I say, 6 *from* 6, there remains 0; then 5 *from* 5 there remains 0; 2 *from* 7 there remains 5; then 0 *from* 1, that is nothing *taken out of* 1, there still remains 1; then 4 *from* 4, there remains 0, and 2 *from* 4, 2.

Note, I shall give you an example or two with, and without cyphers, that you may endeavour to avoid that idle custom.

Ex.

Ex. 5, *with cyph. without* Ex. 6, *with cyph. without*

From 24526

24526

84257

84257

Take 00012

12

84215

84215

Ans. 24514

24514

00042

42

Tyro. I own that the examples without the cyphers look best, and I understand very well what you have shewn me.

Philo. I shall leave you then 3 or 4 examples for practice.

Yards

Ells

Hundreds

From 47162

94785

347621

Take 12412

3104

301

Tyro. Now, sir, be pleased to shew me how to subtract or manage a sum, when the lower figure is sometimes larger than the top one; for I think that it appears difficult to me at present.

Philo. Never fear; you will soon find it easy, if you do but observe the following rule.

2. When the lower figure is larger than the top one, the rule is,

Take the lower figure out of what you do by, which (in whole numbers) you know is ten, and to that difference, or remainder, add also the top figure, and that is the true difference, which place under the first row. This is what is called *borrowing in subtraction*, therefore, remember that you are always to carry 1 to the next lower figure for so doing: an example or two will make it quite plain.

	Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.
	£.	Ells	Yards	Bushels.
From	762	85420	760410	5217624
Take	145	17273	457146	3471276
Difference	617	68147		

Here in *ex. 1.*, I say 5 out of 2 I cannot have; therefore I take 5 out of 10, (which is what I do by here) and there remains 5, and the top figure 2 makes 7, which I place under the first row, and carry 1 to the next lower figure; saying, 1 that I carry to 4 makes 5, which taken out of 6, that is, 5 from 6 there remains 1; but now I carry nothing to the next figure, because the lower figure being less than the top-one, I had no occasion to borrow; therefore, I only say in the last row, 1 from 7, there remains 6: again in *ex. 2.* I say, 3 from 0 I can't, but three from 10 there remains 7, and 0 is 7 still; then I carry 1 to 7 is 8, from 2 I can't, but 8 from 10 there remains 2, and the top figure 2 makes 4: then I carry 1 to 2, which is 3, saying 3 from 4 there remains 1; but now I do not carry any to the next figure, because I did not borrow, but only say, 7 from 5 I can't take, but 7 from 10 there remains 3, and the top 5 makes 8. *Lastly*, I carry 1 to 1 is 2 from 8 there remains 6.

Tyro. This is plain enough.

Philo. This is the easiest way for a learner, but there is another method which is more practicable and expeditious, if you mind and learn it.

Another way to subtract, when the lower figure is larger than the top one.

When you cannot take the lower figure from the top-one, then count the top figure ten more than it really is. Thus, if the top figure be 2 call it 12; if 3 call it 13; if 5 call it 15; if 8 call it 18, &c. and then

then take the lower figure from it, and you have the true *answer* or *difference*: but always remember to carry 1 to the next figure for so doing. This is called *borrowing ten*, and your carrying 1 to the next figure is paying of it again.

EXAMPLES.

	£.	Yards.	Ells.
From	75043	41725	172560
Take	27365	17258	87275
Difference	47678	24467	
Proof	75043	41725	

First, I say 5 from 3 I can't; but calling 3, 13, I say 5 from 13, there remains 8; then I carry 1 to six is 7 from 4 I can't, but I say 7 from 14 there remains 7; then I carry 1 to 3 is 4 from 0 I can't, but 4 from 10 there remains 6. Again I carry 1 to 7 is 8 from 5 I can't, but 8 from 15 there remains 7. *Lastly*, I carry 1 to 2 is 3 from 7 and there remains 4.

 *Note*, Tho' the bottom figures in *subtraction* may be larger than the top-ones; yet you are to remember the last lower figure is never larger than the top one when there is an equal number of figures.

Tyro. I understand you very well, sir.

Philo. Then I shall only leave you an example or two, to try at as you have leisure.

More EXAMPLES.

	£.	lb.	Miles.
From	417215	62170071	417621700
Take	241729	7210943	631720
Difference			

Of SUBTRACTION.

SECTION II.

Of MONEY.

Tyro. **H**OW is subtraction of money performed?
Philo. By taking or subtracting every denomination in the lower line, out of, or from the upper sum, as will appear more plain by the 2 following rules,

Rule 1. When the lower figures in the *farthings*, *pence*, or *shillings*, are smaller than the top-ones; then only subtract or take one from the other, and the remainder is the true *answer*, or *difference*, which is placed under each row to which it belongs.

Rule 2. When the lower figures in the *farthings*, *pence*, or *shillings*, are larger than the top-ones; then subtract or take the lower figure out of what you do by; that is, take the lower *farthings* out of 4, the lower *pence* out of 12, and the lower *shillings* out of 20, taking in the top figure besides; so shall this be the true *difference*, or *answer*; but remember, that you are always to carry 1 to the next figure for borrowing, as you did in *whole numbers*.

Tyro. Please to give me an example or two, I shall soon understand it.

Philo. You cannot miss if you mind the rule well.

Ex. 1.

Ex. 2.

Ex. 3.

	£.	s.	d.		£.	s.	d.		£.	s.	d.		
From	9	-	8	-	11				643	-	14	-	6 $\frac{3}{4}$
Take	7	-	5	-	6				231	-	11	-	2 $\frac{1}{4}$
Rem.	2	-	3	-	5				412	-	3	-	4 $\frac{1}{2}$
Proof.	9	-	8	-	11				643	-	14	-	6 $\frac{3}{4}$

D

I. I

1. I began at the pence in example 1, saying, 6 from 11 pence there remains 5 pence, which I place under the pence; then I say 5 shillings from 8, there remains 3 shillings; and 7 pounds from 9, there remains 2. So the difference is 2*l.* 3*s.* 5*d.*

P R O O F.

The work is proved like *whole numbers*, by adding the difference to the *lower* or *less* number: thus, 5 pence and 6 is 11 pence; then 3 shillings and 5 make 8 shillings; and 2 added to 7 make 9 pounds.

2. In example 2, I say, 3 from 5, there remains 2 pence: then 11 from 15, there remains 4; then 7 from 8 there remains 1; and 2 from 4 there remains 2. *Ex.* 3. having farthings, I say, 1 farthing from 3, there remains 2 farthings, which I set down thus $\frac{1}{2}$; then 2 from 6, there remains 4 pence; then 11 from 14 shillings, there remains 3. Lastly, for the pounds I say, 1 from 3, there remains 2: then 3 from 4, there remains 1; and 2 from 6 there remains 4: so is the difference or answer 4*12l.* 3*s.* 4*d.* $\frac{1}{2}$. Now if you add this 4*12l.* 3*s.* 4*d.* $\frac{1}{2}$ to the lower sum 23*1l.* 11*s.* 2*d.* $\frac{1}{4}$ you will find they amount to the same as the top sum, *viz.* 643*l.* 14*s.* 6*d.* $\frac{3}{4}$.

Note. After this manner is every sum done and proved, in *subtraction*.

More examples for practice.

From 471 - 11 - 2 $\frac{1}{4}$	576 - 19 - 11 $\frac{1}{2}$	409 - 11 - 5 $\frac{3}{4}$
Take 120 - 10 - 2	132 - 13 - 4 $\frac{1}{4}$	304 - 11 - 3 $\frac{1}{2}$
Remains		

2. When

2. When the shillings, pence, and farthings, are larger in the lower line than in the top-one.

EXAMPLE 4.

EXAMPLE 5.

EXAMPLE 6.

	(10)	(20)	(12)	(10)	(20)	(12)	(10)	(20)	(12)	(4)
	£.	s.	d.	£.	s.	d.	£.	s.	d.	
Lent	10	- 4	- 6	42	- 11	- 6	453	- 13	- 5	$\frac{1}{4}$
Received	7	- 10	- 8	17	- 14	- 10	326	- 15	- 8	$\frac{3}{4}$
Due	2	- 13	- 10	24	- 16	- 8	126	- 17	- 8	$\frac{1}{2}$
Proof	10	- 4	- 6	42	- 11	- 6	453	- 13	- 5	$\frac{1}{4}$

Here I say 8 pence from 6 pence I can't; but 8 from 12 (which is what I do by at pence) there remains 4, and the top 6 makes 10: then I carry 1, because I borrowed, saying, 1 that I carry to 10 is 11 shillings, which from 4 I can't take; but 11 from 20 (which is what I do by) there remains 9, and the top 4 makes 13. *Lastly*, I carry 1 to 7 is 8 from 10 there remains 2.

P R O O F.

I prove these examples the same way as before, by adding the *difference* to the *less sum*, saying, 10 pence, and 8 is 18 pence, that is, 6 pence above a shilling, or 12 pence; and 1 carry 1 to 13 is 14, and 10 is 24, which is 4 above 20 shillings, or 11. *Lastly*, I carry 1 to 7 is 8, and 2 makes 10.

Now, *Tyro*, pray try you at *example 6*, where there are farthings.

Tyro. In *example 6*, I say first 3 farthings from 1 farthing I can't; but 3 farthings from a penny, or 4 farthings, there remains 1, and the top farthing makes 2 farthings, which I set down thus $\frac{1}{2}$: then I carry 1 to 8 pence is 9 pence, which from 5 pence I can't have; but 9 from 12 there remains 3, and the top 5 makes

makes 8 pence: again, I carry 1 to 15 is 16, from 13 I can't; but 16 from 20 there remains 4, and the top 13 is 17 shillings. *Lastly*, I carry 1 to 6 is 7 from 3 I can't, but 7 from 13 there remains 6. Or 7 from 10 there remains 3, and the top 3 is 6: then I carry 1 to 2 is 3 from 5 there remains 2; but now carry none because I did not borrow; therefore I only say 3 from 4 there remains 1.

P R O O F.

I prove this, as before directed, saying, 2 farthings and 3 is 5 farthings, which is 1 farthing above a penny, and I carry a penny to 8 is 9, and 8 is 17 pence, 5 above 12, and I carry 1 to 17 is 18, and 15 is 33, which is 13 above 20, and I carry 1 to 6 is 7, and 6 is 13, which is 3 above 10, and I carry 1 to 2 is 3, and 2 is 5; then I say 1 and 3 is 4.

Philo. You have done very well indeed; I shall only set you an example or two for practice.

E X A M P L E S *for Practice.*

	£.	s.	d.		£.	s.	d.
<i>Borrowed</i>	217	- 10	- 0	<i>Debtor</i>	1000	- 00	- 0
<i>Paid</i>	109	- 15	- $8\frac{1}{4}$	<i>Creditor</i>	910	- 15	- $6\frac{1}{2}$

To pay

Balance

Tyro. These I can do very well; therefore wish you would shew me how to balance an accompt, or make out a small reckoning.

Philo. That I will.

3. *Practical questions for business.*

<i>Quest. 1. A borrowed of B</i>	_____	100 - 00 - 0
	£. s. d.	
<i>A paid him at one time</i>	21 - 14 - 6	
<i>At another 10 guineas, viz.</i>	10 - 10 - 0	
<i>At another 20 guineas, viz.</i>	21 - 00 - 0	
<i>At another time</i>	5 - 15 - 0	
<i>Sold him goods amounting to</i>	18 - 14 - 9	
	77 - 14 - 3	
	In all	77 - 14 - 3

What is still due to B?

Answer 22 - 5 - 9

Rule. I add up together all the sums that *A* paid at different times, and find they amount to 77*l.* 14*s.* 3*d.* which I place under 100*l.* and subtracting it therefrom, I find that there is still due to *B* 22*l.* 5*s.* 9*d.* *answer.*

*Quest. 2. A Blacksmith delivered a bill to a Farmer of 45*l.* 14*s.* 6*d.* $\frac{1}{4}$, and the Farmer has paid him in part as under.*

		£. s. d.
By cash	_____	20 - 00 - 0
By 10 Bushels of malt	_____	1 - 15 - 0
By a load of hay	_____	1 - 16 - 0
By meat at several times	_____	2 - 7 - 6
By 14 bushels of oats	_____	2 - 11 - 0
By 20 bushels of wheat	_____	4 - 7 - 3
		32 - 16 - 9

		£. s. d.
<i>Blacksmith's bill</i>	_____	45 - 14 - 6 $\frac{1}{4}$
<i>Farmer's bill</i>	_____	32 - 16 - 9

Balance due to the *Blacksmith* 12 - 17 - 9 $\frac{1}{4}$ *answer.*

Quest. 3. A *steward* collected as much money for rents as amounted to _____ $\text{£. } 4000$
 And remitted to his master as under. _____

By cash at one time	_____	500
At another	_____	500
At another	_____	210
At another	_____	420
By tax bills discounted	_____	41 - 15 - 6 $\frac{3}{4}$
By repairs done to the estate	_____	641 - 14 - 9
By other charges	_____	15 - 13 - 6

Laid out in all $\text{£. } 2329 - 3 - 9\frac{3}{4}$

Which takes from the top $\text{£. } 4000$
 there remains due to his master

$1670 - 16 - 2\frac{1}{4}$

Tyro. I understand you well.

Philo. Then I will leave you one *question* for practice, and you may do the work at leisure.

Quest. 4. Two persons *A* and *B* have a reckoning to settle as follows: *A* lent *B* $\text{£. } 300$, and some time after lent him 100 guineas more: *B* paid him at 3 several times, each 100 guineas, and at another time gave him a *draught* or *note* upon *C*, for $\text{£. } 50$, and sold him as many goods as came to $\text{£. } 34-4-6$: now I demand how the balance stands between them. *Ans.* there is still due to *A* $\text{£. } 5-15-6$. To prove it.

Rule. Add the two sums that *A* lent to *B* together, and you will find them $\text{£. } 405$; then set all that *B* paid in cash, and the *draught* and goods, all under one another, and add them together, which makes $\text{£. } 399-4-6$: this being done, set $\text{£. } 399-4-6$ under $\text{£. } 405$, and *subtract* it therefrom, and you will find there remains $\text{£. } 5-15-6$ due to *A* on balance.

Tyro. I will try it directly, and am sure it is easy enough, but your answer will be some help to me I own.

Philo. As every accompt of *debtor* and *creditor* is settled after this manner, I shall give no more examples, expecting

expecting by this time you are capable of setting yourself questions; and if you are not, it is highly necessary to look over the examples once more, then will you be duly qualified for those questions I shall set you at the end of *subtraction*: and therefore we will proceed to *weights and measures*, where you will find some examples done for your instruction, and some left undone for your practice.

AVOIRDUPOIS WEIGHT.

	Tons	C.	qr.	lb.		C.	qr.	lb.	oz.	dr.
Bought	17	- 11	- 1	- 15	Bought	47	- 1	- 17	- 11	- 14
Sold	14	- 17	- 3	- 21	Sold	19	- 0	- 25	- 10	- 15
Unfold	2	- 13	- 1	- 22	Unfold					

Now, *Tyro*, I shall shew you this first *example*, but no more; because all other sums in every one of the rules are done in the same manner, and there is no occasion for further instruction.

First, I say 21 from 15 I can't, but 21 from 28 (which is what I do by at *lbs.*) there remains 7, and 15 makes 22: then I carry 1 to 3 is 4 from 1 I can't: but 4 from 4 (which is what I do by at *qrs.*) there remains 0, but the top 1 is 1, which I set down. Again, I carry 1 to 17 is 18 from 11 I can't; but 18 from 20 there remains 2, and the top 11 makes 13. *Lastly*, I carry 1 to 4 is 5 from 7, there remains 2, and the work is done.

Tyro. I thank you Sir; examples now without words, will be sufficient.

Note, If you forget what you do by, turn to your tables in *addition*.

Of SUBTRACTION.

TROY-WEIGHT.

	lb.	oz.	dwt.	gr.		lb.	oz.	dwt.	gr.
Bought	14	5	15	14	Bought	214	10	17	11
Sold	9	9	13	17	Sold	109	10	17	15
Rem.	4	8	1	21	Remains				

DRY-MEASURE.

	Loads	bush.	pecks	pints		Chal.	bush.	pecks
Bought	42	17	2	12	Bought	291	21	2
Sold	17	34	2	14	Sold	173	27	3
Unfold	24	22	3	14	Unfold			

WINE-MEASURE.

	Butts	hhd.	gall.	pints		Hbds.	gall.	pints
Bought	14	1	47	0	Bought	64	35	1
Sold	9	0	51	3	Sold	17	46	2
Unfold	5	0	58	3	Unfold			

WINCHESTER-MEASURE.

	Bar.	gall.	pints		Butts	hhd.	bar.	gall.
Brewed	124	21	4	Brewed	21	1	1	20
Sold out	92	27	6	Sold out	15	1	0	26
Unfold	31	29	6	Unfold				

Note, I have reckoned 36 gallons to the barrel, that being customary in selling beer in most places, as I said before.

CLOTH-

CLOTH-MEASURE.

	Yards	qrs.	N.	Ells	Eng. qrs.	N.	Ells	Fl. qrs.	N.						
Bought	147	-	1	-	0	47	-	1	-	3	47	-	2	-	0
Sold	96	-	3	-	2	18	-	2	-	3	29	-	2	-	1
Rem.	50	-	1	-	2										

LONG-MEASURE.

	Deg.	leag.	miles	fur.	rods	yards	feet	inc.	B.	corns							
From	471	-	14	-	1	-	3	-	17	-	2	-	1	-	10	-	1
Take	167	-	17	-	1	-	5	-	21	-	3	-	1	-	10	-	2
Rem.	303	-	16	-	2	-	5	-	35	-	3 $\frac{1}{2}$	-	2	-	11	-	2

LAND-MEASURE.

	<i>Acres</i>	<i>roods</i>	<i>poles</i>	<i>yds.</i>	<i>Acres</i>	<i>roods</i>	<i>poles</i>
<i>From</i>	471	-	2	-	15	-	3
<i>Take</i>	196	-	2	-	26	-	4
<i>Rem.</i>	274	-	3	-	28	-	$4\frac{1}{2}$

T I M E.

	<i>Yrs.</i>	<i>mths.</i>	<i>wks.</i>	<i>days</i>	<i>hours.</i>	<i>days</i>	<i>hours</i>	<i>min.</i>	<i>sec.</i>
<i>From</i>	219	-	10	-	3	-	3	-	17
<i>Take</i>	193	-	11	-	3	-	4	-	20
<i>Rem.</i>	25	-	11	-	3	-	5	-	21

Note. In the first example I reckon 13 months to a year.

Tyro. Sir, I am obliged to you for the pains you have taken; I understand you very well, and will try at all those sums you have left undone.

Philo. That is enough, we will finish *subtraction* then with *Section 3.*

SECTION III.

Containing some useful QUESTIONS to exercise the learner in both RULES.

Quest. 1. **K**ING Harry the eighth died 1547; I demand how many years it is, this being now 1770. *Answer* 223 years.

Quest. 2. Suppose this present year 1770, you were 19 years old, what year was you born in. *Ans.* 1751.

Quest. 3. A boy had 1000 marbles, and he lost at 3 different times at play, each 175, and at another time 150; how many has he still in hand? *Ans.* 325.

Rule. Set down 175 three times, add 150 under that; and then add all of them together, and take the sum out of 1000, and the answer will be 325 left. This rule serves for all that follows.

M O N E Y.

4. A lent B £. 500, and B paid him 4 times, each £. 120. 10s. What is due to A? *Ans.* £. 18.

5. What sum of money must I add to £. 58-14-6 $\frac{3}{4}$, to make it up £. 100? *Ans.* £. 41-5-5 $\frac{1}{4}$.

6. A proof to question 5. What sum must I take out of £. 100, to have the remainder £. 58-14-6 $\frac{3}{4}$. *Answer* £. 41-5-5 $\frac{1}{4}$.

7. A Collector of the excise received £. 2040-14-5 $\frac{1}{4}$, and remitted or paid at 3 several times, each £. 500, and at another time 100 guineas: what has he still in hand? *Ans.* £. 435-14-5 $\frac{1}{4}$.

AVOIR-

AVOIRDUPOIS-WEIGHT.

8. Bought 6 *ton*, 14 *Cwt.* of iron, and sold at one time, 3 *ton*, 11 *C.* 2 *qrs.* and at several other times, sold by retail 15 *C.* 3 *qrs.* 17 *lb.* What remains unfold?
Ans. 2 *ton*, 6 *C.* 2 *qrs.* 11 *lb.*

TROY-WEIGHT.

9. A gentleman delivered to a silversmith 2 *lb.* 5 *oz.* 11 *dwt.* of silver; and he received a silver cup, which weighed 11 *oz.* 14 *dwt.* and at another time 6 large spoons, weighing 1 *lb.* 2 *oz.* 3 *dwt.* 14 *grs.* What weight of silver has the silversmith still in hand: *Ans.* 3 *oz.* 13 *dwt.* 10 *grs.*

LIQUID-MEASURE.

10. A common brewer has 2 goils of new beer, containing 32 butts, 2 barrels, 24 gallons of strong; and 15 butts 1 barrel of small. And he sent to one victualler 8 butts 2 barrels, to another 11 butts, and started into his own storehouses 5 butts 2 barrels, 27 gallons of the strong; and of the small 7 butts, 2 barrels; he also sold out by retail of the small, 4 butts, 23 gallons; I demand what remains of the brewing, both of the goil of strong, and the goil of small? *Ans.* 7 butts, 33 gallons of strong, and 3 butts, 1 barrel, 13 gallons of small.

Note, I have added the answers, that you may know when you are right or wrong; if you cannot do all these questions as yet, it is of no great signification; you must not stop your progress in going forward, because of that. Now follows

DIA-

DIALOGUE IV.

SECTION I.

MULTIPLICATION.

Tyro. WHAT is multiplication?

Philo. Multiplication is a compendious or short way of addition, and teaches us to tell the product of any sum in one line only, which would require several additions.

Tyro. What else is required?

Philo. There are three things to be carefully observed in multiplication.

First. The *multiplicand*, which stands a top, and is that number given to be multiplied.

Secondly. The *multiplier*, or sum you multiply by.

Thirdly, The *product* or *answer*, which is the *multiplicand* multiplied by the *multiplier*.

Tyro. Please to explain this a little more.

Philo. Let us take any two numbers, suppose 8 and 3. I set down 8, and place 3 under it; so is 8 the *multiplicand*, 3 is the *multiplier*, and 24 is the *product*, or *answer*, because 3 times 8 make 24. Is this plain enough.

Tyro. Yes, Sir, I understand you now.

Philo. Very well, *Tyro*, then I would have you get the following table by heart, by all means, notwithstanding it is too customarily omitted.

MUL-

MULTIPLICATION - T A B L E.

Once 1 is 1

2 times, or twice	{	2	4	5 times	{	5	25	9 times	{	9	81
		3	6			6	30			10	100
		4	8			7	35			11	121
		5	10			8	40			12	144
		6	12			9	45				
3 times	{	7	14	6 times	{	6	36	12 times	{	10	100
		8	16			7	42			11	121
		9	18			8	48			12	144
						9	54				
4 times	{	3	9	7 times	{	7	49	12 times	{	7	84
		4	12			8	56			8	96
		5	15			9	63			9	108
		6	18							10	120
		7	21							11	132
5 times	{	8	24	8 times	{	8	64	12 times	{	12	144
		9	27			9	72				
6 times	{	4	16	9 times	{	9	81	12 times	{		
		5	20								
		6	24								
		7	28								
		8	32								
7 times	{	9	36	10 times	{	10	100	12 times	{		
8 times	{			11 times	{	11	121	12 times	{		
9 times	{			12 times	{			12 times	{		

Table of twelves.

Note 1. You should be very careful to get the *table* by heart, as I said before ; saying thus, 2 times 2, or twice 2 is 4, twice 3 is 6, twice 4 is 8, twice 5 is 10, &c.

Note 2. You should also at leisure learn to *box* the *table* well : that is, to say it backwards, forwards, or any way. Thus you see twice 5 is 10, that is, 5 times 2 is 10 : Again, 3 times 9 is 27 ; so also is 9 times 3

the

the same. So 6 times 12 is the same as 12 times 6, and 5 times 9 the same as 9 times 5.

1. To multiply by single figures.

Rule. Multiply every figure in the *multiplicand* by the figure in the *multiplier*, carrying 1 for every 10, as in addition of whole numbers, and you have the *product* or *answer*.

EXAMPLE 1.

Multiplicand 17

Multiplier 5

Product 85

EXAMPLE 2.

Multiplicand 78

Multiplier 6

Product 468

I begin with the *multiplier* 5, saying, 5 times 7 is 35, and set down the 5, and I carry 3; then I say 5 times 1 is 5, and 3 that I carried is 8; so is the *product*, or *answer*, 85. Again,

In *Ex. 2.* I say, 6 times 8 is 48; that is 8, and I carry 4; then 6 times 7 is 42, and 4 I carried is 46: so is this *product* 468.

To prove the work by Addition.

I set the *multiplicand* 17 down 5 times; and the *multiplicand* 78 I set down 6 times, and adding them together, I find 85, and 468, as before; which you may try on your slate at leisure.

Tyro. I see plainly that *multiplication* saves the trouble of many *additions*.

Philo. There is another way to prove *addition*; but as it is of no manner of signification, we will lay aside those puzzling curiosities, and proceed to

Ex-

EXAMPLE 3.	EXAMPLE 4.	EXAMPLE 5.
Multiply 472 by 8	24970 by 9	and 590407 by 7
Answer 3776	Ans. 224730	Ans. 4132849

Tyro. I say 8 times 2, (or twice 8) is 16, 6 and I carry 1; then 8 times 7 is 56, and 1 is 57; that is, 7, and I carry 5; then 8 times 4 is 32, and 5 I carried is 37. In *Ex. 4*, I say, 9 times (0) nothing is 0; then 9 times 7 is 63, that is, 3 and I carry 6; then 9 times 9 is 81, and 6 I carried is 87, that is 7, and I carry 8; then 9 times 4 is 36, and 8 is 44, that is, 4 and I carry 4; and *lastly*, 9 times 2 is 18, and 4 I carried is 22. Also, in *Ex. 5*, I say 7 times 7 is 49, that is 9, and I carry 4; 7 times 0 is 0, but 4 is 4 still; then 7 times 4 is 28, that is 8 and I carry 2; then 7 times 0 is 0, but 2 is 2; then I carry none, but say, 7 times 9 is 63, that is 3, and I carry 6, and *lastly*, 7 times 5 is 35, and 6 is 41.

Philo. Very well done indeed! I leave these two examples for practice.

Multiply 49997296 by 9	and 94171470 by 8
Answer	Answer

Tyro. Please to give me an example or two to multiply by 12 in one line; for I know it is done much quicker, and it is as easy as to make two of it.

EXAMPLES with twelves.

Multiply 42576 by 12	and 994079 by 12	and 99807 by 12
Answer 510912	Ans. 11928948	Ans.

Firſt,

First, 12 times 6 is 72, that is, 2 and I carry 7; then 12 times 7 is 84, and 7 I carried is 91, that is 1, and I carry 9; then 12 times 5 is 60, and 9 I carried is 69, that is 9, and I carry 6; then 12 times 2 is 24, and 6 is 30, that is 0, and I carry 3; and *lastly*, 12 times 4 is 48, and 3 is 51. And thus by having the *table of twelves*, perfectly by heart, every sum will be easy to you.

Tyro. It is easy enough, I see: please now to shew me how you multiply by 2, 3, 4, or more figures.

Philo. This you will soon understand.

2. Of multiplying by several figures.

Rule. When there are several figures in the *multiplier*, then you begin at the first figure in the units place, and multiply it through, or into every figure of the *multiplicand*, as you have done before. This being done, multiply every figure in the *multiplicand* by the second figure of the *multiplier*; only observe to set the first figure of the second row under the *tens place* of the first row; and thus you go on with the third figure in the *multiplier*, placing the *units place* of each succeeding row, under the *tens place* or the row that is above it, till you have gone through every figure of the *multiplier*. Then draw a line under all the rows of figures, and add them together, and the sum is the true *product* or *answer*.

Note. Be sure you remember that you set your figures right under one another, or else in a large sum you will be puzzled to add the work together.

Ex-

EXAMPLES with several figures.

EXAMPLE 1.	PROOF.
Multiply 89	Multiply 47
by 47	by 89
623	423
356	376
Answer 4183	The same 4183

In *example 1*, I say, 7 times 9 is 63, that is 3, and I carry 6; then 7 times 8 is 56, and 6 I carried is 62, which I set down; so is the first line or row finished: Then I take the second figure of the *multiplier*, saying, 4 times 9 is 36, which 6 I set under the 2, or *tens place* of the first line and carry 3; and then I say, 4 times 8 is 32, and 3 that I carried is 35, placing the 5 under the 6, and the 3 quite out towards the left-hand. *Lastly*, I add these up in order, as they stand, saying, 3 is 3; then 6 and 2 is 8; again, 5 and 6 is 11, that is 1, and carry 1 to the 3 is 4.

To prove MULTIPLICATION.

It is a common way to prove *multiplication* by the cross; but it is subject to so many errors, that in short, it is no proof at all to a learner, but rather a corruption. The best way, therefore is this: take the *multiplicand* and set it below, and the *multiplier* a-top, that is, change the *multiplicand* into the *multiplier*, and proceed as before directed, and if the *product* be the same as before, your work is entirely right. See the next examples wrought at large both ways.

EXAMPLE 3.
Multiplicand 895
Multiplier 798

7160
 8055
 6265

Product 714210

PROOF.
Multiplicand 798
Multiplier 895

3990
 7182
 6384

The same 714210

First, I say, 8 times 5 is just 40; therefore I set down the 0, and I carry 4; then 8 times 9 is 72, and 4 I carried is 76, that is 6, and I carry 7; then 8 times 8 is 64, and 7 I carried is 71. Now I take the second figure, saying, 9 times 5 is 45; which 5 I place under the second figure, or *tens place* of the first line, and carry 4, saying, 9 times 9 is 81, and 4 is 85, which 5 I place under the figure 1, and carry 8 to the next figure, saying, 9 times 8 is 72, and 8 that I carried is 80. And now I come to the last figure of the *multiplier*, saying, 7 times 5 is 35, which 5 I place under the second figure of the last line, and carry 3 to the next figure; saying, 7 times 9 is 63, and 3 I carried is 66, that is 6 and I carry 6; then 7 times 8 is 56, and 6 is 62. *Lastly*, I add these up in order as they stand, and find the *product* 714210.

The proof of this example is worthy your observation, *Tyro*; for be the sum ever so large, if you change the *multiplicand* into the *multiplier's* place, and multiply right, you will find the *product* always the same.

Tyro. I see it plainly, sir; and I could not have thought *multiplication* had been so easy.

Philo. Nothing easier, when the *table* is once well learned. I shall now give you two more examples, and leave the rest undone for practice.

EXAMPLE 4.

$$\begin{array}{r}
 \text{Multiply} \quad 913876 \\
 \text{by} \quad 8759 \\
 \hline
 8224884 \\
 4569380 \\
 6397132 \\
 7311008 \\
 \hline
 8004639884
 \end{array}$$

EXAMPLE 5.

$$\begin{array}{r}
 \text{and} \quad 749076 \\
 \text{by} \quad 1795 \\
 \hline
 3745380 \\
 6741684 \\
 5243532 \\
 749076 \\
 \hline
 1344591420
 \end{array}$$

EXAMPLES for practice.

$$\begin{array}{r}
 \text{Multiply} \quad 4567879 \\
 \text{by} \quad 45769 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{Multiply} \quad 9567950 \\
 \text{by} \quad 32796 \\
 \hline
 \end{array}$$

 *Note, Remember, Tyro, that when you multiply by several figures, that you always set the first figure of the second line under the second figure of the first line, and the first figure of the third line under the second figure of the second line: and thus go on, leaving one figure out towards the right-hand every line, setting the next line one figure more towards the left-hand, and every figure under one another in their proper place.*

Tyro. I understand you well, sir.

Philo. Then I will set you but one example more, which you may prove yourself.

Multiply

<i>Multiply</i>	987654321
<i>by</i>	123456789
	8888888889
	7901234568
	6913580247
	5925925926
	4938271605
	3950617284
	2962962963
	1975308642
	987654321
<i>Answer</i>	121932631112635269

P R O O F.

<i>Multiply</i>	123456789
<i>by</i>	987654321

3. Of cyphers between the figures, &c.

The most difficult thing in this rule, is, when the *multiplier* has some figures, some cyphers; but if you be careful to mind the work of an example or two, you will soon understand it, as I shall explain them in words.

EXAMPLE 1.

<i>Multiply</i>	49657
<i>by</i>	30705
	248285
	3475990
	1489710
<i>Answer</i>	1524718185

EXAMPLE 2.

<i>Multiply</i>	7564965
<i>by</i>	5003007000
	52954755000
	2269489500
	3782482500
<i>Answer</i>	37847572849755000

Note,

Note, Pray observe, that you read the following instructions once or twice over, if you do not understand the work.

First, I begin at the 5, and multiply all the top figures by it, as before: now, as the next figure of the *multiplier* is a cypher, it is of no signification to multiply by *that*, because it will produce nothing but a line of cyphers: therefore, I bring it down, that is, I set a cypher under the figure 8, and then begin to multiply by the 7, saying, 7 times 7 is 49, 9 and I carry 4; which 9 I set down in the same line, on the left-hand of the cypher, and go on still to multiply by the 7; then as the next figure is a cypher again, I set down a cypher under it, as I did before, leaving the 9, and the cypher which stands by it, in the last line, both standing out towards the right-hand. This being done, I only multiply by the 3, and set it in the same line, and add all up, and the work is done. Again,

In *example 2*, there are 3 cyphers at the beginning of the *multiplier*, therefore I set 3 cyphers under the line, right under them: then multiply by the 7 as in all other sums; saying, 7 times 5 is 35, &c. Then I come to 2 cyphers more in the *multiplier*, and therefore, set 2 cyphers right under them, and then multiply by the 3, and place the product by the side of the 2 cyphers: then, as there are two cyphers, I again set down two cyphers, thus 00, under them, and then multiply by the 5, placing the product by the side of them. *Lastly*, I add the sum up as it stands, and it is done.

Note, These examples are proved like the others, by setting the *multiplier* a top, and the *multiplicand* under it.

More examples for practice.

$$\begin{array}{r} \text{Multiply} \quad 570965 \\ \text{by} \quad 2400500 \\ \hline \end{array}$$

$$\begin{array}{r} \text{and} \quad 71900072 \\ \text{by} \quad 5000960000 \\ \hline \end{array}$$

And now, *Tyro*, I shall finish this section with shewing you something of *contractions*, or *compendiums*.

4. Of CONTRACTIONS.

When any number of figures are given to be multiplied by 10, 20, 30, 40, 80, 90, 120, 1200: then set the cyphers out, and multiply by the figure or figures, and place the *product* by the side of the cyphers, as before, gives the *answer*.

$$\begin{array}{r} \text{Multiply} \quad 275 \\ \text{by} \quad 20 \\ \hline \end{array}$$

$$\begin{array}{r} \text{and} \quad 3275 \\ \text{by} \quad 90 \\ \hline \end{array}$$

$$\begin{array}{r} \text{and} \quad 6759 \\ \text{by} \quad 1200 \\ \hline \end{array}$$

$$\text{Answer} \quad 5500$$

$$\text{Ans.} \quad 294750$$

$$\text{Ans.} \quad 8110800$$

2. When you are to multiply by 1, the *answer* will be the same as the *multiplicand* itself is. If you are to multiply by 10, add 1 cypher to the *multiplicand*; if by 100, add 2 cyphers; if by 1000, add 3 cyphers: See the work two ways.

$$\begin{array}{r} \text{Multiply} \quad 1753 \\ \text{by} \quad 100 \\ \hline \end{array}$$

$$\begin{array}{r} 1753 \\ 1000 \\ \hline \end{array}$$

$$\begin{array}{r} 1753 \\ 10000 \\ \hline \end{array}$$

$$\text{Answer} \quad 175300$$

$$\text{Ans.} \quad 1753000$$

$$\text{Ans.} \quad 17530000$$

Or

Or rather thus :

Multiply 1753 by 1 answer the same, viz. 1753
 by 10 add 1 cypher, ansf. 17530
 by 100 add 2 cyphers, ansf. 175300
 by 1000 add 3 cyphers, ansf. 1753000
 by 10000 add 4 cyphers, ansf. 17530000

And thus for as many cyphers as you please.

Tyro. The examples are quite plain and easy.

Philo. I am glad that you understand them; for now I shall shew you *that* which will serve you for all common business, in casting up any sort of goods; and pray be careful to mind it well; for there is not a more useful thing in all common *arithmetic*, it being a short way of working the rule of 3, without *division*.

Questions performed by Multiplication.

Tyro. How are these questions performed?

Philo. By the following rule, viz.

Multiply the price by the given number, or quantity, and carry 1 for every 4, 12, and 20, as you do in *addition of money*, and 1 for every 10 in the pounds, as in *whole numbers*.

An example or two well explained will soon make it easy to you.

£. s. d.

Quest. 1. What cost 3 ells, at 0 - 5 - 3 an ell?

3

£. 0 - 15 - 9

Here I multiply the price by the quantity; saying 3 times 3 is 9 pence, which I place under the pence; and then I say, 3 times 5 shillings is 15, which I place under the shillings, and the answer is 15 shillings, and 9 pence.

☞ You

☞ You must remember, that 3 ells, at 5*s.* 3*d.* an ell, is the same as 3 yards, or 3 gros, or 3 gallons, or any other name whatsoever; for it is only 3 times 5 shillings, and 3 pence.

2. What come 5 gros of }
any thing to, at } £. 0 - 6 - 8 a gros?

$$\begin{array}{r} 5 \\ \hline \text{£. } 1 - 13 - 4 \text{ Ans.} \end{array}$$

Here I multiply the price by 5, saying, 5 times 8 is 40 pence, (which is 3 shillings and 4 pence) therefore I set down 4 under the pence, and carry 3, and then 5 times 6 shillings is 30, and 3 I carried is 33 shillings, which is 1*l.* 13*s.* that is 13, and I carry 1; and then I say, 5 times 0 is 0, but 1 is 1.

3. How much does 7 times 8 shillings and 9 pence amount to? Or,
What cost 7 reams of paper at } £. 0 - 8 - 9 a ream?

$$\begin{array}{r} 7 \\ \hline \text{£. } 3 - 1 - 3 \text{ Ans.} \end{array}$$

Here I say, 7 times 9 is 63 pence, which is 5 shillings, and 3 pence, that is 3, and I carry 5; then 7 times 8 is 56 shillings, and 5 I carried is 61; that is 3*l.* 1*s.* or, which is the same, it is 3 twenties, and 1 over; therefore, I set 1 under the shillings, and carry 3 to the pounds, saying, 7 times 0 is 0, but 3 is 3: so is the answer 3*l.* 1*s.* 3*d.*

Note, Tho' I have set cyphers in the place of pounds in these examples, and have also set £. *s.* *d.* over the sums, yet it is better to leave out the cyphers, and more practicable, to make only a great £ before the pounds, as in the following examples.

4. What

4. What cost 9 gallons of brandy, or any thing else, at ———— £. — 8 - $3\frac{1}{4}$ a gallon

$$\begin{array}{r} \text{£. } 3 - 14 - 5\frac{1}{4} \end{array}$$

Here I multiply the price by 9, beginning at the farthing, say, 9 farthings is 2 pence farthing, that is 1 farthing, and I carry 2 pence; then 9 times 3 is 27 pence, and 2 pence I carried is 29 pence, which is 2 shillings, and 5 pence; therefore, I set down 5 under the pence, and carry 2, saying, 9 times 8 is 72 shillings, and 2 I carried is 74. Now 74 shillings is 3*l*. 14*s*. therefore, as there is nothing in the pounds to multiply, I set down 3*l*. 14*s*. viz. the 14 under the shillings place, and 3 towards the left-hand.

Tyro. This is very pretty, and very easy: please to try me with an Example.

Philo. There is no doubt but you will do it.

5. A Gentleman gave ten poor Widows three half crowns a-piece: how much did he give in all? or thus, What cost 10 bushels of any thing, at $\frac{1}{2}$ £. — 7 — 6 a bushel

$$\begin{array}{r} \text{£. } 3 - 15 - 0 \end{array}$$

Tyro. I set down 3 half crowns, that is, 7*s*. 6*d*. and multiply it by 10, saying, 10 times 6 is 60 pence, that is just 5 shillings; therefore, I set down nothing (0) and carry 5; then I say, 10 times 7 is 70, and 5 I carried is 75 shillings, which is 3*l*. 15*s*. So is the answer £. 3 — 15.

6. What comes 12 months } £. 1 - 8 - 6 $\frac{1}{4}$ a month?
wages to, at } 12

Ans. £. 17 - 2 - 3

Or how much is 12 times £. 1 - 8 - 6 $\frac{1}{4}$

Here I say, 12 farthings is 3 pence, that is 0, and I carry 3; then 12 times 6 is 72, and 3 I carried is 75 pence, which is 6s. and 3 pence; I therefore set down 3, and carry 6; then I say, 12 times 8 is 96 shillings, and 6 I carried is 102 shillings, which is 5l. 2s. set down 2, and carry 5; then I say 12 times 1 is 12, and 5 I carried is 17l. So is the answer £. 17 - 2 - 3.

2. Of double figures in the shillings.

When you come to double figures, such as 14, 17, 18 or the like, multiply them like *whole numbers*, and then cast out the twenties, that is, count how many times 20 you can have in the number, and it is done. There are several other ways I shall shew you, and always take *that* which appears easiest to you, till perfect.

7. What cost 5 sheep, at £. 1 - 17 - 6 each?
Or, What is 5 times £. 1 - 17 - 6

Ans. £. 9 - 7 - 6

Here I say 5 times 6 is 30 pence, is 2s. 6d. that is 6 and I carry 2; then I multiply the whole 17 by 5, saying 5 times 7 is 35, and 2 I carried is 37, 7 and I go 3, then 5 times 1 is 5, and 3 is 8, that is 87 shillings, which is 4l. 7s. that is 7, and I carry 4; then 5 times 1 is 5, and 4 is 9.

Or thus :

I multiply first the 7 by the 5, setting down the product on a slate, or a piece of paper, saying 5 times

7 is 35, and 2 I carried is 37 shillings, or 1*l.* 17*s.*: then I say 5 times 10 is 50 shillings, or 2*l.* 10*s.* and add this to 1*l.* 17*s.* it makes 4*l.* 7*s.* as before. See the next example.

8. What cost 7 gallons, at $\text{£. } 1 - 15 - 9\frac{1}{4}$ a gallon?

$$\begin{array}{r} 7 \\ \hline \text{£. } 12 - 10 - 4\frac{3}{4} \text{ Ans.} \end{array}$$

Here I say, 7 farthings is 1 penny, 3 farthings, therefore I set down $\frac{3}{4}$ and carry 1; then 7 times 9 is 63, and 1 is 64 pence, that is 5*s.* and 4*d.* 4 and I carry 5; then 7 times 5 is 35, and 5 is 40, 0 and I carry 4; 7 times 1 is 7, and 4 is 11, that is, 110 shillings in all, which is 5*l.* 10*s.* that is 10, and I carry 5; and *lastly*, 7 times 1 is 7, and 5 is 12.

Or thus, 7 times 5 is 35, and 5 is 40 shillings that is 2*l.* which I set down any where on a slate; then I say, 7 times 10 shillings is 3*l.* 10*s.* Now 3*l.* 10*s.* and 2*l.* is 5*l.* 10*s.* as before. Again,

9. What cost 12 quarters of malt, at } $\text{£. } 1 - 14 - 6$
a quarter 12

$$\text{Ans. } \text{£. } 20 - 14 -$$

Here I say 12 6 pences is 6 shillings, that is 0, and I carry 6; then 12 times 4 is 48, and 6 is 54, 4, and I carry 5; 12 times 1 is 12, and 5 is 17; that is 174 shillings, which is 8*l.* 14*s.* that is 14, and I carry 8; then 12 times 1 is 12, and 8 is 20 pounds. Or

By the *second way*, 12 6 pences is 6 shillings, as before, 0, and I carry 6; then 12 times 4 is 48, and 6 is 54, that is 2*l.* 14*s.* then 12 times 10 shillings is 6*l.* Now 6*l.* and 2*l.* 14*s.* is 8*l.* 14*s.* as before, that is 14, and I carry 8; then 12 times 1 is 12, and 8 is 20.

Tyro. I understand both the ways very well.

Philo. Take which you think is easiest, they both answer the same end.

Tyro. But suppose I have a larger quantity than 12, how must I multiply the price by that number?

Philo. By the following rule.

3. *When there are 2 figures to multiply by.*

Rule. When the given quantity is more than 12, and contains any such numbers as can be found in the *Multiplication-table*, then find any 2 numbers, which, when multiplied together, will make the given number, and then multiply the given price by any one of those figures, and that product multiply by the other number, and that product is the answer. Thus, suppose I am to multiply by 15, I find 3 times 5 is 15; therefore, I multiply by the 3 first, and set down the product; then I multiply this product by 5, and that gives me the answer for 15; because 3 times 5, or 5 times 3 make that number. Again suppose I were to multiply by 24, I find 4 times 6 is 24; therefore, I first multiply by 4, and then multiply that product by 6, which is the answer: or otherwise, as three times 8 is 24, I may therefore multiply first by 3 and then by 8. So if the number be 63, I find 7 times 9, or 9 times 7 is 63; therefore, I multiply the price by any one of these figures, and that product I multiply by the other for the true answer.

10. What cost 15 ells of holland, at £. - 7 - 6 an ell?
Here 3 times 5 is 15, multiply first by 3

The price of 3 ells	_____	£. 1 - 2 - 6
Multiply by 5		5

Price of 15 ells	_____	_____	£. 5 - 12 - 6	<i>Ans.</i>
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Work

Work at Length.

First I say 3 six-pences is 1s. 6d. that is 6 and I carry 1; then 3 times 7 is 21, and 1 I carried is 22 shillings, which is 1l. 2s. So is the price of 3 ells 1l. 2s. 6d. Now I multiply this 1l. 2s. 6d. by 5 (because 5 times 3 is 15) saying, 5 times 6 is 30 pence, is 2s. 6d. or 5 six-pences is 2s. 6d. that is 6, and I carry 2; then 5 times 2 is 10, and 2 is 12, which I set down. *Lastly*, I say, 5 times 1 is 5; so the answer for 15 ells, at 7s. 6d. an ell, is £. 5 - 12 - 6.

11. What cost 24 chaldrons }
of coals, at £. 1 - 11 - 6 a chaldron?
4 times 6 is 24, first multiply by 4

The price of 4 chald. $\begin{array}{r} 6 - 6 - 0 \\ 4 \end{array}$

The price of 24 Chald. £. 37 - 16 - 0 *Ans.*
Proved another way — £. 1 - 11 - 6
3 times 8 is 24, multiply by 3

Price of 3 chal. $\begin{array}{r} 4 - 14 - 6 \\ \text{Multiply by } 8 \end{array}$

Price of 24 chald. £. 37 - 16 - 0 as above

12. What cost 35 sheep, at £. 1 - 15 - 7 each?
5 times 7 is 35, multiply first by 5

Price of 5 $\begin{array}{r} 8 - 17 - 11 \\ 7 \end{array}$

Price of 35 £. 62 - 5 - 5 *Ans.*

Of MULTIPLICATION

13. What cost 48 load, at £. 3 - 10 - 6 a load?
6 times 8 is 48, multiply by 6

$$\begin{array}{r} \text{Price of } 6 \\ \text{Multiply by } 8 \\ \hline \end{array} \quad \begin{array}{r} 21 - 3 - 0 \\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Price of } 48 \\ \hline \end{array} \quad \begin{array}{r} \text{£. } 169 - 4 - 0 \end{array}$$

14. What cost 72 pieces of?

Holland, at } £. 2 - 3 - 5 a piece?
6 times 12, or 12 times 6 is 72, by 12

$$\begin{array}{r} \text{Price of } 12 \\ \text{Multiply by } 6 \\ \hline \end{array} \quad \begin{array}{r} 26 - 1 - 0 \\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Price of } 72 \\ \hline \end{array} \quad \begin{array}{r} \text{£. } 156 - 6 - 0 \text{ Ans.} \end{array}$$

15. What cost 108 quartern?

loaves, at } £. 0 - 0 - 5 $\frac{3}{4}$ a piece?
12 times 9 is 108, multiply by 12

$$\begin{array}{r} \text{Price of } 12 \\ \hline \end{array} \quad \begin{array}{r} 0 - 5 - 9\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} \text{Price of } 108 \\ \hline \end{array} \quad \begin{array}{r} \text{£. } 2 - 11 - 9\frac{3}{4} \end{array}$$

Note 1. I have here indulged you with cyphers, because I would do every thing that is easy to your understanding; but really, *Tyro*, it is better without them. Set down now 5 pence 3 farthings any where.

$$\begin{array}{r} 5\frac{3}{4} \\ 12 \\ \hline - 5 - 9 \\ 9 \\ \hline \text{£. } 2 - 11 - 9 \text{ Answer} \end{array}$$

Note 2. This is the only way, if you have a mind to do things quick, and off hand; for tho' there be a form in some schools to the contrary, and you may hitherto have been instructed in that way; yet, as I observed before, when you come to do a sum, you are not bound to set it down in the very form you were taught at school; for while you are doing *that*, another will answer the question over and over.

Tyro. I understand you, sir, very well, and thank you for your care; but it is only use, as you observed; for I think one full as easy as the other, and I grant it is much better for the quick dispatching of business.

16. What comes the cloath-
ing of 144 soldiers to, at } £. 3 - 12 each man?
12 times 12 is 144, multiply by 12

12 cost	43 - 4	
12	again by 12	
144 cost	£. 518 - 8	<i>Ans.</i>

Tyro. You need not give me any more examples: but pray, suppose the quantity, or given number, be such as does not fall under, or cannot be found in the *Multiplication table*, such as 17, 26, 38, 59, or the like; how then?

Philo. This is as easy as the other, as appears by the following rule.

4. *Of Numbers that are not to be found in the Multiplication-table, and how to work by them.*

Rule. When the given number does not fall under the common course of the table, such as 17, 38, 59, or the like; then take any two figures that will come the nearest to the number (but not exceeding it) and

work with them as in the former examples, and then add the price to that product as many times as there are odd numbers, and the work is done.

We will give you examples of 17, 38, and 59.

17 what cost 17 grofs of corks, at £. 10 - 6½ a grofs?
4 times 4 is 16, and 1 is 17. Mult. by 4

Price of 4 is	2 - 2 - 2
4	4

Price of 16 is	8 - 8 - 8
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Add the price of 1 grofs, viz.	10 - 6½

Price of 17 is	£. 8 - 19 - 2½ Ans.
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18. What cost 38 loads of hay, at £. 1 - 11 - 6 a load?
6 times 6 is 36, and 2 is 38. Mult. by 6

Price of 6 is	9 - 9 - 6
6	6

Price of 36 is	56 - 14 -
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Mul. the price 17. 11s. 6d by 2 gives	3 - 3 - add

Price of 38 is £.	59 - 17 - Ans.
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19. What is the value of 59 ports £. 1 - 16 - each
7 times 8 is 56, and 3 is 59. Mult. by 7

Value of 7 is	12 - 12 -
8	by 8

Value of 56 is	100 - 16 -
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Then multiply 17. 16s. by 3 is	5 - 8 - add

Value of 59	£. 106 - 4 -
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This

This may be done another way.

As you said before, 7 times 8 is 56, and 3 added make 59. So now you may say, 6 times 10 is 60; and then take, or subtract the value of 1 piece out of that product, and the answer will be the same as 59.

1 port is		£. 1 - 16 -		
10 times 6 is 60, first	by	6		
Value of 6 is		10 - 16		
10		10		
Value of 60 is		108 -		
Subtract 1 piece, viz.		1 - 16 - subtract		
59	<i>Ans.</i>	106 - 4 - as above		

Note 1. As some boys learn sooner by one method than another, some masters find it easier to teach them to calculate by even tens, and add the odd numbers after. Thus if it were 43, then multiply by 10, then by 4, gives 40. and then add the 3 odd numbers. So 64 is $10 \times 6 \times 5 = 65$, and thus for any numbers.

Note 2. If you are perfect in what has already been shewn you, it will be very easy to perform any sum in common arithmetic, relating to business, not only within the compass of the table, but in much larger numbers. As for instance; Suppose it was required to tell the value of 1000 moidores: or what will it cost to cloath 1000 men, at 1*l.* 7*s.* a man? Here 10 times 10 make 100, and 10 times 100 make 1000. Multiply therefore 1*l.* 7*s.* by 10; then that product by 10; and this last product by 10 will give the answer, viz. £. 1350.

QUESTIONS *undone for practice.*

1. What comes 33 dozen of candles to, at 4s. 9d. a dozen? *Answer* £. 7 - 16 - 9. Multiply by 3, and then by 11.

2. How much is 46 pistoles, at 17s. 6d. each? or what cost 46 pieces of *Irish*, at 17s. 6d. a piece. *Answer* £. 40 - 5. Multiply by 5, then by 9, and add 17s. 6d. more for the odd 1.

3. What cost 63 barrels, at £. 1 - 5 - 9 a barrel? *Ans.* £. 81 - 2 - 3.

4. What comes 49 months salary to, at 3 guineas a month? *Ans.* £. 154 - 7.

5. What comes 105 gallons of rum to, at 7s. 6d. $\frac{1}{2}$ a gallon? *Ans.* £. 39 - 11 - 10 $\frac{1}{2}$. 10 times 10 make 100, and then multiply 7s. 6d. $\frac{1}{2}$ by 5, the odd 5 gallons and add it to the last product.

6. If I spend 1d. $\frac{3}{4}$, or 7 farthings a day, how much is that a year, allowing 365 days? *Answer* £. 2 - 13 - 2 $\frac{3}{4}$

First, 10 times 10 make 100; then multiply the answer of 100 by 3, it gives you the expences of 300 days *viz.* 2l. 3s. 9d. Now there are but 65 days behind; therefore set down 7 farthings in another place, and multiply it by 8, and that product multiplied by 8 again makes 64, and then add once 1d. $\frac{3}{4}$ for the odd day, gives 9s. 3d. $\frac{3}{4}$ for the 65 days, which added to the 300 days gives £. 2 - 13 - 2 $\frac{3}{4}$. *Secondly*, or thus, which is shorter, 11 times 11 is 121; then multiply the product of 121 by 3, gives 363, and then add 2 times 1d. $\frac{3}{4}$, *viz.* 3d. $\frac{1}{2}$ to that product, gives £. 2 - 13 - 2 $\frac{3}{4}$, as before.

As I have added the answers to these questions, I hope, *Tyro*, you will try to prove them at large: for if you mind and make yourself master of this rule, you will be able to cast up any common thing as quick, or quicker than by any other method, and with far greater ease.

As for *cross multiplication*, viz. multiplying money by money, or feet and inches by feet and inches, it is better let alone for the present; therefore, we will proceed to *division*.

DIALOGUE V.

SECTION I.

Of DIVISION.

Tyro. **W**HAT is *division*, and what does it teach?

Philo. *Division* is just the reverse of *multiplication*; for, as any sum is encreased as many times as the figure you multiply by; so in *division*, the number is decreased, or divided into as many parts, as the value of the figures you divide by.

Tyro. What is to be learned or observed in this rule?

Philo. There are four things very necessary to be known in *division*, viz.

First, The *Dividend*, which is the sum given to be divided.

Secondly, The *Divisor*, or the number you are to divide by.

Thirdly, The *Quotient*, or answer, which shews you how many times the *Divisor* is contained in the *Dividend*: or into how many parts the *Dividend* is divided.

Fourthly, The *Remainder*, which is a fractional part of the *Quotient*: but this does not concern you as yet.

Tyro. Am I to get this by heart?

Philo. You are not bound to get it just word for word; but you should understand well what the first 3 mean.

Tyro. Be so kind, sir, as to explain them a little more for me.

Philo. Observe then : let us take any two numbers, suppose 24, and 6 : now 24 is the *Dividend*, and 6 is the *Divisor*. Then I ask how many times 6 can I have in 24, and the answer is 4 ; which 4 is called the *Quotient*. So also, suppose it was required to divide 108 by 12 or into 12 parts, then every part will be 9 ; for 12 times 9 is 108. Now 108 is the *Dividend* or sum to be divided ; 12 is the *Divisor*, or what you divide by, and 9 is the *Quotient*.

Tyro. Now, sir, I apprehend you quite well.

Philo. Then I will proceed to some examples in single figures ; but pray observe the following rule.

1. *Of dividing by single figures in one line only.*

Rule. First ask how many times the *Divisor* is contained in the first figure of the *Dividend*, and if the *Divisor* be larger than the first figure in the *Dividend*, then seek how many times you can have it in the first two figures in the *Dividend*, and set the figure down accordingly : and if any thing remains from the first figure in the *Dividend*, carry it to the second ; and if any thing remains in the second figure, carry it to the third ; remembering always in this short *Division*, that if one remains you call it 10, if two remains you call it 20, if 5 50, and so on, carrying the remainder of one figure to another in your mind.

Note, To make the work both shorter and easier, remember that 2's is read *two's*, 3's *three's*, 5's is *five's*, 12's is *twelve's*, and so for any other figure : thus, the 7's in 14, is read thus, the *sevens* in 14 ; and the 6's in 24, is the *sixes* in 24 ; which is the same as if I should ask, how many *sixes* can I have in 24, but only shorter and more convenient.

An example or two will, with care, make it familiar to you.

Ex-

Ex. 1.	Dividend	Ex. 2.	Dividend
Divisor	3)39	Divisor	4)168
Quotient	13 Ans. 3	Quotient	42 Ans. 4
Proof	39	Proof	168

Now observe: In *ex. 1*, I ask how many *three's* I can have in 3; or I say the 3's in 3 is once; therefore I set down one under the 3 in the *dividend*, and as there remains nothing over, I ask how many 3's I can have in 9; or say, the 3's in 9 is 3 times 3, and nothing over; therefore, I set down 3 under the 9, and it is done. Again,

In *ex. 2*, I say the 4's in 1 I can't have; but taking the next figure to it, *viz.* 6, I say the 4's in 16 is just 4 times; therefore, I set down 4 under the 6 in the *dividend*, and as there is nothing remains, I only say, the 4's in 8 is twice, therefore I set down 2 under the 8, and it is done. Now to prove it I multiply 42, the *quotient*, by 4, the *divisor*, and find it 168 like the *dividend*.

Ex. 3.	Dividend	Ex. 4.	Dividend
Divisor	8)37192	Divisor	9)245052
Answer	4649 8	Answer	27228 9
Proof	37192	Proof	245052

Now observe, *Tyro*, in *ex. 3*, I divide by 8, saying, the 8's in 3 I can't; but the 8's in 37 is 4 times 8 is 32, and 5 over; I therefore set down 4 under the 7, and carry 5 to the next figure, which is 1, which I now call

51; (for what you carry from one figure you must always place before the next figure) then I say the 8's in 51 is 6 times 8 is 48, and 3 over; which 3 I now carry to the 9, and it is 39; therefore, I say the 8's in 39 is 4 times 8 is 32, and 7 over; this 7 I now place before the 2, and it is 72; then I say the 8's in 72 is just 9 times, and the work is done. *To prove it*, I multiply the *quotient* or answer by the *divisor* 8, and find the *product* the same as the *dividend*. In *Ex. 4*, I say, the 9's in 2 I can't; but the 9's in 24 is 2 times, or *twice* 9 is 18, and 6 over, which 2 I place under the 4, and carry 6 to the next figure, which is 5, and call it 65; then I say the 9's in 65 is 7 times 9 is 63, and 2 over, which 2 I place before the cypher (0) and it is 20; then I say the 9's in 20 is *twice* 9 is 18, and 2 over, which I carry to the 5, and it is 25; then I say the 9's in 25 is *twice* 9 is 18, and 7 over, which I set before the last figure 2, and it is 72; then I say, the 9's in 72 is just 8 times. *To prove it*, I multiply the *answer* by 9, and the *product* will be the same as the *dividend*.

EXAMPLE 5.

Divide by 7)417296

Answer 59613.5*
 7

Proof 417296

EXAMPLE 6.

Divide by 9)9471636

Answer 1052404
 9

Proof 9471636

* Here in *example 5*, there is 5 remains at last: therefore, I set it at the end of the answer, parting it with a stroke, thus, -5; and when I prove the work I multiply by 7, and take the remainder 5 in, saying, 7 times 3 is 21, and 5 is 26, 6 and I carry 2, &c.

Ex-

EXAMPLE 7.

$$\begin{array}{r} 8 \overline{) 716206} \\ \hline \end{array}$$

Answer $\begin{array}{r} 89525 \cdot 6 \\ 8 \\ \hline \end{array}$

Proof $\begin{array}{r} 716206 \\ \hline \end{array}$

EXAMPLE 8.

$$\begin{array}{r} 5 \overline{) 12631908} \\ \hline \end{array}$$

Answer $\begin{array}{r} 2526381 \cdot 3 \\ 5 \\ \hline \end{array}$

Proof $\begin{array}{r} 12631908 \\ \hline \end{array}$

More examples for practice.

$$\begin{array}{r} 7 \overline{) 697220716} \\ \hline \end{array}$$

$$\begin{array}{r} 9 \overline{) 407217943} \\ \hline \end{array}$$

Tyro. I understand what you have shewn me very well; but pray, before you come to long *division*, give me an example at large to divide by *twelve* the short way.

Philo. I will: let it be required to divide 1479908 by 12. I set it down thus:

EXAMPLE 11.

$$\begin{array}{r} 12 \overline{) 1479908} \\ \hline \end{array}$$

$$\begin{array}{r} 123325 \cdot 8 \\ \hline \end{array}$$

EXAMPLE 12.

$$\begin{array}{r} 12 \overline{) 59904965} \\ \hline \end{array}$$

$$\begin{array}{r} 4992080 \cdot 5 \\ \hline \end{array}$$

Here I divide by 12, saying, the 12's in 14 is *once* 12, and 2 over, and this 2 I carry to the 7 and it is 27; then I say the 12's in 27 is *twice* 12 is 24, and 3 over, that is 39; then I say the 12's in 39 is 3 times 12 is 36, and 3 over, that is 39 again; therefore, I set down another 3, and carry the 3 that is over to the next figure, which is a cypher, calling it 30; then I say the 12's in 30 is *twice* 12 is 24, and 6 over, which is 68; now the 12's in 68 is 5 times 12 is 60, and 8 over, which I place after the sum, thus, -8, and it is done.

In *Ex.* 12, I say the 12's in 5 I can't; but the 12's in 59 is 4 times 12 is 48, and 11 over, which 11 carried to, or joined to the next figure 9 is 119; then I say the 12's

in

in 119 is nine times 12 is 108, and 11 over again, which I now carry to, and join to the next figure 0, and call it 110, then say the 12's in 110 is 9 times, and 2 over, which 2 I join to, or carry to the 4, and it is 24: now the 12's in 24 is just *twice*, and nothing over; therefore I now begin a-fresh at the 9, saying, the 12's in 9 I can't, but the 12's in 96 is just 8 times: *lastly*, I say, the 12's in 5 I can't have; therefore, I set down a cypher under the last figure 5, and drawing a short line, set the 5 that remains after it, thus -5.

Now *to prove it*, I multiply back by 12, and take in the 5, saying, 12 times 0 is 0, but 5 is 5; then 12 times 8 is 96, 6 and I carry 9, &c. till I go through the whole.

 *Note*, Multiplication is an infallible proof for *division*; for if you multiply the *quotient* by the *divisor*, you will have the same figures as are in the *dividend*, always remembering to take in the remainder with the first figure you begin to multiply.

Tyro. I humbly thank you, Sir, I am quite satisfied with what you have shewn me; and now, if you please, I shall be obliged to you to shew me how to divide by several figures.

Philo. You will soon learn it with care: you must note there are 3 or 4 ways to work *division*; but as my intent is not for curiosity, but improvement, I shall only shew you that method which is most *natural* and *practicable*; you may at any time learn the rest.

2. Of dividing by 2, 3, or more figures.

Rule. First *seek*, or ask how many times the figures in the *divisor* are contained in the same number of figures in the *dividend*, and put that figure in the *quotient*. Secondly, multiply now the *divisor* by the said figure in the *quotient*, and place it under those figures in the *dividend* that you began to work with, always observing, that the *product* be not larger than the figures

figures in the *dividend*; for if they are, you must rub out, or cancel the figure in the *quotient*, and put one of a less denomination. *Thirdly*, this being done, *subtract* now the *product* from that part of the *dividend* it stands under, and to the remainder bring down the next figure in the *dividend*, placing, or joining it to the last figure of the *remainder*. Then *seek* how many times the *divisor* is contained in these figures; then multiply the *divisor* by the said figure; then subtract again; and *lastly*, bring down the next figure in the *dividend*, as before; and thus proceed till you have no more figures in the *dividend* to bring down, and the work is done.

☞ *Note 1.* Every time you *subtract*, observe whether the *remainder* be larger than the *divisor*; for if it be, you must put a larger figure in the *quotient*.

Note 2. Whenever you take a figure down from the *dividend*, and join it to the *remainder*, and that is still less than the *divisor*; then always put a cypher in the *quotient*, and bring down another figure in the *dividend*.

An example or two at large will make it easier.

EXAMPLE I.

<i>Dividend</i>	
<i>Divisor</i> 15)19260(1284 <i>quotient, or answer</i>	
15	<i>Quotient</i> 1284
————	<i>multiply by 15 divisor</i>
42	————
30	6420
————	1284
126	————
120	<i>Product</i> 19260 <i>dividend</i>
————	
60	
60	
————	
0	

Now

Now observe : having 2 figures in the *divisor*, I ask how many times they are contained in the 2 first figures of the *dividend*, (*viz.* 19) and find it *once*; therefore I set 1 in the *quotient*. Secondly, I multiply the *divisor* 15 by 1, saying, *once* 5 is 5, *once* 1 is 1, and place it under the 19. Then, *thirdly*, *subtract* 15 from 19, and there remains 4; and *lastly*, I bring down the next figure in the *dividend*, (*viz.* the 2) and join it by the side of the remainder 4, and it is 42; and now I begin as at the first, and ask how many times 15 are contained in 42, and find it *twice*; therefore I set down 2 in the *quotient*: then I multiply the *divisor* 15 by 2, which is 30, and place it under 42; this done, I *subtract* 30 from 42, and there remains 12; and then again, I bring down the next figure of the *dividend* (*viz.* 6) and place it by the side of 12, and it is 126. Then I ask how many times 15 I can have in 126, and find it 8 times; therefore, I put an 8 in the *quotient*, and multiply 15 by it, which is 120, which I place under 126. Then I *subtract* 120 from 126, which is 6, and bring down the last figure in the *dividend*, which is a cypher (0) and it makes 60; then I seek how many times 15 I can have in 60, and find it 4 times; then I multiply 15 by 4, and find it just 60, which I place under the other 60, and the work is done.

R U L E. 2.

Note 1. When there are several figures in the *divisor*, it is easier for a learner to ask how many times the first figure of the *divisor* is contained in the first figure of the *dividend*, and place the times in the *quotient*; then multiply the whole *divisor* by the *quotient* figure, and if the *product* be more than the figures which belong to the *dividend*, you must make trial of a less figure, and put it in the *quotient*.

Note

Note 2. If the first figure of the *divisor* be larger than the figure in the *dividend*, then take 2 figures in the *dividend*, and seek how many times the first figure of the *divisor* is contained in them.

Note 3. You must remember, that in making trial how often the first figure in the *divisor* is contained in two figures of the *dividend*, it will sometimes appear to be 10, or 12 times; but observe, it never can be above 9 at most, and oftentimes not so many as it appears to be.

The same example farther demonstrated.

Let it be required to divide 19260 by 15.

First, I set the *dividend* down on a slate, and make a couple of crooked lines at the ends of it, in the first of which I place the *divisor*, thus, 15)19260(and the other is to place the *quotient* in.

Secondly, I ask how many times the first figure of the *divisor* is contained in the first figure of the *dividend* and find it *once*, therefore I place a 1 in the *quotient*, and multiply the whole *divisor* by it, and place the product under the two first figures of the *dividend*, and subtract it therefrom, and it will stand thus:

Second work.

$$\begin{array}{r}
 15 \overline{) 19260} (1 \\
 \underline{15 } \\
 \text{Remains} \quad 42
 \end{array}$$

Thirdly, To this remainder 4 I bring down the next figure, *viz* 2, (always making a dot under the figure I bring down) and it is 42; then I ask how often the first figure 1 in the *divisor* is contained in 4, and it is 4 times; but upon trial, I find 4 times 15 is 60, therefore,

therefore, as 60 is more than 42, I must take a less figure; I therefore make trial of a 2, and find twice 15 is 30; which I place under 42, and subtract it, and there remains 12, which stands thus:

Third work

$$15)19260(12$$

$$\underline{15 }$$

$$42$$

$$\underline{30}$$

$$\text{Remains} \quad 126$$

Fourthly, I now make a dot under the 6, and bring it down by the side of the 12, and it is 126; then I ask how many of the first figures of the *divisor* I can have in the first two figures of the *dividend*, and find it 12 times; but according to *note 3*, it never goes above 9 times; therefore, I multiply 15 by 9, and it is 135; now I can't take 135 out of 126, therefore 9 times is too much and I make trial of a less figure, to wit, 8, which I put in the *quotient*, and multiply 15 by it, which is 120, and place it under 126, and there remains 6, which stands thus:

Fourth work.

$$15)19260(128$$

$$\underline{15 }$$

$$42$$

$$\underline{30}$$

$$126$$

$$\underline{120}$$

$$\text{Remains} \quad 6$$

Lastly,

Lastly, I make a dot under the cypher (o) and bring it down by the side of the 6, thus, 60; then I ask how many times the first figure of the *divisor* is contained in 6, or the *ones* in 6 is 6 times; but upon trial, I find it will go but 4 times; therefore I place a 4 in the *quotient*, and multiplying 15 by it, find it to be 60, which I place under the other 60, and there remains o, and the work is done, as under.

Last work.

Divisor 15)19260(*1284 quotient*

$$\begin{array}{r}
 15 \dots \\
 \hline
 42 \\
 30 \\
 \hline
 126 \\
 120 \\
 \hline
 60 \\
 60 \\
 \hline
 0
 \end{array}$$

Proof

1284 *quotient*
15 *divisor*

$$\begin{array}{r}
 6420 \\
 1284 \\
 \hline
 \end{array}$$

Product 19260 *dividend*

 *Note*, The reason of making dots under every figure you take down, is because you may not mistake which figure comes next in course; but when you are quite perfect you need not trouble yourself with them.

Tyro. This is a plain demonstration, and I see now the manner of working *division*, though, I own, I am not perfect yet.

Philo. I shall give you an example or two more then, and shew you at the same time how to prove it by *addition*.

Ex-

EXAMPLE 2.

Divide by 375) 24172196 (64459 Answer

*2250....

1672

*1500

1721

*1500

2219

*1875

3446

*3375

Remains

*71

Proof

24172196

64459

375

322295

451213

193377

24172125

71 add rem.

24172196 proof

The P R O O F.

This example is proved the same as the last, by multiplying the *quotient* by the *divisor*, and afterwards adding the *remainder* to it.

P R O O F by ADDITION.

Though *multiplication* be an infallible proof of *division* yet in large sums, *addition* is much shorter and easier, and when the work is right, is as infallible as the other. If indeed it be objected, that a boy may *vamp*, or alter the figures (as in proving *multiplication* by the cross) in order to make the sum come right, there is the same reason to suppose without his Master's inspection) that he may alter his *multiplication* sum, when he comes to add it up, that it may fall right with the

di-

dividend. Besides, as in the very work of every *division* sum, the *divisor* is naturally multiplied into every figure of the *quotient*, what need it to be done again? And I persuade myself, that those masters, that have made use of this method of proving *division*, will acknowledge, that it is not only much easier to the scholar, but also much more improving; because he must set his figures in order, right under one another, or else he will be puzzled to add the sum up: he cannot therefore avoid improving himself (in some measure) by this method.

To prove Division by Addition.

Rule. Add the Remainder of all the different products of the *divisor* together, in order as they stand, and their sum will be the same as the *dividend*. Or thus: add the remainder (if any) and all the lower lines together in order as they stand, and you will find their sum the same as the *dividend*, if the work be right.

Note, in *example 2*, I have made stars (thus*) against those figures which are to be added together; which you see is the lower row of the two; for the top row is never added.

Two

Two examples with their proofs.

EXAMPLE 3.

Divide by 5267)821695074(156008 answer

*5267.....

29499

*26335

31645

*31602

43074

*42136

Remains

*938

Proof

821695074

EXAMPLE 4.

Divide by 9758)94076257(9640 answer

*87822...

62542

*58548

39945

*39032

Remains

*9137

Proof

94076257

Here in the first example, I take all the lower lines of the two, against which this mark (*) is placed, and beginning at the remainder, I say 8 and 6 is 14, 4 and I carry

I carry 1, to 3 is 4, and 3 is 7; then 9 and 1 is 10, that is 0, and I carry 1 to 2 is 3, and 2 is 5; then 4 and 5 is 9, then 6 and 3 is 9, and 7 is 16, 6 and I carry 1, to 1 is 2, and 3 is 5, and 6 is 11, that is 1, and I carry 1, to 3 is 4, and 6 is 10, and 2 is 12, that is 2, and carry 1, to 2 is 3, and 5 is 8. And thus you may proceed to prove the other example.

Tyro. I confess I could not have thought it had been so easy; and it is (in effect) proving it by Multiplication, as you observed.

Philo. I am glad you understand it; and I hope you will take such care as to have no occasion to prove the work at all. Persons in business, *Tyro*, cannot go thro' these forms; if they suspect they have done wrong, they look over the work a second time, and that is a sufficient proof in general.

Of CONTRACTIONS.

Tyro. What is the use of contractions?

Philo. When the divisor consists of several cyphers after a figure, or figures, then cut them all off, or separate them from the figures with a dash of your pen or pencil; and also remember at the same time to cut off as many cyphers or figures in the dividend; then work the sum as if such cyphers never had been there at all, and you will have the same answer.

Ex. 1.

$$\begin{array}{r} 1|000)9250|000 \\ \hline 9250 \text{ Ans.} \end{array}$$

Ex. 2.

$$\begin{array}{r} 4|00)5476|00 \\ \hline 1369 \text{ Ans.} \end{array}$$

Ex. 3.

$$\begin{array}{r} 12|00)54762|00 \\ \hline \text{Ans. } 4563-6 \end{array}$$

Here cut off all the cyphers in the divisor, and as many cyphers in the dividend, and divide only by the single figures, and if any thing remains I set it after it. As in example 3, there is 6 remains, which I set after the answer, thus, — 6.

Tyro. This is easy enough; and now, if you please, I should be glad to know what you mean by *Division of Parts*.

SECTION I.

DIVISION of Parts.

Philo. **D***VISION of Parts* is the dividing by any two single Figures in their Parts, which 2 Figures multiplied together will be equal to the *Divisor*.

Tyro. I do not rightly apprehend you.

Philo. You remember, *Tyro*, in *Questions of Multiplication*, that when any number was given in the table, you found two such figures, which, when multiplied together, would make that number; so here also you do the same, only with this difference, that you here divide by them instead of multiplying. Thus, suppose I was to divide by 24, by 32, by 48, or by 72, I first divide by 3, then by 8; for 3 times 8 is 24, and the last Quotient is the answer. So if I divide by 48, I divide that number first by 8, and then the *Quotient* I divide by 6, and have the proper answer, &c.

EXAMPLE I.

Divide 16488 by 24. Here 6 times 4 is 24.

First by 6) 2748 Quotient by 6

Then by 4) 687 Quotient by 24

EXAMPLE 2.

Divide 125216 by 72. 6 times 12 is 72

First by 6) 21036 Quotient by 6

Then by 12) 1753 Quotient by 72

Tyro. I understand you ; this is only then the reverse of *Multiplication*.

Philo. Nothing more, as you may more plainly see by what follows.

2. Of dividing MONEY.

Division of Money is just the reverse of *Multiplication of Money*; for as in *Questions of Multiplication*, the money is encreased so many times, according to the given number; so here the money is decreased, or divided into as many parts as the given number is.

It will be easier to your comprehension to take an example in *Multiplication*, and prove it by *Division*.

If 1 ell cost £. 1 - 3 - 8 What cost 40 ells?

$$\begin{array}{r}
 8 \\
 5 \\
 \hline
 40
 \end{array}
 \begin{array}{r}
 9 - 9 - 4 \\
 5 \\
 \hline
 \text{£. } 47 - 6 - 8 \text{ Ans.}
 \end{array}$$

PROOF by DIVISION.

1. If 40 ells cost £. 47 - 6 - 8, what cost 1 ell?

Here I divide back by the same figures I multiplied by, carrying the pounds to the shillings, and the remainder of the shillings to the pence, doing by *twenties*, and by *twelves*, as in *Addition*.

by $5) \underline{\underline{£47 - 6 - 8}}$

Then by $8) 9 - 9 - 4$ Price of 8

$\underline{\underline{£. 1 - 3 - 8}}$ Price of 1 *Ans.*

Here I divide first by 5, saying, the 5's in 47 is 9 times 5, and 2 over, which is £.2; this £.2 I carry to the 6s. and it is £.2 - 6, or 46s. then I say the 5's in 46 is 9 times 5 is 45, and 1 over, that is 1 shilling, which I carry to the 8 pence, and it is 1s. 8d. or 20 pence, then I say the 5's in 20 is 4 pence. Now I divide this £.9 - 9 - 4 by 8, saying, the 8's in 9 is *once*, and £. 1 over, which I carry to the 9 shillings, and it is £.1 - 9, or 29 shillings; then I say, the 8's in 29 is 3 times 8 is 24, and 5 shillings over, which I carry to the pence, and it is 5s. 4d. or 64 pence; then I say the 8's in 64 is 8 times. So is the price of 1 ell £. 1 - 3 - 8; as appears also by the work of *Multiplication*.

EXAMPLE 2.

Suppose £ 88 - 4 - 0 be divided among 56 poor widows, how much is the share of each?

by $7) \underline{\underline{£88 - 4 - 0}}$ Here 7 times 8 is 56.

by $8) 12 - 12 - 0$ Share of 8

Ans. 1 - 11 - 6 Share of each.

If you understand the nature of multiplying any sum by any figure, you see you may as readily divide it back again.

Tyro. Sir, I see the way of working it very plainly. Have you any thing farther to add.

Philo. Nothing more, as you say you are perfect; only some questions for to exercise you.

3. QUESTIONS to exercise DIVISION.

1. Divide 822306485 by 1715. *Ans.* 479479.
2. Divide 150348045 by 285. *Ans.* 527537.
3. Divide 47198714 by 6357. *Ans.* 7425, and 4346 remains.

QUESTIONS in MONEY.

1. Divide 1*l.* 16*s.* 8*d.* into 5 parts. *Ans.* 7*s.* 4*d.*
2. Divide 4*l.* 8*s.* 1*d.* $\frac{1}{2}$ into 9 parts. *Ans.* 9*s.* 9*d.* $\frac{1}{2}$.
3. Divide 25*l.* 5*s.* 4*d.* into 32 parts. *Ans.* 15*s.* 9*d.* $\frac{1}{2}$.
4. A gentleman left by will 100 pounds among 3 score (or 60) poor persons, to be paid every *Christmas-Day*, how much is each to receive?

I leave this last question wholly for the learner to do. And now, *Tyro*, you are come to that rule, in which you will exercise all the other *four*; therefore, if you be not perfect, you will be at a loss very often.

DIALOGUE VI.

SECTION I.

Of REDUCTION.

Tyro. **W**HAT is *Reduction*, and what does it teach?
Philo. *Reduction* is a rule compounded of all the foregoing rules; being a proper exercise for the better perfecting you in them. It teaches to *reduce* things of one denomination into another, and is of excellent use in common affairs of life.

Tyro. How many parts is this rule divided into?

Philo. Two. 1. *Reduction ascending.* 2. *Reduction descending.*

Tyro. What do you mean by *Reduction ascending*?

Philo. *Reduction ascending* is when things of a small are reduced, or brought into things of a greater denomination; as farthings into pence, pence into shillings, shillings into pounds, or ounces into hundred weights: And this remember is done by *Division* only, by dividing by as many of the less, as make one of the greater denomination.

Tyro. What do you mean by *Reduction descending*?

Philo. *Descending* is just the reverse of the other; for by *Reduction descending*, things of a great are reduced or brought into things of a less denomination; as pounds into shillings, shillings into pence or farthings; Cwts. into pounds, tons into quarters, miles into yards, yards into inches, and the like; and this remember is always done by *Multiplication* only, by multiplying the greater denomination by as many as it contains of the less denomination.

Tyro. Then I find *Reduction descending* is the easier of the two, as it is performed by *Multiplication* only.

Philo. It is so: And that is the reason it is commonly taught first.

Note, I shall prove every sum of *Reduction descending*, by *Reduction ascending*, and then you will see the nature of both.

1. Of REDUCTION descending.

Tyro. How do you say this is performed?

Philo. By this rule. Multiply every denomination by as many as the next less denomination contains, and you have the answer, which is called *Reduction descending*. Then to prove the work, divide back by the same figures you multiplied by, and this is called *ascending*.

A few questions will make it plain to you.

Quest. 1. In 12 pounds how many shillings?

$\text{£. } 12$
Multiply by 20 Shillings in a £.

Ans. 240 Shillings

PROOF *ascending.*

In 240 shillings how many £. sterling?
Here I divide back by 20, as before I multiplied.

$2|0)24|0$ Shillings

$\text{£. } 12.$ *Ans.*

Here I cut off the cyphers in the *Divisor* and *Dividend* with a dash of my pen, and divide by 2 only, and the answer is 12.

Quest. 2. In $\text{£. } 23 - 14s.$ how many shillings and pence.

$\text{£. } 23 - 14s.$
First by 20 and take in the 14.

Then by 474
 12

5688 Pence.

PROOF.

In 5688 pence how many shillings and £. 's sterling.

Divide by $12)5688$ Pence

Then by $2|0)47|4$ Shillings

$\text{£. } 23 - 14$ as above
F 4

Here

Here I divide by 12, and it brings the pence into shillings; then I divide by 20, cutting off the cypher, and cutting off the 4 also: Then I divide by 2 only, saying, the 2's in 4 is *twice*, the 2's in 7 is 3 times 2 and 1 over, which I place before the figure 4, that I also cut off, and it is 14 shillings.

This is a *general rule*; observe, that when you cut off any figure, or figures, what remains in dividing must be placed before them, and is the true remainder.

Tyro. This is very plain I own.

Philo. Then we will proceed to *Question 3.*

Quest. 3. In £. 45 - 17 - 6 $\frac{3}{4}$ how many shillings, pence, and farthings?

$$\begin{array}{r}
 \text{£. 45 - 17 - 6}\frac{3}{4} \\
 \text{by } 20 \\
 \hline
 917 \text{ Shillings} \\
 \text{by } 12 \\
 \hline
 11010 \text{ Pence} \\
 \text{by } 4 \\
 \hline
 \end{array}$$

Ans. 44043 Farthings

Here I multiply £. 45 by 20, and take in the 17 shillings, saying, 0 is 0 but 7 is 7; then twice 5 is 10, and the 1 by the side of the 7 shillings is 11, that is 1, and I carry 1; then twice 4 is 8, and 1 is 9, which is 917 shillings. Again, I multiply the shillings by 12, and take in the odd sixpence; and *lastly*, I multiply the pence by 4, and take in the odd 3 farthings, so is the work done.

PROOF.

PROOF *by* Reduction ascending.

In 44043 farthings, how many pence, shillings, and pounds *sterling*?

Here I divide only back by the same figures I multiplied by, and it is done.

Divide by 4)44043 farthings

by 12)11010 pence, and 3 farthings over

by 20)917 shillings, and 6 pence over

£. 45 - 17 - 6 $\frac{3}{4}$ as above

Tyro. I understand it very well indeed.

Philo. Then I need not instruct you any more till some difficulty offers.

Quest. 4. In £99 - 19 - 11 $\frac{3}{4}$, how many shillings, pence, and farthings?

£.99 - 19 - 11 $\frac{3}{4}$
Multiply by 20

1999 shillings

by 12

23999 pence

by 4

95999 farthings

In 95999 farthings, how many pence, shillings, and pounds

Divide by 4)95999 farthings

by 12)23999—3

by 20)1999—11

£. 99 - 19 - 11 $\frac{3}{4}$

5. In 30 guineas, how many shillings and pence?

30 guineas

21 shillings make 1 guinea

30
60

630 shillings
12

7560 pence

P R O O F.

In 7650 pence, how many shillings and guineas?

12)7560 pence

21)630(30 shill. *ans.*
63
00

And thus you see, *Tyro*, that any sum, of any name or denomination, may be reduced to another.

Quest.

Quest. 6. In £. 472 - 15 shillings, how many crowns, shillings, groats, and pence?

$\text{£. } 472 - 15$
Multiply by $\frac{4}{1}$ the crowns in a £. and count the 15s.
 for 3 crowns, saying, 4 times 2 is
 Crowns 1891 8, and 3 is 11.
 5 shillings 1 crown

 Shillings 9455
 3 groats 1 shilling

 Groats 28365
 4 pence is 1 groat

 Pence 113460 *anf.*

P R O O F:

In 113460 pence, how many groats, shillings, crowns and pounds.

Divide by 4) 113460 pence
 by 3) 28365 groats
 by 5) 9455 shillings
 by 4) 1891

£. 472-3 crowns over, viz. 15 shillings, as above.

Note 1. To bring pounds into pence at one operation, multiply by 240, because 240 pence make 1 pound.

To bring pounds into farthings at one operation, multiply by 960, the farthings in a pound. On the contrary.

Note

Note 2. To bring pence into pounds, *divide by 240.*
To bring farthings into pounds, *divide by 960.*

A VOIRDUPOIS-WEIGHT.

Quest. 7. In 17 C. 3 qrs. 15 lbs. how many Cwt. qrs. and lbs.

	C.	qr.	lb.	
	17	- 3	- 15	
by	4 qrs. make 1 Cwt.			
	71	qrs.		
by	28	lb. 1 qr.		
	573			
	143			
	2003	lbs.	<i>Ans.</i>	

P R O O F.

In 2003 pounds weight, how many quarters of hundreds, and hundred weight?

28)2003(71 qrs.	qrs. lb.
196	4)71 - 15
43	17 - 3 - 15 as before
28	C. qrs. lb.
15 lb. over	

Quest.

Quest. 8. In 17 C. 3 qrs. 15. lb. how many qrs. lbs. and oz.

$$\begin{array}{r}
 \text{C. qrs. lb.} \\
 17 - 3 - 15 \\
 \text{by } 4 \\
 \hline
 71 \text{ qrs.} \\
 28 \\
 \hline
 573 \\
 143 \\
 \hline
 2003 \text{ lb.}^* \\
 16 \\
 \hline
 12018 \\
 2003 \\
 \hline
 32048 \text{ oz. ansf.}
 \end{array}$$

Note, The following method is very useful in many cases to reduce hundred-weights into pounds, being both short and expeditious, viz.

* Set down the hundreds 4 times under one another in the following manner, and add the odd pounds besides, gives the answer.

$$\begin{array}{r}
 17 \text{ Cwt.} \\
 17 \\
 17 \\
 17 \\
 17 \\
 99 \text{ lb. in 3 qrs. 15 lb.} \\
 \hline
 2003 \text{ lb.}^*
 \end{array}$$

P R O O F.

In 32048 ounces, how many lbs. qrs. and Cwts.
Divide this back again by 16, 28, and 4, and you will have 17 C. 3 qrs. 15 lb.

Rice Price 56 Reg

TROY.

TROY-WEIGHT.

Quest. 9. In 5 lb. 2 oz. 14 dwts. of silver, how many oz. dwts. and grs.

	lb.	oz.	dwts.
	5	2	14
12 oz. 1 lb. Troy by	12	and take in in	2 oz.
	62	oz.	
20 dwts. 1 oz. by	20	and take in	14 dwts.
	1254	dwts.	
24 grs. 1 dwt. by	24		
	5016		
	2508		
	30096	Grains	Ans

P R O O F.

Divide back by the same figures, 24, 20, and 12, and have a regard to the remainder, you'll find it 5 lb. 2 oz. 14 dwts. Do you now understand what has been shewn you?

Tyro. I do very well; and I can see that *reduction ascending* is only the proof to the other, and that they prove one another in every respect.

Philo. If you are perfect in what I have shewn you, there is no occasion to run through all the weights and measures, for they are done after the same manner.

Tyro. I know it, Sir; but every example is a fresh encouragement.

Philo. True; and I am as ready to give them.

D R Y-

DRY-MEASURE.

Quest. 10. In 14 loads, 15 bushels, how many bushels and pecks?

L.	B.
14 - 15	
40 bushels 1 load, <i>mult. by</i>	40 and take in the 15 bush.
575	
4	Pecks 1 bushel
2300	Pecks <i>Ans.</i>

P R O O F.

In 2300 pecks, how many bushels and loads?
 Divide 2300 by 4, and that Quotient by 40, and 15 will remain; which is 14 loads, 15 bushels.

LIQUID-MEASURE.

Quest. 11. In 12 Hogsheads of wine how many gallons and pints?

	12 Hogsheads
<i>by</i>	63 Gallons in 1 hoghead of wine
	36
	72
	756 Gallons
<i>by</i>	8 Pints 1 Gallon
	6048 <i>Ans.</i>

PROOF.

P R O O F.

In 6048 pints how many gallons and hogheads?
 Divide this back again by 8, and by 63, and you
 will have just 12 hogheads.

B E E R - M E A S U R E.

Quest. 12. In 18 butts of beer how many hog-
 heads and gallons?

$$\begin{array}{r}
 18 \text{ butts} \\
 \text{by } 2 \text{ hogheads } 1 \text{ butt} \\
 \hline
 36 \text{ hhds.} \\
 \text{by } 54 \text{ gallons } 1 \text{ hoghead of beer} \\
 \hline
 144 \\
 180 \\
 \hline
 1944 \text{ gallons } \textit{Ans.}
 \end{array}$$

P R O O F.

In 1944 gallons of beer, how many hogheads and
 butts? *Ans.* 18 butts.

C L O T H - M E A S U R E.

Quest 13. In 26 yards, 3 quarters, how many
 quarters and nails?

$$\begin{array}{r}
 \text{Yards } \textit{qrs.} \\
 26 - 3 \\
 \text{by } 4 \text{ quarters } 1 \text{ yard, and take in the } 3 \text{ } \textit{qrs.} \\
 \hline
 107 \text{ } \textit{qrs.} \\
 \text{by } 4 \text{ nails } 1 \text{ } \textit{qr.} \\
 \hline
 428 \text{ nails.}
 \end{array}$$

P R O O F

P R O O F.

In 428 nails, how many quarters and yards?

Divide 428 by 4, and then by 4 again. *Ans.* 26 yards 3 qrs.

Note, This and the two following questions, are of great Service in the *Rule of Three Direct*; where it is often required to reduce yards and ells into quarters, and the contrary.

Quest. 14. In 26 ells *English*, 3 quarters, how many quarters and nails?

	Ells qrs.
	26 - 3
by 5, and take in 3 qrs.	5 qrs. 1 ell <i>English</i> .
	133 qrs.
	by 4 Nails 1 qr.
	532 Nails

P R O O F.

In 532 nails, how many quarters and ells *English*?

Divide 532 by 4, and by 5, you'll have 26 ells *English*, 3 qrs.

Quest. 15. In 26 ells *Flemish*, 2 qrs. how many quarters and nails?

	Ells Fl. qrs.
	26 - 2
by 3 qrs. 1 ell <i>Flemish</i> , take in 2 qrs.	80 qrs.
	by 4 Nails 1 qr.
	320 Nails. <i>Ans.</i>

P R O O F.

P R O O F.

In 320 nails, how many quarters and ells *Flemish*?
Divide 320 by 4, and by 3, you'll have 26 ells *Flemish*, 2 qrs.

L O N G - M E A S U R E.

Quest. 16. In 927 miles, how many yards, feet, and inches?

In 1 mile are	927 Miles 1760 Yards
	55620 6489 927
	1631520 Yards 3 Feet 1 yard
	4894560 Feet 12
	58734720 Inches

P R O O F.

In 58734720 inches, how many feet, yards, and miles.

Divide back by the same figures, you'll have at last 927 miles.

Note, I shall give you a fuller example in this rule, when we come to *Compound Reduction*, there being *Division* required when you are to bring rods into yards.

L A N D -

LAND-MEASURE.

Quest. 17. In 54 acres how many roods and poles?

4 roods 1 acre,	by	54 acres	or multiply 54 by 160, the
		4	rods in an acre give 8640
		216	rods.
40 rods 1 rood,	by	40	
		rods 8640	

P R O O F.

In 8640 rods or poles, how many acres?

Divide by 40 and then by 4, or divide 8640 by 160, and it gives 54 acres.

S Q U A R E - M E A S U R E.

Quest. 18. In 217 square yards, 5 feet, how many square feet, inches, and square quarters?

Square yards Feet

	217 - 5
9 square feet 1 yard,	by 9
	1958 square feet
144 inches 1 foot	by 144
	7832
	7832
	1958
	281952 square inches
16 square qrs. 1 inch	by 16
	1691712
	281952
	4511232 square quarters.

P R O O F.

OF REDUCTION.

P R O O F.

In 4511232 square quarters, how many square inches, feet, and square yards?

Divide this number back by 16, you have square inches; then by 144, you have square feet, and *lastly*, by 9, you'll have 217 yards, 5 feet.

Note, That 12 times 144, or 1728 solid inches make a *solid foot*; so that you are to multiply *solid feet* by 1728, to bring them into *solid inches*, and on the contrary, to bring solid inches into *solid feet*, you must divide by 1728.

T I M E.

Quest. 19. How many days, hours, minutes, and seconds are expired since the birth of our Lord and Saviour *Jesus Christ*, supposing it 1767 years, 217 days, 17 hours, and 35 minutes, and allowing just 365 days to a year.

Years days hours minutes

1767 - 217 - 17 - 35

365

8842

10603

15303

645172 days

24

2580695

1290345

15484145 hours

60

929048735 minutes

60

55742924100 seconds

PROOF

P R O O F.

In 55742924100 seconds, how many minutes, hours, days and years?

Divide this by 60, 60, 24, and 365, and you will find 1767 years, 217 days, 17 hours, 35 minutes.

† *Note*, According to your tables in time, 365 days 6 hours make a year; therefore, as many years as are given, you must multiply them by 6, and add them to the hours.

Quest. 20. If a lad be just 12 years old, how many hours are since expired?

$$\begin{array}{r}
 12 \text{ years} \\
 * \quad 365 \\
 \hline
 4380 \text{ days}
 \end{array}$$

$$\begin{array}{r}
 24 \\
 \hline
 17520 \\
 8760 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 105120 \text{ hours} \\
 \text{add} \quad 72 \text{ odd hours} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 105192
 \end{array}$$

$$\begin{array}{r}
 12 \text{ years} \\
 \dagger 6 \text{ odd hours in 1 year} \\
 \hline
 72 \text{ odd hours in 12 years}
 \end{array}$$

* *Note*, Though I set 365 under 12, yet I multiply 365 by 12, because it is done in one line.

SECTION II.

Of COMPOUND REDUCTION.

Tyro. **W**HY do you call this *compound reduction*?
Philo. Because it is compounded of *multiplication* and *division*, and cannot be done without both.

Tyro. What is its particular use?

Philo. It teaches us to reduce any sort of foreign money into pounds *sterling*, and the contrary; so that it is useful in all business and traffic, as appears by the following table and examples.

A select ALPHABETICAL TABLE of foreign coins.

NAMES.	Value	Sterling.	What Country.
Abashee	_____	16 pence	Persia
Asar	_____	6s. 8d.	Ditto
Asper	_____	3 farthings	Turky
Besse	_____	3 halfpence	Persia
Capan	_____	3 pence	East-India
Capeck	_____	1 penny	Muscovy
Cohan	_____	30 shillings	Japan
Christiana	_____	16 pence	Sweden
Copflake	_____	1 shilling	Germany
Crusado	_____	6s. 2d.	Ditto
Ditto	_____	2s. 10.	Portugal
A Crown	_____	4s. 6d.	France
Ditto	_____	5s. 3d.	Florence
Ditto	_____	7s. 6d.	Rome
Ditto	_____	1l. 2s.	Barcelona
A Dina	_____	1l. 10s.	Aleppo
A Dollar	_____	4s. 6d.	Italy
A Lion Dollar	_____	4 shillings	Aleppo
A Cross Dollar	_____	4s. 2d.	Holland
A Specie Dollar	_____	5 shillings	Ditto
A Zealand Dollar	_____	3 shillings	Ditto

NAMES.	Sterling.	Country.
Old Philip's Dollar ———	5 shillings	Holland
Leopold's Dollar ———	4s. 3d.	Ditto
Rudolphus's Dollar	4s. 4d.	Ditto
Prince of Orange's Dollar	4s. 4d.	Ditto
Maxamilian's Dollar ———	4s. 5d.	Ditto
Ferdinando's Dollar ———	4s. 3d.	Ditto
Dollar ———	2s. 3d.	Dantzick
Dollar ———	2s. 3d.	Sweden
A Rix-Dollar of the empire	4s. 3d. $\frac{3}{4}$	Germany
A Ducat ———	4s. 8d.	Hungary
Ditto ———	4s. 8d.	Poland
A Valentia Ducat ———	5s. 3d.	Spain
A Saragoza Ducat ———	5s. 6d.	Ditto
A Barcelona Ducat	6 shillings	Ditto
Ducat ———	5 shillings	Naples
A Ducatoon ———	6s. 3d.	Holland
A piece of Eight in common	4s. 6d.	Spain
A Mexico piece of Eight —	4s. 6d. and 4s. 4d. $\frac{1}{2}$	Ditto
A Peru piece ———	4s. 5d.	Ditto
A Florin ———	4s. 4d.	Ditto
Ditto ———	3s. 4d.	Germany
Ditto ———	2s. 6d.	Sicily
Ditto ———	2 shillings	Holland
Greven ———	1 shilling	Muscovy
Guilder, or Gilder	3s. 8d.	Germany
Ditto gold ———	4s. 9d.	Ditto
Ditto of Noremberg ———	7s. 1d.	Ditto
Ditto ———	5 shillings	Portugal
Ditto ———	18 pence	Dantzick
A Harpur ———	9 pence	Irish
A Livre is 20 Sous ———	18 pence	France
A Mark ———	13s. 4d. not current	England
Ditto ———	16 shillings	Denmark
Ditto ———	2 shillings	Germany
Mark-Lups ———	3s. 9d. $\frac{3}{4}$	Poland
Messe ———	15 pence	East-India Mill-

NAMES.	Sterling.	Country.
Mill-Ree (is 1000 Rees) -	6s. 9d. nearly	Portugal
A Moidore, marked 4000, or 4 times 6s. 9d. viz. -	27 shillings	Ditto
An Obb or Cobb $\frac{1}{2}$ a Harpur	4d. $\frac{1}{2}$ penny	Irish
A Pattacoon, or Pettavoon	4s. 8d.	Spain
A Ruble ——— ———	10 shillings	Muscovy
Seraph ——— ———	5 shillings	Turkey
Timph ——— ———	7 pence	Poland
Tical ——— ———	2l. 16s. 3d.	China
Tari ——— ———	5 pence	Sicily
Toman gold ——— ———	3l. 6s. 8d.	Persia
Zachin ——— ———	7s. 6d.	Venice
Zelot ——— ——— ———	7d $\frac{1}{2}$ penny.	Turkey

Here follows another useful table.

*A TABLE of the just Weight of the customary gold coins
used in England.*

	Dwts.	Grains.	
A Port	9	- 5	Note 1. These all weigh something more than is set down in the table; but as they are fractional parts I have omitted them.
$\frac{1}{2}$ Port	4	- 14	
A Moidore	6	- 22	
$\frac{1}{2}$ Moidore	3	- 11	
Guinea	5	- 9	
$\frac{1}{2}$ Guinea	2	- 16	
A Pistole	4	- 8	

Note 2. If a guinea don't want above 7 grains, a moidore above 9, and a six and thirty above 12 grains of weight, you may safely take them.

Note 3. Scales and Weights for this purpose are made by Mr John Kirk, in St. Paul's Church Yard, London.

Tyro. I am obliged to you, sir.

Philo. And now, *Tyro*, I will give a Variety sufficient for any careful learner to do any question relating to the common

common course of business: but before you begin, I would desire you to mind the two following observations.

Observation 1. When one sum of money, or any weight or measure, is to be brought into another sum of a different name or denomination, then always remember, that before you divide, you make the *Divisor* and *Dividend* of the same name. That is, if the *Dividend* be pounds, pence, yards, shillings, &c. your *Divisor* must also be of the same name.

Observation 2. Take notice likewise, that your *Remainder* is always of the same name as your *Dividend* and *Divisor*, be it what it will. These observations kept in memory, will be of great service to you in the *Rule of three direct*, and in all common rules of arithmetic.

1. Questions in Compound Reduction.

In 324 moidores how many pounds?

324 moidores
1 moidore 27 shillings

2268

648

Shillings 21087418 shillings

£. 437 - 8 shillings

2. A Merchant at London delivers to his correspondent as much broad-cloth as comes to £. 547 - 14 - 7; and he is to receive the same in cross dollars, at 4s. 2d. each. How many must he receive?

Rule. Bring the money into pence, and divide by the pence in 1 dollar.

<i>s. d.</i> 4 - 2 12 50	£. 547 - 14 - 7 20 10954 shillings 12
---	---

Pence in a dollar 510) 131455 pence

26291 cross dollars. *Ans.*

3. How many crusadoes, at 6*s.* 2*d.* each, must be paid for 1000 guilders of *Noremburg*, at 7*s.* 1*d.* each?

<i>s. d.</i> Crusado 6 - 2 12 74 pence 110 74 360 296 640 592 48 pence	1000 guilders 85 74) 85000 (1148 74 	<i>s. d.</i> 7 - 1 guilder 12 85 pence
--	--	---

Ans. 1148 crusadoes and 4 shillings

Quest.

Quest. 4. How many rix dollars, at 4*s.* 3*d.* $\frac{1}{4}$ each, may I receive for £. 1750 - 15 - 6 $\frac{1}{2}$ sterling?

Rule. Bring the sterling money into farthings, and divide by the farthings in 1 dollar.

<i>s.</i> <i>d.</i> 4 - 3 $\frac{1}{4}$ 12 51 4 205	£. 1750 - 15 - 6 $\frac{1}{2}$ 20 35015 shill. 12 420186 pence 4
--	--

Farthings in 1 dollar 205) 1680746 (8198 rix doll. *Ans.*
 1640 and 156 farth.

 or 3*s.* 3*d.*

407 205 2024 1845 1796 1640

by 4) 156 farthings over *

by 12) 39 pence

3 - 3 pence

Tyro. * The Remainder I see is farthings as well as the *Dividend*; and I suppose you divide it by 4, and by 12, to bring it into pence and shillings: Do you not?

Philo. You say right; and all other questions are done after the same manner.

Quest. 5. In 276 crusadoes, at 6s. 2d. each, how many Hungarian ducats, at 4s. 8d. each?

s. d.
4 - 8
12
—
56

276 crusadoes
74 pence in a crusado.

—
1104
1932
—

s. d.
6 - 2
12
—

Pence in a ducat 56) 20424 (364 ducats. *Ans.* 74 d.

168
—

362

336
—

Ans. 364 ducats, and
40 pence over, or
3s. 4d.

264

224
—

40 pence over

Quest.

Quest. 6. In £. 347 - 15 - 7, how many French crowns, at 4s. 6d. $\frac{1}{8}$ each?

$$\begin{array}{r}
 \text{£. } 347 - 15 - 7 \\
 \underline{20} \\
 6955 \text{ shillings} \\
 \underline{12} \\
 \text{A crown } 54 \frac{1}{8} \text{ d.} \\
 \underline{8} \\
 83467 \text{ pence} \\
 \text{8 eighths in 1 penny} \\
 \underline{433 \text{ eighths}} \\
 433)667736(1542 \text{ Ans. } 1542 \text{ French} \\
 \underline{433} \text{ crowns and } 6d. \frac{2}{8} \frac{1}{4}. \\
 2347 \\
 \underline{2165} \\
 1823 \\
 \underline{1732} \\
 916 \\
 \underline{866} \\
 8)50 \\
 \underline{6} \frac{2}{8} \frac{1}{4}.
 \end{array}$$

Here I multiply the pence by 8, and the dividend and the divisor for an eighth of a penny, the remainder also is 8ths of a penny; therefore, I divide it by 8, and it is 10 pence and 3 8ths over.

Note 1. As you do not understand fractions as yet, it may not be amiss to give you a word or two by way of information. You see a *French* crown in the above example is valued at 54 d. 1 eighth, that is, 54 pence, and 1 eighth of a penny, which 1 eighth is called a fraction; the 1 which stands a-top, is called the numerator, and the 8 is the denominator: so also $\frac{3}{7}$ is read 3 sevenths, $\frac{5}{11}$ is 5 elevenths, and $\frac{3}{5}$ is 3 fifths. Here 2, 5, and 3 are numerators, and 7, 11, and 5, are the denominators.

Of REDUCTION.

Note 2. The Rule to work by fractions in all such cases as this, is thus: Multiply the whole number by the *denominator*, that is, the lower figure of the fraction, and take in the top figure, or *Numerator*.

Quest. 7. A merchant sends to his correspondent as much corn as comes to £. 1575, to receive the same in ducatoons, at 6s. 3d. $\frac{3}{5}$ each; how many ought he to receive?

s. d. 6 - 3 $\frac{3}{5}$ 12 75 by 5 and take in the top 3 378 fifts	£. 1575 20 31500 shillings 12 378000 pence by 5 brings d. into 5ths divisor 378) 1890000 (5000 ducatoons 1890 Ans. 000
--	---

Quest. 8. In £. 1000 how many 7 pence halfpennies?

£. 1000 20 20000 shillings 12 240000 pence 4 960000	Or thus : 960 farth. 1000 32000
---	--

Farthing: in 7 $\frac{1}{2}$ are 310)960000 0 32000 Ans.	310)960000(0 32000 Ans.
--	---

Quest.

Quest. 9. In 100 *Portugal* pieces, at 36 shillings each, how many pounds, half crowns, and crowns?

Piece 36 shillings

100

2)0360|0 shillings

180 pounds

8 half crowns 1 pound

2)1440 half crowns

720 crowns

Here you see I multiply, or divide always by as many of the *less* as make one of the *greater*, according as the nature of the question requires.

Quest. 10. In 432 moidores, how many shillings, crowns, groats, and farthings?

432 moidores
27 shillings 1 moidore

3024
864

Shillings in 1 crown 5) 11664 shillings

Crowns 2332 4s. over, or 12 groats
Groats in 1 crown 15 and take in 12 groats

11662
2333

Farthings in 1 groat 34992 groats
16

209952
34992

559872 farthings

Tyro. I see now, Sir, the nature of the rule plainly; but pray how do you manage when there is a sum of money to be paid in pieces of different value, and that there shall be as many in number of one sort as of another; that is, an equal number of every sort?

Philo. This is very useful in many respects, and is done by this general rule, *viz.* Add all the pieces together; then bring the sum of money to be paid, and the sum of all the pieces into one name or denomination; and *lastly*, divide the sum to be paid by the total of all the pieces, and the quotient is the number of each, one with another.

Quest.

Quest. 11. How many shillings, 6 pences, 4 pences, 3 pences, 2 pences, pence, halfpence, and farthings, of each a like number, will discharge a debt of £. 335 - 8 - 4? *Ans.* 2800 of each.

I set down all the pieces one under another, as follows:

A shilling is 12

6

4

3

2

1

$\frac{1}{2}$
 $\frac{1}{4}$

Pence

28 $\frac{3}{4}$

4

Farthings 115

£. 335 - 8 - 4

20

6708 shillings

12

80500 pence

4

115)322000(2800 pieces of each
230

(fort

920

920

00

Here I bring £. 335 - 8 - 4 into farthings, and divide by 115, the farthings in all the pieces, and the answer is 2800 of each.

OBSERVATION 1.

1. When any pieces of money are to be brought into pounds *sterling*, you should consider what relation those pieces bear to a pound *sterling*; that is, what part of a pound it consists of; for by this the Work will be rendered much shorter, and you will not have occasion to

reduce the work into pence, but of this I shall shew you more fully in the *Rule of Three Direct*.

Quest. 12. In £ 270 how many nobles, at 6s. 8d. each?

270

3 nobles, or 3 times 6s. 8d. is 1 pound

810 *Ans.*

To bring nobles into pounds, divide by 3.

Quest. 13. In £. 270 how many marks at 13s. 4d.

Here, as 13s. 4d. is 2 thirds of a pound, I multiply the pounds by 3, and divide by 2, as follows.

270

3

2)810

405 marks. *Ans.*

Tyro. This is much shorter than to bring the pounds and the marks into pence, and have to divide besides.

Pbilo. To be sure it is, and when you want to bring marks into pounds, then multiply by 2, and divide by 3; for you must have less pounds than marks, because a pound is larger than a mark. And thus, *Tyro*, by your care and observation you may work many sums shorter than the common form of rules in general laid down; for it is impossible (without a large volume indeed, and even not then) to lay down rules and examples to make a careless learner understand: but a diligent learner, that is desirous to know, and make himself perfect, will not only observe the rules given him,

him, but will also try and contrive methods of his own, and is not satisfied with knowing things by halves.

Quest 14. Suppose a bill of exchange was accepted at *London*, for £. 580, for value received at *Amsterdam*, how many *Flemish* pounds ought to be remitted at *Amsterdam*?

$$\begin{array}{r}
 33 - 4 \text{ one F. pound} \\
 12 \\
 \hline
 \text{Divisor } 400 \text{ pence}
 \end{array}
 \qquad
 \begin{array}{r}
 \text{£. } 580 \\
 20 \\
 \hline
 11600 \\
 12 \\
 \hline
 4 \overline{) 1392} \overline{) 00}
 \end{array}$$

Ans. 348 *Flemish* pounds

Do you understand it?

Tyro. I do; for let the question be what it will, I have nothing to do but make them both of a name, and multiply, or divide by as many of the less, as make one of the greater; then the quotient (if a division sum) will be the answer, and the remainder the same as the dividend.

Philo. Very well; then I will set you one sum more for trial.

Quest 15. A General of an army, (consisting of 5000 men) after a very sharp engagement lost 2380 men; but coming off victorious, he, for their valiant behaviour, gave 100 guineas to be equally divided among them, and the remainder (if any) to be given to a little errand-boy: How much did each man receive?

Tyro. I proceed thus: I take 2380 men that were lost, out of 5000, and there remains 2620 that escaped. Then I reduce 1000 guineas into farthings, and find

it

it 1008000, and dividing this by the number of men that escaped, *viz.* 2620, I find the quotient 384 farthings, that is just 8 shillings each man, and 192 farthings remaining over, which is 4 shillings for the little errand boy, as appears by the following work.

5000 foldiers

2380 killed

Divisor 2620 escaped

1000 guineas

21

21000 shillings

12

252000 pence

4

Men

2620)1008000(384 farthings each

786

2220

2096

1240

1048

4)384 farthings

12) 96 pence

8 shillings each. *Ans.*

4)192 farthings remain

12)48 pence

4 shillings for the little boy

Philo. Very well done indeed.

LONG-

LONG-MEASURE.

Quest. 16. How many furlongs, rods, yards, feet, inches, and barley-corns will reach round the world, supposing it (according to the best calculation) to be 25020 miles.

$$\begin{array}{r}
 25020 \text{ miles } * \\
 \underline{8} \\
 200160 \text{ furlongs} \\
 \underline{40} \\
 8006400 \text{ rods} \\
 \underline{11 \text{ half yards in 1 rod } \dagger} \\
 \text{by } 2) 88070400 \text{ half yards} \\
 \underline{\hspace{1.5cm}} \\
 44035200 \text{ yards} \\
 \underline{3} \\
 132105600 \text{ feet} \\
 \underline{12} \\
 1585267200 \text{ inches} \\
 \underline{3} \\
 4755801600 \text{ barley-corns}
 \end{array}$$

* According to this calculation of 25020 miles round the globe, $69 \frac{1}{2}$ miles make 1 degree. See the note in page 49 long-measure.

† Note, That $5 \frac{1}{2}$ yards make 1 rod; but because I cannot well multiply by $5 \frac{1}{2}$, I therefore multiply by 11 and they are $\frac{1}{2}$ yards; and then divide by 2 to bring them into yards.

Another

Another way.

As 1760 yards make a mile, if you multiply 25020 by 1760, you will have the same answer in yards; and if you multiply the yards by 36, you will have the inches, &c.

SECTION III.

QUESTIONS to exercise REDUCTION.

1. IN £. 56 how many crowns and six-pences? *Ans.* 224 crowns, and 2240 sixpences.
 2. In 8568 pence, how many shillings, and guineas? *Ans.* 714 shillings, and 34 guineas.
 3. In £. 12 - 2 shillings, how many groats, three-pences, and six-pences? *Ans.* 726 groats, 968 three-pences, and 484 six-pences.
 4. In 750 pounds, how many moidores and guineas? *Ans.* 555 moidores, and 15 shillings, and 714 guineas, and 6 shillings over.
 5. In 500 ports, at 36s. each, how many pounds, half crowns, crowns, and groats? *Ans.* £. 900, 7200 half crowns, 3600 crowns, and 54,000 groats.
 6. In £. 500 how many pieces, at 7d. $\frac{1}{2}$, and pieces at 7s. 6d. each? *Ans.* 16,000 pieces, at 7d. $\frac{1}{2}$. And 1333 pieces, at 7s. 6d. each, and $\frac{1}{3}$ of a piece, or 2s. 6d.
 7. A merchant at Holland draws a bill upon his correspondent in London, for 2500 ducatoons, at 6s. 2d. $\frac{3}{5}$ each, how many pounds sterling ought he to receive at London. *Ans.* £. 777 - 1 - 8.
- Rule.* Bring a ducatoon into 5ths of a penny, (as in page 138) and multiply the number of ducatoons by them; then bring a pound sterling also into fifts. (*viz.*

1200 fifths) and dividing by 1200, you will have the $\text{£. } 777 \frac{1}{12}$; that is, $\text{£. } 777$ one twelfth of a pound, or 1s. 8d.

8. A rich *Nobleman* has 5 villages, in every village 3 streets, in every street a dozen houses, in every house 5 rooms, in every room 2 bureaux, in every bureau 12 drawers, in every drawer 4 bags, every bag valued at 150 guineas; which he is going to *exchange* for $\text{£. } 3 - 12$ shilling pieces, how many must he receive in all?

Rule. Multiply all the numbers by one another as they follow in order, and your last product will be guineas. Bring these guineas into shillings, and divide by 72, the shillings in a $\text{£. } 3 - 12$ piece, you'll have the answer, which is 3780000 pieces, at $\text{£. } 3 - 12$ shillings each.

And thus, *Tyro*, may you by diligence and observation do any sort of sum.

Tyro. Sir, I see it plainly, and will try to do them all, and prove their answers the first opportunity.

Philo. Then I will now proceed to the most beautiful and most useful rule in all common *arithmetic*, viz. the *Rule of three direct*, commonly called the *Golden rule of three*.

DIALOGUE VII.

SECTION I.

The single Rule of three direct, commonly called the GOLDEN RULE.

Tyro. **W**HAT do you mean by the Rule of three direct?

Philo. The Rule of three is the rule of proportion; because it shews what relation or proportion one number bears to another.

Tyro. What is given, and what is required in the Rule of three?

Philo. Three numbers are given to find a fourth, which is the answer.

Tyro. How is this fourth number found?

Philo. By this one general rule. Multiply the second number by the third, and divide by the first.

Tyro. How does the proportion of these 4 numbers stand or what relation do they bear to each other?

Philo. As the first is to the second number, so is the third to the fourth number. For if you multiply the fourth number, or answer by the first the product will be the same as the third, multiplied by the second; but more of this by and by.

Tyro. How am I to work the Rule of three direct?

Philo. By the following rule, which you ought so far to get by heart, as to understand it without book.

A general R U L E.

1. When any 3 numbers are given, place them one after the other, in order as they stand, and this is called stating the question.

This being done, multiply the *second* by the *third*, and divide by the *first*, and you have the *fourth* number, or answer.

Let the three numbers be 4, 8, and 12.

As 4 : 8 :: 12 to a 4th number, viz. 24.

$$\begin{array}{r} 8 \\ - \\ 4 \overline{)96} \\ - \\ 24 \text{ Ans.} \end{array}$$

Thus 4 : 8 :: 12 : 24, thus read, as 4 to 8, so is 12 to 24.

Here you see the *fourth* number, or answer, is 24. Now, Tyro, it is worth your while to observe what *relation* or *proportion* these bear to each other.

OBSERVATION 1.

As the *first* is to the *second*, so is the *third* to the *fourth*. That is, as 4 is to 8, so is 12 to 24 : for 4 is the half of 8, and 12 is the half of 24.

OBSERVATION 2.

As the *first* is to the *third*, so is the *second* to the *fourth*. That is, as 4 to 12, so is 8 to 24 : for 12, the *third* number, is 3 times more than 4, the *first* number; and 24, the *fourth* number, is 3 times more than 8, the *second* number. Or, in other Words, thus : the *third* divided by the *first*, is equal to the *fourth* divided by the *second*. And thus you will find the *proportion* holds good,

good, take the numbers which way you will, as by the following *observations*.

OBSERVATION 3.

The *first* number is equal to the product of the *second* and *third*, divided by the *fourth*; that is 96 divided by 24 is equal to 4.

OBSERVATION 4.

The *second* is equal to the product of the *first* and *fourth*, divided by the *third*; that is, 96 divided by 12, is equal to 8.

OBSERVATION 5.

The *third* is equal to the product of the *first* and *fourth*, divided by the *second*; that is, 96 divided by 8 is equal to 12.

OBSERVATION 6.

The *fourth* is equal to the product of the *second* and *third*, divided by the *first*; that is, 96 divided by 4 is equal to 24 *.

Tyro. This is very pretty indeed! and will this *proportion* hold so in any other 3 numbers.

Philo. No doubt of that; if you will try any 3, you will find they will answer to the foregoing *observations*.

This is only to shew you, *Tyro*, the nature of *proportion*, relating to numbers, without any particular name; but as for working the *rule of three direct*, when the numbers are called by the names of *pounds, yards, ells, ounces, pence*, or the like; that indeed is the effect of practice, and may very soon be learned by the following instructions, or directions.

* See more of this in my *Young algebraist's companion*, dialogue 7, section 2.

SECTION II.

Containing some full and necessary directions for the better performing this rule.

Tyro. **Y**OU have already told me, that there are always three numbers given to find a fourth; but in what order am I to place them, when they are of a different name, or denomination. That is, *how am I to state the question?*

Philos. Do you but mind the following instructions, and read your questions over slow, and considerately, and you will soon know how to *state* it, or place the numbers in their order; for I shall first of all give you directions to work common easy sums, and then shew you how to proceed when the *first* and *third* numbers are not alike, and when there is required two or three statings.

SECTION III.

DIRECTIONS to state and work easy Questions in the Rule of three direct.

First, **A**LWAYS remember that when 3 numbers are given, *two* of these numbers always are of one and the same name and denomination, *viz.* either money, weight, or measure, which *two* numbers out of the *three*, must be your *first* and your *third* number; and the other in course must be placed in the middle, which *middle* number will always be of the same name as the answer. Thus if the *second* or *middle* number be money, weight, or measure; so also will the *fourth* number or answer be.

2. If at any time the *first number* or the *third*, be of a higher or greater denomination than the other, then make them both of one denomination by the rules of *reduction*. Thus, if the *first number* be yards, and the *third number* ells, reduce them both into quarters, by multiplying the yards by 4, and the ells by 5; so also if the *first number* be *lbs.* and the *third* *Cwts.* and *grs.* you must reduce the *Cwts.* and *grs.* into the *lbs.* that they may not only be of one name, (weight, measure, &c.) but likewise the very same name, viz. pounds pence, yards, quarters, &c.

3. If your *second number* be a compound quantity, such as pounds, shillings, and pence, or *Cwts.* *grs.* and *lbs.* you must reduce it into pence, or into *lbs.* by the rule of *reduction*; and remember, that after you have multiplied the *second* by the *third*, and divided by your *first*, the quotient will be of the same name as you reduced your *second number* into, be it what it will.

4. In all questions in the *Rule of three*, where there is only 3 numbers given, you have already been told, that *two* of those *three* will always be of one name, and must be your *first* and *third* numbers. Now, it is very easy to know which of those *two* must be your *third* number: for whenever a question is asked, or whatsoever number follows the question, that is your *third* number. Thus, what cost 24 yards? or how much does 96 *lb.* cost? Here 24, or 96, will be your *third* number; and so for any other question, as you will see by the two first examples, as follows.

Quest. 1. If 3 yards cost 7 shillings, what cost 24 yards?

Yds. shill. yds.
If 3—7—24

7 multiply

Divide by 3)168

by 2|0) 5|6 shillings

2l. 16s. ans.

Proof, or proportion.

As 3 : 7 :: 24 : 56. For 3 times 56 is 168, and 7 times 24 is 168. And thus for any other question.

Quest. 2. If 8 ells cost 11 shillings, what cost 96 ells?

Ells shill. ells

If 8—11—96

11

8)1056

2|0)13|2

£. 6 - 12 Ans.

Note, Here you see that the question is asked, what cost 24 yards, or what cost 96 ells; therefore, you may be sure these are to be your third number. Then your *first* is plainly known by being of the same name; and your *middle*, or *second* number, must be known of course, which will always be the same name as the answer. See observation 1.

Rule. Having stated the question, I multiply the *second* by my *third* number, or the *third* by the *second*, and divide by the *first*, and the answer is 56 shillings, which is the same name as the *second* number; then I divide

divide those shillings by 20, and have the answer. The second question is done the same.

Tyro. I see the nature of it plainly; but there is a way to prove sums in the *Rule of three*, by stating them backwards, is there not?

Philo. Yes, and a very good method it is to confirm you in the truth of what you have done; for the question will then be turned quite the reverse way.

First proof, or variation.

If 24 yards cost 56 shillings, what cost 3 yards?

<i>Yards</i>	<i>shill.</i>	<i>yards</i>
If 24	56	3
	3	
24)	168	(7 shillings. <i>Ans.</i>
	168	
	0	

Here you see the *fourth* number in the original question, is the *first* number in this question; the answer of the original is the *second* number; and the *second* number in the original is the *answer* here.

Second

Second proof, or variation.

If 7 shillings buy 3 yards, how many can I have for 56 shillings?

Here I find 2 numbers out of the three are money, and the number following the question (*how many*) is 56 shillings; therefore, 56 shillings is my third number, which is thus stated.

Shill. Yards Shill.

If 7—3—56

3

7)168

24 yards. *Ans.*

So that you see, by minding which *two* of the *three* are of the same name, it is no difficult matter to state any common question, and others more difficult will naturally follow by use and practice.

Tyro. I understand it very well indeed, and heartily thank you.

Philo. Then I shall proceed to give you more useful examples, and when any thing occurs that you don't rightly apprehend, ask me, or read the instructions over again.

Quest.

The RULE of THREE direct.

Quest. 3. If 3 lb. of tobacco, (or any other thing) cost 2s. 9d. what cost 255 lb?

lb. s. d. lb.
If 3 — 2 — 9 — 255
12 33 second number

93

765
765

3)8415

12)2805 pence

2|0)23|3-9

£. 11 - 13 - 9 Ans.

P R O O F.

Pence lb. Pence
If 2805 — 255 — 33

33

765

765

2805)8415 (3 lb. Ans.

8415

0

☞ The second number being shillings and pence, I bring them into pence by multiplying by 12, and taking in the odd 9 pence; and after I have multiplied the third number 255 by 33, and divided by my first, which is 3, the quotient is of the same name still, viz. pence, which

which I divide by 12, to bring into shillings, and by 20 to bring them into pounds, as above: and thus for any common sum. See *observation 2.*

Quest. 4. If an ounce of tobacco cost 5 farthings, what cost 2 C. 3 qrs. 21 lb.?

oz. farth. C. qrs. lbs.
If * 1 — 5 — 2 — 3 — 21

4

11 qrs.

28

89

24

329 lbs.

16

1974

329

5264 oz.

5 second number

4) 26320 farthings

12) 6580 pence

2|0) 54|8-4 d.

£. 27 - 8 - 4. *Ans.*

* *Note,* Because the first number is ounces, I bring the third number also into ounces, to answer it; but after I have multiplied by my second number 5, they are all farthings; and as my first number is only an unit (or 1) I do not divide by that; for 1 neither multiplies nor divides: I therefore only divide the farthings by 4, 12, and 20, and the question is done. See the proof the next question.

Quest. 5. If 2 C. 3 qrs. 21 lb. (or 5264 ounces)
cost £. 27 - 8 - 4, what cost an ounce?

oz. £. s. d. oz.
If 5264 — 27 - 8 - 4 — 1

20

—

548

12

—

6580

4

—

5264)26320(5 farth. *Ans.*

26320

—

0

☞ Take notice that
whenever your *first*
number is large, and
the *third* small (as in
this example) that you
always reduce the *mid-*
dle or *second* number,
to its lowest denomi-
nation.

Quest.

Quest. 6. Bought a silver cup and tankard, which weighed 3 lb. 11 oz. troy, and cost me £. 8 - 18 - 2½; what did they lie me in per ounce?

Here, as the question is, *What did they cost per ounce?* You must know your answer is to be in money; so also must your *second* number be, and the other two numbers will naturally follow, thus:

lbs.	oz.	£.	s.	d.	oz.
If 3 - 11	—	8 - 18 - 2½	—	1	

12 oz. 1 lb. troy 20

47 oz. 178 shillings

12

2138 pence

4

74)8554(182 farthings. *Ans.*

47

4)182 farthings

385

376

12) 45-2

94

3s. 9d. ½ per oz. *Ans.*

94

0

Quest. 7. A gentleman has an estate which brings him in clear £. 2000 a year, and the charges he is at, one day with another, for house-keeping, and other expences, is £. 5 - 15s. I demand what he loses or saves every year?

Rule. See what £. 5-15 s. *per* day amounts to in a year, and take it out of his yearly income £. 2000 is what he saves; but if his expences be more, take the £. 2000 from that, and the remainder is what he loses.

<i>Day</i>	<i>£.</i>	<i>s.</i>	<i>days.</i>
If 1	5	- 15	365
	20		115

115	1825
-----	------

4015

2 0	4197 5	shillings
-----	--------	-----------

	Yearly expences	£. 2048 - 15
Yearly expences	_____	£. 2048 - 15
Estate	_____	2000 -

He loses yearly	_____	£. 48 - 15 <i>Ans.</i>

Quest.

Quest. 8. A farmer agreed with his servant to thresh all the corn he had, and the servant was to receive a guinea for every 7 quarters: now, he threshed in all 15 loads 1 quarter, and has received of his master, at different times, by cash and goods, 9 guineas; I demand how the reckoning stands between them?

Rule. First, see what 15 loads 1 quarter come to, at a guinea, or 21 shillings per quarter, and then subtract 9 guineas out of it.

$\begin{array}{rcl} \text{qrs.} & \text{s.} & \text{loads qr.} \\ \text{If } 7 & \text{---} 21 & \text{---} 14 \text{---} 1 \\ & & 5 \text{ qrs. 1 load} \end{array}$

$\begin{array}{r} 76 \text{ qrs.} \\ 21 \\ \hline \end{array}$

$\begin{array}{r} 76 \\ 152 \\ \hline \end{array}$

$\begin{array}{r} 7 \overline{) 1596} \\ \hline \end{array}$

210 22|8 shillings

£. 11 - 8 *Anf.*

Servant's wages for threshing — £. 11 - 8
 He received 9 guineas, *viz* — 9 - 9

Due to the servant — £. 1 - 19 *Anf.*

The RULE of THREE direct.

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Quest. 10. Bought 4 bags of hops, which weighed as under,

	C. grs. lb.			
No	1	—	2	— 3 — 11
	2	—	1	— 2 — 27
	3	—	2	— 1 — 17
	4	—	1	— 3 — 15
				8 - 3 - 14

And gave after the rate of 5 guineas per cwt. what do they come to?

	C.	grs.	lb.
If 1	—	—	—
112	—	5	5
		20	
112 lb.		105	

C.	grs.	lb.
8	3	14
4		
	35	grs.
	28	
	284	
	71	
	994	lbs.
	105	
	4970	
	9940	

First number 112) 104370'931 shillings divided by 20, is 46-11. So that the answer is £. 46-11-10 $\frac{1}{2}$.

	357
	336
	210
	112
	98
	12
112)	1176 (10 pence
	112
	56
	4
112)	224 (2 Farthings
	224
	0

* * * Note

The RULE of THREE direct.

** Note, I reduce the first and third into *lbs.* and the second into shillings, and the quotient is shillings; and the remainder 98 I multiply by 12, and divide by the same divisor, (*viz.* 112) and it gives 10 pence in the quotient; and the next remainder I multiply by 4, and it gives 2 farthings.

* * See a shorter way than either this, or page 121, to bring *cwts. qrs. and lbs.* into *lbs. viz.* in example 1, in tare and tret.

Quest. 10. There is a ship worth £. 2000, of which I have the $\frac{7}{32}$ parts, I demand what my share is worth?

First, according to the common order of the rule of three, I say,

Parts		Parts
If 32 be	£. 2000	what is 7
	7	
	32)14000(437 pounds	
	128	
	120	
	96	
	240	
	224	<i>Ans. £. 437—10s.</i>
	16 remainder	
	20 shillings in a £.	
	32)320(10 shillings	
	320	
	0	

Note,

Note, I multiply the remainder 16 by 20, and divide by the same *divisor* 32, and the quotient is shillings. Had any thing remained I should now have multiplied it by 12, and divided by 32, and the quotient would have been pence, &c.

Note, It often happens, that dividing by the parts is both shorter and easier than dividing by the whole: nothing but practice and observation can make a man master of this; for ten thousand rules without these are of very little account.

O B S E R V A T I O N.

Here, according to the nature of questions of *multiplication* or *division* of money, I perform the above question with ease.

$$\begin{array}{rcl} \text{Parts} & & \text{Parts} \\ \text{If } 32 \text{ ---} & \text{---} \text{ } \text{£ } 2000 & \text{---} 7 \\ & & 7 \end{array}$$

$$8 \text{ times } 4 \text{ is } 32 \text{ by } 8) 14000$$

$$4) 1750$$

$$\text{Ans. } \text{£ } 437 - 10$$

Here I divide by 8, and by 4; when I divide by 4, I find 2 remain, which I call 2 pounds; then I say the 4th part of £. 2 is 10 shillings. And now you may see the beauty of *multiplication* and *division* of money, if you do but observe it well.

Tyro. I humbly thank you, Sir, and see there is nothing like knowing the former rules perfectly.

Philo. As I said before, many questions may be performed by *multiplication*, and *division* of money,

The RULE of THREE direct.

if they be rightly applied, and much shorter and easier, than the common order of the *Rule of Three*. I shall give you one instance, and no more, which if well observed, will be sufficient; for according to the nature of the question, a great many figures may be saved as well as trouble.

Quest. 11. Bought a box of superfine tea, which weighed 2 qrs. 7 lb. and gave £. 73 - 12 - 7½ for it; what is 5 lb of it worth?

First, and common way.

qrs.	lb.	£.	s.	d.	lb.
If 2	7	73	12	7½	5
28		20			

63

1472 shillings

12

17671 pence

4

70686 qrs.

5 third number

63)353430(5610 farthings

315

384

378

63

63

0

Farthings

4)5610

12)1402½

2)0)1116-10

Ans. 5-16-10½

Short

Quest. 12. If 3 yards cost 8s. 3d. what cost 96 ells English?

<i>Yards</i>	<i>s.</i>	<i>d.</i>	<i>Ells Engl.</i>
If 3	8	3	96
4	12		5

12 qrs. 99 480 qrs.
99

4320

4320

First number 12) 47520

12) 3960 pence

2|0) 33|0

£ 16 - 10 *Ans.*

Rule. Here, because the *first* number is yards, and the *second* ells *English*, I bring them both into quarters, by multiplying the yards by 4, and the ells by 5, and then proceed as before.

First

First proof, or variation.

Quest. 13. If 96 ells *English* cost £. 16 - 10s. what cost 3 yards?

Ells	£.	s.	Yards.
If 96	16	10	3
5	20		4
480 qrs.	330	shillings	12 qrs
	12		
	3960	pence	
	12	third number	

First number 480) 47520) 99 pence. Ans. or 8s. 3d.

4320

4320

4320

0

Here you see the *fourth* number or answer, is the same as the *second* number, in the last question. See the next question.

Quest.

Second Variation, or Proof.

Quest. 14. If I buy 3 yards of holland for 8s. 3d. how many ells *English* may I have for £. 16—10.

☞ Here, because the answer is to be in measure, therefore measure must be my second number, thus :

s. d. Yards	£. s.	
If 8 - 3 — 3 —	16 - 10	
12	20	
—	—	
99 pence	330 shillings	
	12	
	—	
	3960 pence	
	3 second number	
	—	

First number 99) 11880 (120 yards

99	
—	
198	Yards.
198	120
—	
0	4 qrs 1 yard
Qrs. 1 ell	5) 480 qrs.
	—
	96 ells. <i>Ans.</i>

☞ Here you see I do not bring the second number, yards, into quarters, as I did before in the other two questions; for it is better to work with it as it is; because it saves a great many figures, and the answer will be 120 yards, which I reduce into ells very easily, by multiplying by 4, and dividing by 5; and if you consider all the three different ways of working this question, it will be of great service to you.

Quest.

Quest. 15. A Draper bought 12 pieces of holland, each piece containing 23 ells *English*, and gave after the rate of 3*s.* 4*d.* per ell *Flemish*; what did they come to?

First 23 ells 1 piece
12 pieces

267 ells in all

<i>Ells Flem.</i>	<i>s. d.</i>	<i>Ells Eng.</i>
If 1 —————	3 - 4 —————	276
3 qrs. 1 ell <i>Fl.</i> 12	12	5
3 qrs.	40 pence	1380 qrs.
		40
		3)55200
		12)18400 pence
		2 0) 153 3 - 4
		£. 76 - 13 - 4 <i>Ans.</i>

Tyro. I am obliged to you; and now, if you please, I will try at one.

Philo. With all my heart; I like to see you in this mind.

Quest.

Quest. 16. A Woollen-Draper bought 4 packs of cloth, each pack containing 3 parcels, each parcel 6 pieces, and each piece 40 yards, and gave 3*l.* 10*s.* for every 6 yards one with another; what do they come to?

Tyro. First, I see how many yards there are in all, by multiplying them together as they stand; thus:

4 packs	Yards £. s.	Yards
3	1 <i>l</i> 6—3—10	2880
12 parcels	—20	70
6	70	6)201600 shillings
72 pieces		2 0) 3360 0
40		1680 <i>Ans.</i>
2880 yards in all		

Philo. Very right; and you may prove this sum several ways at leisure, which will be of service. First, £. 3 - 10s. every 6 yards is 1*l.* 1*s.* 8*d.* for 1 yard. Therefore say, if 1 yard cost 1*l.* 1*s.* 8*d.* what cost 2880, and it will be the same answer. And again, you know 2880 yards, at £. 1 per yard, is £. 2880 ; therefore say, If 20 shillings be 1*l.* 1*s.* 8*d.* what will £. 2880 be; and you will have 1680 for answer, as before.

Quest.

Quest. 17. How many dozen of candles, at 5s. 3d. per dozen, may I have in *exchange* for 3 pieces of *Irish*, each long 20 ells, at 3s. 9d. per yard?

Here is two statings in this question; first, see what the *Irish* comes to, and as the question asks, how many dozen of candles, therefore 1 dozen must be the *middle* number of the second stating.

First, 3 pieces, each 20 ells, is 60 ells in all.

Yard	s.	d.	Ells.	s.	d.	Dozen	£.	s.	d.
If 1	3	9	60.	If 5	3	be 1	14	1	3
4	12		5	12			20		
4	45		300	63			281		
	300						12		
4)	13500			63)	3375	(53 dozen			
							315		
12)	3375	pence					225		
							189		
210)	2811	3							
							36		
£ 14	1	3	<i>Irish</i> cost				12	in 1 doz.	
							63)	432	(6 candles
								378	
							54		

Answer 53 dozen and 6, that is, 53 dozen and a half. The remainder 54, and all other remainders are placed fraction-wise over the *divisor*, thus, $\frac{54}{63}$. So that the true answer is 53 dozen and 6 candles, and 54 sixty-three parts of another candle, or $\frac{6}{7}$, which wants but $\frac{1}{7}$ of another whole one.

Tyro. I understand it all but the $\frac{6}{7}$ ths, how do you make that out?

Philo. You will see better when you come to *Vulgar Fractions*; for $\frac{6}{7}$ is the same as $\frac{54}{63}$. For divide the Numerator 54 by 9, and it is 6, for a new Numerator, and divide 6 the Denominator also by 9, and it is 7. But the answer without the fraction is near enough, and you must not mind these curiosities at present.

Quest. 18. A Woollen-Draper bought a certain quantity of serge and broad-cloth for £. 500: the quantity of broad-cloth he bought was 500 yards, at 12s. 6d. per yard. Now he bought 3 times as many yards of serge as he did broad-cloth. I demand how much each cost, and what the serge cost per yard?

First, I find what the broad cloth comes to.

<i>Yard</i>	<i>s.</i>	<i>d.</i>	<i>Yards</i>		£.
If 1—	12	6—	500	Both of them cost	500 -
	12			Broad cloth cost	312 -
	150			The Serge cost	187 - 10
	500				
12)	75000				
210)	62510		shill.		

£ 312 - 10 *Ans.* broad cloth

Here you see the broad-cloth cost in all £ 312 - 10s. which taken out of £. 500 (what they both cost) the serge then must cost in all £ 187 - 10s. Now the question says, he had 3 times as many yards of serge as he had of broad-cloth, which is in all 1500 yards, (that is, 3 times 50) of serge. Then to find how much the serge cost per yard, I say,

If

Yards £. s. Yard
If 1500 cost 187 - 10 --- 1
20

3750
12

First number 15|00)450|00(30 pence, or 2s. 6d. *Ans.*
45 the large cost per yard.

0

Tyro. Sir, I am highly obliged to you; and I perceive, that it is nothing else but considering the nature of the question, and two statings are as easy done as one. But pray, if I may be so free, is not Interest cast up, or done by the *Rule of three*?

Pbilo. There is a shorter way of doing it, but it may be done very easily; as also, *exchange*, *loss*, and *gain*, *barter*, *fellowship*, and many such like rules, though they bear those different names, are all but the *Rule of proportion*: however, to satisfy you, I will give you an example or two of each, to prepare you the better to work them.

The RULE of THREE direct.

1. An EXAMPLE in INTEREST.

Quest. 19. What is the interest of £. 360; for three years, at £. $3\frac{1}{2}$ (or £. 3 - 10s.) *per cent. per annum?*

Note, Remember that *per cent.* signifies £. 100; so that in other words it is thus: What comes the interest of £. 360 to for three years, at £. $3\frac{1}{2}$ for every £. 100? Say,

£.	£.	s.	£.
If 100	— 3 —	10	— 360
	20		70
	—		—
	70		1100)252100
			—
			210)2512
			—
			£ 12—12 for 1 year <i>Ans.</i>
			3 years multiply
			—
			£ 37 - 16 for 3 years <i>Ans.</i>

And after this manner any sum, at any rate *per cent.* may be done.

Note, When there are odd months you must allow in proportion, thus: Suppose it had been six months more, you see one year, or twelve months, is £. 12-12s. therefore, six months is half of it, *viz.* £. 6-6s. and three months is £. 3-3s. more.

Quest.

Quest. 19. A person left his niece £. 650, to be paid to her when she came to proper age, with interest upon it at 4 *per cent.* Now it lay in the executor's hands 219 days, what had she got to receive principal and interest together?

First, I find the interest for one year.

$\begin{array}{r} \text{£. } \text{£.} \quad \text{£.} \\ \text{If } 100 - 4 \text{ — } 650. \\ \quad \quad \quad 4 \\ \hline \text{£ } 26 \text{ } 100 \end{array}$	$\begin{array}{r} \text{Days } \text{£.} \quad \text{Days} \\ \text{If } 365 \text{ be } 26 \text{ — } 219 \\ \quad \quad \quad 26 \\ \hline \end{array}$	$\begin{array}{r} \\ \hline 1314 \\ 438 \\ \hline \end{array}$
--	---	--

I cut off 2 cyphers, which is the same as divided by 100, and the answer is £ 26 for one year's interest.

$$\begin{array}{r} 365 \overline{) 5694} (15 \text{ £} \\ \underline{365} \\ 2044 \\ \underline{1825} \\ 219 \\ \underline{20} \end{array}$$

$$\begin{array}{r} \text{Principal — } \text{£ } 650 - \\ 129 \text{ days interest is } 15 - 12 \\ \hline \end{array}$$

Ans. She has £ 665 - 12 to receive.

$$\begin{array}{r}) 4380 (12 \text{ shil.} \\ \underline{365} \\ 730 \\ \underline{730} \end{array}$$

2. An EXAMPLE in bartering, or exchanging. See question 17.

Quest. 21. Two merchants *A* and *B* barter, or exchange with one another in traffic. *A* has sugar, which he sells at £. 4 per cwt. for ready money; but in barter he will have £. 4 - 13 - 4. *B* has wine at £ 13 per hogshead: now the question is, how much *B* ought to advance his wine in barter, to equal the advance of *A*'s sugar.

First, Sugar — £ 4 - 13 - 4 in barter
 For ready money — 4 - - -
 —————
 £ - 13 - 4 advance

Then, If $\begin{matrix} \text{£.} & \text{s.} & \text{d.} & \text{£.} \\ 4 & 13 & 4 & 13 \end{matrix}$ advance

12

—
 160

13

—
 480

160

First number 4)2080

—
 12) 520 pence

—
 2)0 413

—
 £ 2 - 3 - 4 Ans.

Ans. *B* must advance his wine £ 2 - 3 - 4: So that in barter his wine is worth £ 15 - 3 - 4 per hogshead.

3. EXAMPLES in profit and loss, or loss and gain.

Quest. 22. Bought 3 pipes of wine for 100 guineas, one of which leaked or ran out 23 gallons; the remainder I sold at the rate of 3 half crowns per gallon, what was my gain or loss?

First, 1 pipe is	126 gallons	<i>Gal. s. d.</i>	<i>Galls.</i>
	<u>3</u>	If 1-7-6	355
		12	90
3 pipes is	378 gallons		
	<u>23 leaked out</u>	90	12)31950
	355 gallons left		210)26612-6
			Sold for £ 133-2-6

Made of the whole £ 133 - 2 - 6

Cost 100 guineas, viz. 105 - -

Gain £ 28 - 2 - 6 *Ans.*

Quest.

The RULE of THREE direct.

Quest. 23. What is gained *per cent.* that is, what is gained in laying out £ 100, when 1 shilling brings in 14d. $\frac{3}{4}$?

First,	14d. $\frac{3}{4}$	d.	d.	£.
1 shilling	12	If 12 gain 2 $\frac{3}{4}$ — 100		
			4	20
1 shilling gains	2 $\frac{3}{4}$			
		11	2000 shillings	
			12	
			24000 pence	
			11	
		12	264000	
		4	22000	
		12	5500	
		2	0 45 8-4	

Gained *Ans.* £ 22 - 18 - 4

Quest.

Quest. 24. A Gentleman that saw very plainly he could not live upon the common interest of his money, set up as common Brewer, and laid out £ 1000 for that purpose, in fitting up an office, and buying utensils; and hops, malt, excise, wages, and other charges at the year's end amounted to £ 4000 more; in which time he sold as much beer as came to £ 6875. I demand what he gained in all, and how much he gained per cent.

First, beer amounted to — £ 6875
His charges are — 5000

Gained in one year — 1875

£. £. £.
If 5000 gain 1875 — 100

5|000(187|500

£ 37 — 2500 remains
20

Ans. He gained in all 5|000)50|000
£ 1875, which is £ 37-10s. 10 shillings
per cent.

* *Note*, I cut off three cyphers in the *divisor*, and 3 figures in the *dividend* to answer them; then I divide 187 by 5, and it is 37, and 2 over, which, I place before the 500 that I cut off, makes 2500 remain, which I multiply by 20, and cut off 3 cyphers, and divide by 5 only, gives 10 shillings.

4. EXAMPLES *in company called Fellowship.*

Quest. 25. Two tradesmen or merchants, *A* and *B*, enter into partnership, and purchase one common stock. *A* put in £ 1750, and *B* put in £ 1050; and they gain the first year of trading £ 800 clear; what is the share of each, according to the sum each advanced at first?

Rule. In all questions of this nature the rule is, Add together what every person first put into stock; and then say, as the *whole stock* is to the *gain or loss*; so is what every person separately put into his particular *gain or loss*. That is, the *whole stock* added together is your *first* number; the *gain or loss* your *second* number; and what every man separately put in is your *third* number; and the different answers, or *fourth* number, will be every man's proper share.

The work.

<i>A</i> put in	_____	_____	£ 1750
<i>B</i> put in	_____	_____	1050
Whole stock	_____		£ 2800

Secondly, If $\begin{matrix} \text{£.} \\ 2800 \end{matrix}$ gain $\begin{matrix} \text{£.} \\ 800 \end{matrix}$ — $\begin{matrix} \text{£.} \\ 1050 \end{matrix}$ B's stock.
 $\begin{matrix} 800 \end{matrix}$

$\begin{matrix} 2800 \\ 84 \end{matrix}$ $\begin{matrix} 8400 \\ 84 \end{matrix}$ $\begin{matrix} 100 \\ 84 \end{matrix}$ $\begin{matrix} 300 \\ 84 \end{matrix}$ £. B's
 gain.

00

Ans. A's Share of the Gain is — £ 500

B's Share is — — — 300

Gain — — — 800 proof

Note, If the gain or loss be pounds, shillings, and pence, reduce it accordingly, and your answer will be of the same name; and if there be remains, add them together, and they will be always equal to the divisor if the work be right, and you must carry one to the next denomination when it happens so.

Quest. 26. A person broke, in debt for £ 1890; but he gives up all his effects to his creditors, which amounted to £. 661 - 10s. how much then must each creditor receive in the pound?

Note, Had each person's debt been specified, there must have been as many different statings, but in this case, but one, thus:

$\begin{matrix} \text{£.} & \text{£.} & \text{s.} & \text{£.} \\ \text{If } 1890 & \text{be } 661 & 10 & \text{— } 1 \end{matrix}$
 20

$\begin{matrix} 1890 \\ 13230 \end{matrix}$ 13230(7 shillings in the pound. Ans.
 13230

0

P R O O F.

Debt. £ 1890

 7

20)1323|0

 £ 661 - 10 effects

Now, suppose for example's sake, that any of the creditor's debt or demand was £ 58, what must he receive? Say, if £ 1 be 7 shillings, what will £ 58 be, and you will find it £ 20 - 6 for his part; and so for the others, be they more or less.

Tyro. I am extremely obliged to you, for your explaining things to me in this manner.

Philo. I have no other aim but your advantage; nor should I have been so long and particular, but that your knowledge in this *rule* is the foundation of all the higher rules, and even of the *mathematics* itself, thousands of examples more, 'tis true, might be given you; but as they depend upon what you have seen, and your constant practice, I shall give you but one more, which in some measure varies from the former ones.

Quest.

Quest. 27. A woollen-draper bought of a clothier, an equal quantity of four sorts of cloth, which cost him £ 367 - 10 s. Some cost 18 s. 6 d. some 14 s. 3 d. some 11 s. 6 d. and some 4 s. 9 d. per yard: how many yards of each had he?

Rule. The method of working all sums of this sort is this: add all the different prices together into one sum, which is the *first* number; then 1 yard, or 1 lb. &c. will be your *second* number, and the whole sum laid out your *third* number.

The work.

First, I add all the different prices thus;

s.	d.
18	6
14	3
11	6
4	9
20	

£.	s.	£.	£.	s.	£. 2 - 9 -
If 2	-	9	-	1	-
367	-	10	-	20	-
20					

49 shillings

49)7350(150 yards of each. *Ans.*

49

245

245

0

Tyro. I perceive the nature of it plainly

Philo. Indeed, *Tyro*, if you understand these examples, you are able to solve any common question relating to business.

N. B. My intent, *Tyro*, was to have given you here a notion of *timber-measure*, and how to *gauge* a common cask, cooler, or piece of malt, &c. or to measure

any regular piece of ground; it being not only diverting, but also useful in the country, and very satisfactory to parents, when their children have some knowledge of these things. But I shall reserve this for a dialogue by itself, and only leave you a few questions for practice.

QUESTIONS to exercise the learner in the Rule of Three.

28. The rents of a whole parish amount to £ 1750, and a rate is granted of £ 32 - 16 - 3; what is that in the pound? *Ans.* 4 d. $\frac{1}{2}$.

29. A bankrupt is indebted £ 2980 - 10 s. but all his effects amount but to £ 931 - 8 - $1\frac{1}{2}$; what have his creditors in the pound? *Ans.* 6s. 3d.

30. Bought a lighter of coals, containing 33 chaldrons, 12 bushels, for which I gave £ 50; what do they lie me in per bushel? *Ans.* 10d.

31. Bought of a Goldsmith 4 lb. 11 oz. 10 dwts. of plate, at 5s. 4d. per ounce; what does it come to? *Ans.* £ 15 - 17 - 4.

32. What is the interest of £ 631 - 5s. for a year, at 3 per. cent? *Ans.* £ 18 - 18 - 9.

33. What comes the commission of 245 - 6s. to, at $2\frac{1}{2}$ per cent? *Ans.* £ 6 - 2 - $7\frac{3}{4}$, $\frac{1}{5}$.

34. What must I give for the $\frac{6}{32}$ parts of a ship, that is worth £ 635 - 5s. *Ans.* £ 119 - 2 - $2\frac{1}{4}$.

35. Shipped for Barbadoes 500 pair of stockings, at 3s. 6d. per pair, and 1650 yards of baize, at 15d. per yard; and I have received in return 348 gallons of rum, at 6s. 8d. per gallon, and 750 lb. of indigo, at 1s. 6d. per lb. what remains due upon my adventure? *Ans.* 24 - 12 - 6.

Rule. See what the stockings and baize come to, and add them together; then see what the rum and indigo both come to, and subtract it from the other, you will find the answer as above.

36. How

36. How many bricks 9 inches long, and 4 inches wide, will floor a room 18 feet square; that is, 18 feet wide, and 18 feet long?

Rule. First, find the content of the room in square feet, that is, multiply the length by the breadth; then multiply that product by 144, will bring it into square inches: this done, multiply the length of the brick by the breadth, it is 36 inches, which is your divisor. And the answer is 1296 bricks, 2 thirds.

37. How many 10 inch tiles will floor a malt kiln 16 feet long, and 14 feet wide. *Ans.* $322 \frac{56}{100}$. *Rule.* Multiply 10 by itself, is the contents of one tile in square inches; then find the contents in the room in inches, and divide by 100, it is 322, and better than a half.

38. Bought a ton of iron, which cost £ 19 - 10 s. there being in number 32 bars: I desire to know the price of each, and what they weigh one with another. *Ans.* 12 s. 2 d. $\frac{1}{4}$, and each bar weighs 2 qrs. 14 lb.

39. There is a cistern that holds 4 hogshheads of water, (allowing 63 gallons to the hogshhead) in which are placed two pipes, the larger of which will discharge $1 \frac{1}{2}$ gallon every minute; but if they both be set open they will discharge $2 \frac{1}{4}$ gallons every minute: I demand in how long time the cistern will be filled by each pipe alone, and in how long when they are both set open together. *Ans.* Greater pipe will fill it in 2 hours 48 minutes; the small one in 5 hours 36 minutes; and both in 1 hour 52 minutes.

40. There is a steeple, which stands right upon level ground, whose shadow measures 75 yards, at the same time that the shadow of a straight walking stick, which is a yard long, measures 5 feet: I demand the height of the steeple. *Ans.* 45 yards.

41. As I was beating on the forest grounds,
 Up starts a hare before my two greyhounds;
 The dogs be light of foot, did fairly run
 Unto her 15 rods, just 21.
 The distance that she started up before
 Was 4 score, 16 rods just, and no more:
 Now this I'd have you unto me declare,
 How far they ran before they caught the hare?
Ans. 336 rods.

DIALOGUE VIII.

SECTION I.

The RULE of Three Inverse, or Reciprocal Proportion.

Tyro. **W**HAT do you mean by the *Rule of Three Inverse*?

Philo. It is often called the *Rule of three Reverse*; that is, it is contrary to the *Rule of Three Direct*. For, as in the *Rule of Three Direct*, you multiply your *second* number by your *third*, and divide by your *first*; here you multiply your *second* by your *first*, or your *first* by your *second*, and divide by your *third*, and the *fourth* number is the answer.

Note. Your *third* and *first* number must be of the same name or denomination, as in the *Rule of Three Direct*, and the *fourth* number or answer, will be of the same name as the *second* number.

Quest.

Quest. 1. How many yards of shalloon, 3 qrs. wide, will line 4 yards of cloth, which is 5 qrs. wide? *Ans.* 6 yards $\frac{2}{3}$.

As in the *Rule of Three Direct*, so here also, the *question*, or demand lies upon the *third* number.

qrs. yards qrs.
If 5 — 4 — 3
5 first number

3)20
—
Ans. 6 $\frac{2}{3}$

Ans. 6 yards, and 2 remains, which I place a top of the 3, thus $\frac{2}{3}$, and it is 2 thirds. So is the answer 6 yards $\frac{2}{3}$.

Quest. 2. If when wheat is 3 s. 6 d. a bushel, the three-penny loaf weighs 4 lb. 2 oz. what ought it to weigh when the wheat is 5 shillings a bushel? *Ans.* 2 lb. 11 oz.

s.	d.	lb.	oz.	s.
If 3	- 6	— 4	- 2	— 5
12		12 oz.	1 lb. troy	12
—		—		—
42 pence		50		60
		42		
		—		
		100		
		200		
		—		
		6 0	210 0	
		—		
		12)	35 oz.	
		—		

2 lb. 11 oz. *Ans.*

Quest. 3. A regiment of soldiers, consisting of 1000 men, are to have new coats, each containing 3 yards, 2 qrs. of cloth, that is, 6 qrs. wide, and they are to be lined with shalloon that is yard wide; how much cloth will it take for their cloaths, and how much shalloon will line them? *Ans.* 5250 yards.

First, see how much is in their cloaths, thus:

1 coat is 3 Yards 2 qrs.

4

that is 14 qrs.

1000

14000 qrs. in all

Then, if $\begin{array}{ccc} \text{qrs.} & \text{qrs.} & \text{Yard.} \\ 6 & 14000 & 1 \text{ shalloon} \end{array}$

6

4 qrs.

Third number 4)84000

4)21000 qrs.

5250 yards. *Ans.*

Quest.

Quest. 4. An acre of land contains (according to the table) 40 rods in length, and 4 in breadth, what must be the length to make an acre, when the breadth is 15 rods?

Breadth Length Breadth

If 4 ——— 40 ——— 15

4

15)160(10 rods

15

10

5½ yards 1 rod

Rods Yds Feet

15)55(3 yards. Ans. 10 - 3 - 2

45

10

3 feet

15)30(2 feet

30

0

Quest. 5. If £ 100 in 12 months gain £ 4 interest, what principal will gain as much in 8 months? Ans. £ 150.

Months £. Months

If 12 ——— 100 ——— 8

12

8)1200

£ 150 Ans.

Quest.

Quest. 6. An army of soldiers consisting of 800, besiege a town, having with them provisions for 3 months, but as they could not take the town in that time, the General was willing to make the same provisions last 5 months, allowing each man the same daily provision : The question is, how many men must there be that the provision may last them 5 months ?

<i>Months</i>	<i>Men</i>	<i>Months</i>	<i>Men</i>
* If 3	800	5	800 for 3 months
	3		480 for 5 months
	5) 2400		320 discharged
	480	<i>Ans.</i>	

The last QUESTION varied.

* *Note,* This question is set in words on purpose, *Tyro*, to puzzle you ; but you must observe, that in all such sort of sums, the circumstance of what is done is never to be minded : for, suppose I should say thus :

If 800 men in 3 months eat 10000 penny loaves ; how many will eat the same in 5 months, at that rate ?
Ans. 480 men as before : so that the action, or circumstance of what they do is nothing, let it be eating, drinking, or any other thing.

Tyro. I understand you, Sir, and thank you for this observation ; and I think you have given me sufficient examples.

Philo. All other questions are done after the same manner. If you are perfect in these, you will easily do the following ones.

QUESTIONS

QUESTIONS to exercise this RULE.

Quest. 7. If when wheat is sold for 3s. 9d. a bushel, the six-penny loaf weighs 5 lb. 10 oz. how much ought it to weigh when it is half a crown a bushel? *Ans.* 8 lb. 9 oz.

Quest. 8. How many yards of matting, that is 2 feet wide will cover a room 20 feet long and 18 feet wide. *Ans.* 180 feet, or 60 yards.

Quest. 9. How many yards of paper, 3 quarters wide, will be sufficient to hang a room that is 24 yards round, and 4 yards high? *Ans.* 128 yards.

Tyro. These questions are sufficient, sir.

Philo. Then we will proceed to *section 2*.

SECTION II.

Of Plural Proportion, called the Double Rule of Three Direct.

Tyro. WHAT is plural proportion, or the double rule of three direct?

Philo. Plural proportion is when 5 numbers are given to find a sixth; which six numbers, or answer, is found by two statings of the single rule of three direct.

Tyro. This is very difficult I imagine; for how am I to know how to make the first stating.

DEFINITION and RULE.

Philo. In every regular sum in the Double rule of three direct, the first three numbers always shew the condition, or supposition; and the other two, that is, the two

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two last, shew the demand. The answer of the question will always be the same as the *middle* number; and the *first* and *third* must be of the same name in both statings, like as in the *single rule of three direct*.

Tyro. Cannot these questions be done at one stating?

Philo. Yes, by a rule generally called the *rule of three of 5 numbers*, and that with a great deal more ease, as you shall see by and by; but we will give an example here.

Quest. If £ 100 in 12 months gain £ 4 interest; what will £ 175 gain in 9 months. *Ans.* £ 5 - 5s.

First, if £. 100 gain £. 4 what—£. 175

4
—
1|00)7|00
—
£ 7

Here I find £ 175 gains £ 7 in twelve months. Therefore, I say in the second stating

Months £. Months
If 12 — 7 — 9

9
—
12)63
£ 5 — 3 remains
20
—

Same divisor 12)60

—
5 shillings

Or

Or thus :

You may begin with the *second*, *third*, and *fifth* numbers.

Months £. Months
First, if 12—be—4——9 Ans. £ 3.

£. £. £.
Then, If 100—3——175 Ans. £ 5 - 5.

Note, See the proof of this question in the first example of the next rule *inverse*.

Quest. 2. If 600 seamen in one week eat 1500 lb. of beef; how many lb. will serve 120 seamen 12 weeks?
Ans. 3600 lb. See *Example 2*, in the next rule.

All questions in this rule being done after the same manner, I shall leave other examples for the *rule of three of five numbers*, by which they are done with more ease; and shall only give you a notion of stating questions here and in the *double rule of three inverse*.

A RULE to state, or know the nature of a question, and to know the certainty of your three first numbers, and the name of the answer.

You know there are *five* numbers given to find out a *sixth*; mind then you carefully observe what name your answer is to be in: this being done, place your 5 numbers in two rows, *viz.* 3 in the first row, and 2 under them, leaving a blank space under that number which is of the same name as your answer is to be in; so will you see both here, and in the next rule, the order of both these *double rules of three direct and inverse*.

Take

196 *The DOUBLE RULE of THREE direct.*

Take the first question, where your answer is to be in pounds, and rank the number thus, as the question itself runs.

If £ 100—12 months £ 4.

£ 175—9 months *

* Here you see the blank, or vacant place, is *money*; therefore, money must be your middle number of the three, and money will be the answer. *Note*, The blank must fall under the supposition, which here is £ 4.

Second QUESTION.

If 600 seamen—I week—1500 *lb.*

120 seamen—12 weeks *

* This *third* number in the second row, has a blank place under you see, according to the order of the sum; therefore, 1500 *lb.* must be your *second* number in the first stating, and your answer will also be *lbs.*

Tyro. I humbly thank you, sir.

Philo. There are many other ways of varying this rule; but this is sufficient for your purpose at present. See the rule of five numbers.

SEC-

SECTION III.

The Double Rule of Three Inverse, called also Plural Proportion.

Tyro. **W**HAT is the difference between this rule and the last?

Philo. A great deal; for though here be 5 numbers given to find a *sixth*, yet you will find it much more difficult to state the question here than in the other; but observe the following direction.

R U L E.

1. As in the *double rule direct*, so also here are 3 numbers in the supposition, and 2 in the demand.
2. Place the numbers in order, as in the last rule, viz. 3 in one row and 2 in the next, leaving that number with a blank place under, which will be the same name of your answer, observing well where the supposition lies.
3. This being done, you state the question with 3 numbers first, as before; which is wrought according to the *single rule of three inverse*.

☞ If your first stating be *direct*, the second must be *inverse*; for both are never *direct*, nor never *inverse*.

Quest.

198 *The DOUBLE RULE of THREE Inverse.*

Quest. 1. If £ 100 principal * in 12 months gain £ 4 interest, what principal will gain 5 guineas (*viz.* £ 5 - 5s.) in 9 months? *Ans.* £ 175 principal.

* *Note,* The word principal signifies the money put out to interest.

Here you see there are three numbers in the supposition, *viz.* £ 100, 12 months, and £ 4; and 2 in the demand, *viz.* £ 5 - 5s. and 9 months.

Now observe, that one of these two in the demand must be the last number in the *first stating*, and the other the last in the *second stating*, and one must be always *inverse*.

First, inverse.
$$\begin{array}{ccccc} \text{Months} & \text{£.} & - & \text{Months} \\ \text{If } 12 & \text{---} & 100 & \text{---} & 9 \end{array}$$

Multiply £ 100 by 12, and divide by 9, you have £ $133\frac{1}{3}$, *viz.* £ 133 - 6 - 8. Then,

Secondly, direct.
$$\begin{array}{ccccccc} \text{£.} & & \text{£.} & \text{s.} & \text{d.} & & \text{£.} \text{ s.} \\ \text{If } 4 & \text{---} & 133 & - & 6 & - & 8 \text{ --- } 5 - 5. \end{array}$$

Here I reduce the *first* and *last* numbers into shillings, the *second* into pence, and by the *rule direct*, I find the answer £ 175, the principal required.

2. Or thus :

1. *Direct.*
$$\begin{array}{ccccccc} \text{£.} & & \text{£.} & & \text{£.} & \text{s.} & \text{£.} \\ \text{If } 4 & \text{---} & 100 & \text{---} & 5 & - & 5. \end{array} \text{ Ans. } 131-5s.$$

2. *Inverse.*
$$\begin{array}{ccccc} \text{Months} & \text{£.} & \text{s.} & \text{Months} \\ \text{If } 12 & \text{---} & 131 - 5 & \text{---} & 9. \end{array} \text{ Ans. } £ 175,$$

as before.

Quest.

The RULE of THREE of five Numbers. 199

Quest. 2. If 600 seamen in 1 week eat 1500 lb. of beef, how long will 3600 lb. serve 120 men at that rate.

	<i>Men</i>	<i>Week</i>	<i>lbs.</i>
Here, If	600	1	1500
	3600 lb.	*	120 men

First, Stating Inverse. If 600—1—120 *Ans.* 5 weeks.

Secondly, Direct. If 1500—5—3600 *Ans.* 12 weeks.

* See the second question of the last rule.

Tyro. I understand it a little, but cannot say I am perfect in it.

Philo. It is of no great signification at present; for you will see more of it in the next rule.

SECTION IV.

The Rule of three composed of five numbers.

Tyro. **W**HAT difference is there between this and the other two preceding rules?

Philo. Only this, that all questions in the *double rule of three direct and inverse*, are performed here at one stating.

Tyro. So you said before, I remember; please, therefore, to give me a rule to work it by; for, I think that is much better than the trouble of two statings.

Philo. It is so in general, but not always: however, it is easier, and better for the learner upon the whole; for here, as well as in the two former rules, the difficulty lies to know whether the question be *direct*, or

in-

200 *The RULE of THREE of five Numbers.*

inverse. This being known, the manner of stating the question will be easy, and the work as easy.

A standing rule to state questions.

1. As in the *single rule of three direct*, so also here, the *middle number* governs the question; and the answer will be of the same name.

2. Having noted which the *middle number* is, the other 4 are easily known; for the *two* that are in the supposition always are the *two* before the *middle numbers*; the other *two* are the demand under the *third* and *fourth* numbers.

Lastly, Always observe, that your *first number* be of the same name as your *fourth*; and your *second number* the same name as your *fifth*. This being done

1. *The rule for all direct questions is this.*

Multiply your *first number* by your *second*, for a *divisor*, then multiply your *third*, *fourth*, and *fifth* together, and divide their product by the product of the *first* and *second*, and the quotient will be of the same name as the *middle number*, be what it will, and is the answer.

Ex-

The RULE of THREE of five Number. 201

EXAMPLE. The first question of the two last rules.

Quest. 1. If £ 100 in 12 months gain £ 4 interest, what will £ 175 gain in 9 months. Ans. £ 5 - 5s.

£.	Mths.	£.	£.	Mths.
If 100	— 12	gain 4	— 175	— 9
	12			9
1200			1575	
			4	
			12	100
			£ 5 - 3	remains
			20	
			12	60
			5 shillings	

Ans. £ 5 - 5s. See the first example of the last rules.

Quest. If 600 seamen, in 1 week, eat 1500 lb. of beef; how many lb. will serve 120 seamen 12 weeks?

Men	Week	lb.	Men	Weeks
If 600	— 1	— 1500	— 120	— 12
	1			120
600		180000		
		12		
		6	100	21600
		3600	lbs.	Ans.

See Ex. 2, in the double rule direct.

Ans-

Another rule for the more easy stating a question.

1. *Note*, If you observe the two foregoing examples, they are stated just as the numbers follow one another in the words of the question; but it very often happens that the numbers in the sum lie contrary to the order they ought to be placed in; therefore, you should turn the question into other words and it will be easier. Or,

2. Set all the numbers down as they run in the question, writing the names over each; then consider what your answer is to be in, and place that down afresh for the *middle number*; then the *two* where the demand lies must be your *fourth* and *fifth*, and the other *two* your *first* and *second* numbers: the *first* must be the same as the *fourth*, and the *second* the same as the *fifth*. See the next question.

Quest. 3. If £ 4 be the wages of 8 men 10 days, what will it cost me, or what must be the wages of 32 men 12 days? *Ans.* £ 19 - 4s.

Here, as before directed, I place the numbers after one another, thus :

£. Men Days Men Days
If 4—8—10—32—12. wrong stated.

Now, here the *middle number* is days, but should be money, because the question says, what will it cost me, for 32 men 12 days; therefore money must be my middle number, and then, *Tyro*, the true stating will appear easy as follows :

Men Days £. Men Days
Thus *, 8—10—4—32—12. *Ans.* £ 19-4s.

Now here, *Tyro*, the stating runs like the words of the sum, as follows, which is very natural.

* If 8 men in 10 days cost me £ 4 for wages, what will 32 men cost me in 12 days, at that rate? *Ans.* as before.

Quest.

Quest. 4. (A proof of the first question in the last rule)
A person put out 175 pounds to receive interest, and when it had continued 9 months, he received principal and interest together £ 180 - 5s. I demand at what rate *per cent. per annum*, he put his money out at? *Ans.* £ 4 *per cent. per annum.*

Here the words of the question are not like the *stating*. You must observe then to proceed as under, in all questions of this nature.

Rule. Take the principal from the interest and principal together, the remainder will be the interest for the time it continued, and then the stating will follow.

Thus, Principal and interest		£ 180 - 5s.
Principal put out	—	175 -
Interest for 9 months	—	£ 5 - 5s.

£.	Mths	£.	s.		£.	Mths.
If 175	— 9	gain 5 - 5	interest,	what will	100	in 12

Here, according to the rule, bring your *middle number* into shillings; then multiply it by your *fourth* and *fifth*, for a *dividend*; this done, multiply your *first* and *second* together for a *divisor*, and the quotient will be 80 shillings, or £ 4, the rate he put the money out at.

Tyro. I heartily thank you; and I find by this I shall be able to answer Mr. Coker's last question in the *double rule of three direct*, which before always appeared very difficult to me.

Philo. You say very right, *Tyro*, for the question is done after the very same manner; therefore shall give you only two or three examples in the *double rule of three inverse*, and leave you to practise the other examples.

QUESTIONS *Inverse.*

2. The rule for *questions inverse* is as follows: the 3 numbers belonging to the supposition must be your *first*, *second*, and *third* number, and the demand is your *fourth* and *fifth* number. These being observed, the rule is, multiply your *second*, *third*, and *fourth* together for a *dividend*, and your *first* and *fifth* together for a *divisor*, and the *quotient* is the same answer as your *middle number*.

Quest. 1. (*A proof to the very last question, and of question 1 in the last rule*) if £ 100 in 12 months gain £ 4 interest, what principal will 5 guineas gain in 9 months?

Here, as the principal is to be the answer, so the principal (*viz.* £ 100) must be your *middle number*.

£. Months £.	£. s. Months £.
If 4—12 be 100 principal,	what 5 - 5 be 9 <i>Ans.</i> 175.

I state the question according to the first standing rule, making my *first* and *fourth*, and my *second* and *fifth* alike; this done, I bring my *first* number into shillings (*viz.* 80) because the *fourth* must be brought into shillings: this being done, I multiply the *second*, *third*, and *fourth* together for a *dividend*, and the *first* and *fifth* together for a *divisor*, and the *quotient* is £ 175, the principal required.

Quest. 2. If 600 seamen in 1 week eat 1500 lb. of beef, how long will 3600 last 120 seamen?

lb.	Men	Week	lb.	Men
If 1500—	600—	1—	3600—	120

Here I multiply the *second*, *third*, and *fourth* together for a *dividend*, which is 2160000, and my *first* and *fifth* for a *divisor*, which is 180000, and the *quotient* is 12 weeks.

Quest.

Quest. 3. If it cost £ 240 for 60 servants in 8 weeks, how long will £ 140 serve 8 servants? *Ans.* 35 weeks.

Tyro. Sir, these examples are sufficient, I think.

Philo. If you are well grounded in the nature of the rule, that is enough; for you may set yourself sums at any time by way of exercise. And now, *Tyro*, I will give you a notion of *tare* and *tret*.

SECTION V.

TARE and TRET.

Tyro. **W**HAT do you mean by *tare* and *tret*?

Philo. *Tare* and *tret* are allowances made in buying and selling commodities that are liable to loss or waste.

Tyro. What are the general names belonging to this rule.

Philo. They are 6, *viz.* *gross*, *tare*, *tret*, *futtle*, *cloff*, and *neat*.

Tyro. Please to explain these in their order.

Philo. *First*, *Gross* signifies the whole weight of any commodity, or parcel of goods, bag, or box, or cask, and all included thus: suppose I had a box of tea, bag of spice, or cask of oil, which weighs 120 lb. I say it weighs 120 lb. *gross*.

Secondly, *Tare* is an allowance made for the weight of the box, bag, or cask, and what hangs about it, and is to be taken out of the *gross*, and then the remainder is called the *neat*, or *clear weight* of the commodity: thus, suppose the foregoing box, bag, or cask, after the goods are out, should weigh 16 lb then

I say, it has 16 *lb.* *tare*, which I subtract out of the *gross* 120 *lb.* there remains 104 *lb.* *neat*, thus:

Bought a box of tea,	<i>Gross</i>	120 <i>lb.</i>
	<i>Tare</i>	16 <i>lb.</i>
	<i>Neat</i>	104 <i>lb.</i>

Thirdly, *Tret* is an allowance of 4 *lb.* for every 104; that is, 1 *lb.* is allowed for every 26 *lb.* for such commodities are liable to waste, moths, dust, &c.

Note, When there is *tret* in the sum, then after the *tare* is taken from the *gross*, the remainder is called *suttle*, instead of *neat*.

Note 2. To find the *tret* pounds, divide the *suttle* pounds by 26, and the *quotient* in the *tret*, which subtracted from the *suttle* gives the *neat*.

Thus, A box of tea, *Gross* 120 *lb.*
Tare 16

26) 104 (4 *tret*
 104

 0

<i>Suttle</i>	104 <i>lb.</i> which divide by 26
<i>Tret</i>	4 <i>lb.</i>
<i>Neat</i>	100 <i>lb.</i>

Fourthly, *Suttle* pounds are so called (as was said before) when there is *tret* in the sum; for when there is no *tret* there is no *suttle*, and the *tret* pounds are always subtracted out of them, as in the above.

Fifthly, *Cloff* is an allowance of 2 *lb.* for every 3 *Cwt.* for the turn of the scale, and for small draughts; but this is allowed only in some particular things and cases.

Sixthly, *Neat* weight is the *clear* weight of the commodity after all the allowances are made.

Tyro. Sir, I understand you very well; but if I remember, you told me, (in *page 164*) you would shew me a shorter method to reduce *Cwts.* and *qrs.* into *lbs.* than you did in *page 121*.

Philo. I did so; and will now shew you all the ways, and leave you to take your choice.

To reduce hundreds and quarters into pounds.

EXAMPLE I. Reduce 17 C. 3 qrs. 15 lb. into lbs.

	C. qrs. lb.	
<i>First method</i>	17 - 3 - 15	
	4	
	—	
	71	
<i>Common way</i>	28	
	—	
	573	
	143	
	—	
<i>Ans.</i>	2003 lbs.	

	C. qrs. lb.
<i>Second method</i>	17 - 3 - 15*
	17
	17
	17
	Add 99 for the 3 qrs.
	— 15 lbs.
<i>Ans.</i>	2003 lb. See p. 121.

* *Note,* In this method you must always set the C's down 4 times, *viz.* twice under one another, and the other two, each one place more towards the left-hand; then count how many *lbs.* there are in the odd *qrs.* and *lbs.* (which are here 99) and always place them under the *units, tens*, as above, and adding them together, you have the *answer*.

Note, This method is shorter than the *first*, and more particular when there are many figures in the given *hundreds*.

Third method. Multiply by 112, and take in the odd pounds.

$$\begin{array}{rcl} \text{C. qrs. lb.} & & \text{C. lbs.} \\ 17 - 3 - 15 \text{ is} & 17 \text{ and} & 99 \\ & 112 * & \end{array}$$

Ans. 2003

* Here I multiply by 112 in 1 line, thus: I say, 12 times 7 is 84, and 9 odd lbs. make 93, 3, and I carry 9; then 12 times 1 is 12, and 9 I carried is 21, and the other 9 I take in also is 30, that is 0, and I carry 3. *Lastly*, I say, once 17 is 17, and 3 I carried is 20, which I place by the side of the other 2 figures, and it is done.

This is also shorter than the *first method*, but something more difficult till you learn to multiply by 112 very well, as you will see in *Ex. 2*.

 *Fourth method.* The best way, (in my opinion) is this last method, which is this: multiply the C's by 12 only, always remembering to place the two first figures out towards the right-hand; then set the odd pounds under, and add them together as they stand, it is done.

$$\begin{array}{rcl} \text{C. qrs. lb.} & & \text{C. lbs.} \\ \text{Thus, } 17 - 3 - 15 \text{ is} & 17 - 99 \text{ †} & \\ & 204 & \\ & 99 & \\ & \hline & & \end{array}$$

Ans. 2003 lbs. as before.

There is no occasion to set down the 12 under the 17, when you multiply; for it is easy to remember you multiply by 12. I say then 12 times 7 is 84, (which 4 I place 2 figures towards the right hand of the 17) that

that is 4 and I go 8; then 12 times 1 is 12, and 8 is 20, which I place to the left of the 4, so that the 2 falls under the 7; then I set down the 99 odd pounds, and add them together, they make 2003 *lb.*

‡ *Note*, But you may do it yet shorter by use, and full as easy, and that is by taking in the 99 as you multiply, which will save the trouble of adding it afterwards.

$$\begin{array}{r}
 \text{C. } lb. \\
 \text{Thus, } 17 - 99 \\
 \quad 303 \\
 \hline
 2003
 \end{array}$$

Here I say, 12 times 7 is 84, and 9 is 93, 3 and I carry 9; 12 times 1 is 12, and 9 I carry is 21, and the last 9 belonging to the 99 makes 30.

Now, the reason of this will appear plain, if you consider that 17 C. is 1700 *lb.* and 17 times 12 *lb.* therefore, I only set down 1700 *lb.* and add 12 times 17 to it.

$$\begin{array}{r}
 \text{Thus, } 17 \text{ C. is } \text{—} 1700 \\
 \text{More, } 12 \text{ times } 17 \text{ is } 204 \text{ add}
 \end{array}$$

Ans. 1904 *lb.* in 17 C.

$$\begin{array}{r}
 1700 - 99 \text{ lb.} \\
 \text{So also } 303 \text{ is } 12 \text{ times } 17, \text{ and } 99. \\
 \hline
 2003 \text{ as before}
 \end{array}$$

✎ *Note*, Or you may do it shorter yet, only by putting or joining the odd *lbs.* by the side of the C's (making a dot between them for distinction sake) then multiply the C's only by 12, placing 2 figures out from them,

them, as before directed, and you have the answer, which will save the trouble of taking in.

Thus, 17 - 99 I set down 17.99

204

Ans. 2003 lb.

Here I multiply 17 only by 12, always placing the two first figures to the right-hand: then add the two lines together.

Example 2. In 47 C. 1 qr. 26 lb. neat, how many lbs.

C. qr. lb. C. lb.

Here 47 - 1 - 26 is 47.54 multiply 47 only by 12.
47 multiplied by 12 is 564

5318 *Ans.*

Example 3. In 86 C. 1 qr. 17 lb. neat, how many lbs.

C. qr. lb. C. lb.

Here 86 - 1 - 17 is 86.45
86 multiplied by 12 1032

9677 *Ans.*

Here I join the odd 45 lb. close to the 86 C. and then multiply 86 only by 12, placing two figures out as before.

Tyro. Very well, sir, I understand what you have shewn me; but suppose the odd pounds amount to three figures; may I join them to the C's then?

Philo. No, only 2 of them; and place the other figure (which will always be 1) under the units place of the C's, as in the following example.

Ex-

Example 4. Reduce 89 C. 3 qrs. 21 lb. neat, into lbs.

That is, 89 C. and 105 lb. Thus, 89

3 qrs. 21 lb. is $\frac{\quad}{\quad}$ 105 lb.
 89 multiplied by 12 is $\frac{\quad}{\quad}$ 1068

Ans. 10073 lb.

Here 3 qrs. 21 lbs. into 105 lbs. and because there are three figures, I place the 1 under the 9, and thus for all such examples.

Now, *Tyro*, I will prove this last example by the other three methods, and leave you to take your choice as practice or fancy direct.

C. qrs. lb.	C. lb.	C. lb.
89 - 3 - 21	<i>Secondly</i> , 89 - 105	<i>Thirdly</i> , 89 - 105*
4	89	112
	89	
359	89	10073 lb.
28	105	
2873	10073 lb.	
720		

10073 lb. common

* This third Method is certainly very short, if once you can multiply well by 112: for observe, I say, 12 times 9 is 108, (and 5 the odd lbs.) is 113; 3, and I carry 11; then 12 times 8 is 96, and 11 is 107, and the 10 lb. is 117, that is, 7, and I carry 11; *lastly*, I say, *once* 89 is 89, and 11 I carried is 100.

Tyro. Sir, I heartily thank you, I see now how it is done.

Philo. Then I will give you one example at large.

TARE and TRET.

Example 5. Bought 3 hogheads of sugar as under.

		C. qrs. lb.	
Gross No 1	—	11 - 2 - 15	
2	—	15 - 1 - 17	
3	—	17 - 3 - 14	
Tare of all 1 C. 1 qr. 2 lbs.	}	Gross 44 - 3 - 18	
Tret 4 lb. for every 104, at		Tare 1 - 1 - 2	
56 shillings a C. neat, what			
come they to ?		Suttle 43 - 2 - 16	
lb.		Suttle 4888 lbs.	
26)4888	188 tret	Tret 188 lbs.	
26			
228		Neat 4700 lbs	
208			
208			
208			

Then, if 1 C. viz. 112 lb. cost 2l. 16s. what cost 4700 lbs. neat? Ans. £117-10s.

Example 6. Bought 3 chests of tea, each weighing gross, 1 C. 3 qrs. 12 lbs. tare of each 1 qr. 5 lb. at 4s. 9d. a lb. neat, what do they come to?

Here being no tret, the tare taken out of the gross the remainder is neat. Ans. £131-6-9.

Tyro. I am highly obliged to you, sir; and am now ready for the next rule if you please.

Philo. The next thing I shall instruct you in is the rule of practice.

DIA-

DIALOGUE IX.

SECTION I.

Of PRACTICE:

Tyro. **W**HAT do you mean by *practice*?

Philo. *Practice* is only *division of money*, and very much resembles *reduction ascending*; for here you divide by as many of *less* as make one of the *greater*. It is a short contracted way of the *rule of three direct*, and saves the trouble of stating questions, by dividing by the parts contained in the whole.

Tyro. Are not *vulgar fractions* often taught before *practice*?

Philo. They are; but there is no necessity for it: if you can read a fraction that is enough. Thus, $\frac{2}{7}$, $\frac{3}{4}$, $\frac{1}{8}$, or $\frac{1}{12}$, are thus read, 2 *sevenths*, 3 *fourths*, 1 *eighth*, and 1 *twelfth*.

Tyro. I know this very well, sir.

Philo. Then you may proceed to work this rule as soon as you please; but it will be necessary to learn the following tables by heart.

K 5

TABLE

TABLE 1.

TABLE 2.

Even parts of a shilling.

Even parts of a £ Sterling.

Pence

6	} is {	$\frac{1}{2}$	of a shill.
4		$\frac{1}{3}$	
3		$\frac{1}{4}$	
2		$\frac{1}{6}$	
$1\frac{1}{2}$ qrs.		$\frac{1}{8}$	
1		$\frac{1}{12}$	
$\frac{3}{4}$		$\frac{1}{16}$	
$\frac{1}{2}$		$\frac{1}{24}$	
$\frac{1}{4}$		$\frac{1}{48}$	

Shillings

10	} is {	$\frac{1}{2}$	
6-8d.		$\frac{1}{3}$	
5		$\frac{1}{4}$	
4		$\frac{1}{5}$	
3-4d.		$\frac{1}{6}$	
2-6d.		$\frac{1}{8}$	
2		$\frac{1}{10}$	
1-8d.		$\frac{1}{12}$	
1-3d.		$\frac{1}{16}$	
1		$\frac{1}{20}$	

Tyro. I will learn these tables as fast as I conveniently can.

Philo. Very well; then I shall proceed to give you examples under the following cases.

CASE 1.

When the sum is given in farthings, that is 1, 2, or 3 farthings *per lb.* or yard, then only bring these farthings into *pence, shillings, and pounds*, and the work is done.

$\frac{3}{4}$	$\frac{1}{4}$	1753 lb. at $\frac{1}{4}$ a lb.
1d	$\frac{1}{12}$	438 $\frac{1}{4}$
1s. 2 0		3 6-6d.
		£1-16-6 $\frac{1}{4}$ Ans.

Here 1753 lbs. at a farthing a lb. is 1753 farthings; therefore, I only divide by 4, by 12, and by 20, and have £1-16-6 $\frac{1}{4}$ for answer.

$\frac{1}{2}ny$ | $\frac{1}{2}$ | 1753 *lb.* at $\frac{1}{2}ny$ a *lb.* Here 1753 is 1753 half-pence. I divide therefore by 2 to bring them into pence, and by 12 and 20, to bring them into £'s, and the answer is £ 3-13-0 $\frac{1}{2}$, which is just double the first sum, at a farthing a *lb.* and is a proof to it.

1d.	$\frac{1}{12}$	876- $\frac{1}{2}$
1s.	2 0	7 3-
		£3-13-0 $\frac{1}{2}$ <i>Ans.</i>

* $\frac{3}{4}$ | $\frac{1}{8}$ | 1753 *lb.* at $\frac{3}{4}$ per *lb.* Here I say 3 farthings is the 1 eighth of 6 pence, and dividing by 8, I have 219 six-pences, and 3 farthings over; then I divide by 2 (because 6d. is $\frac{1}{2}$ a shilling) and have 109 shillings, and 6d. over; then I divide by 20, to bring them into £'s, and have £ 5-9-6 $\frac{3}{4}$.

6d.	$\frac{1}{2}$	219- $\frac{3}{4}$
1s.	2 0	109-6
		£5-9-6 $\frac{3}{4}$ <i>Ans.</i>

Note, If you add the answers of the *first* and *second* sum together, you will find the amount to £ 5-9-6 $\frac{3}{4}$, which proves the work.

* There is another way to do this. Multiply 1753 by 3 farthings, and the product is farthings; then divide by 4, 12, and 20.

Ed.

	1753 lb. at $\frac{3}{4}$ per lb.
	3
4	5259
12	1314 $\frac{3}{4}$
2 0	10 9-6
	£5-9-6 $\frac{3}{4}$ Ans.
d	Yards
1 $\frac{1}{2}$	1753 $\frac{1}{2}$ at 1d per yd.
2 0	14 6-1
	£7-6-1 Ans.

† Note, I make use of the same number, 1753, because you may prove the work of one sum by another, by adding the products of any two together, thus: the product of 3 farthings and 1 farthing will prove the last sum of a penny.

CASE 2.

When the given price is pence and farthings, then take

the parts for the pence first, and then the parts for the farthings.

d.	Ells	d.
1 $\frac{1}{2}$	1753 at $1\frac{1}{4}$ per ell	
$\frac{1}{4}$	$\frac{1}{4}$	146 - 1d.
		36 - 6 $\frac{1}{4}$ †
2 0	18 2 - 7 $\frac{1}{4}$	

£9-2-7 $\frac{1}{4}$ Ans.

† Here I say 1 penny is the $\frac{1}{2}$ of a shilling; then I say, 1 farthing is the $\frac{1}{4}$ of a penny, and dividing 146 shillings by 4, I have 36 shillings, and 2 remains, which is 2 shillings; then I say, the 4th of 2 shillings is 6d. and the 4th of a penny is $\frac{1}{4}$; then I add them both together, which make 18s. 7d. $\frac{1}{4}$, and dividing by 20, I have £9-2-7 $\frac{1}{4}$ for answer

☞ Always remember that what remains is shillings and pence, and you must also divide such shillings

shillings by the same figure, and you have the true answer.

d. $1\frac{1}{2}$ $\frac{1}{8}$ lb. 1753 at $1\frac{1}{2}$ per lb.

2|0 21|9 - $1\frac{1}{2}$

£10-19- $1\frac{1}{2}$ Ans.

* This you see is just double the sum of 3 farthings.

d. $1\frac{1}{2}$ $\frac{1}{8}$ lb. 1753 at $1\frac{3}{4}$

$\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{6}$ 219 - $1\frac{1}{2}$
36 - $6\frac{1}{4}$ *

2|0 25|5 - $7\frac{3}{4}$

£12-15- $7\frac{3}{4}$

* Note, Here I say 3 half-pence is the $\frac{1}{8}$ th (as before) then I say, 1 farthing is the $\frac{1}{6}$ th of 3 halfpence, and dividing by 6, I have 36, and 3 remains, which 3 I call 3 shillings; then I say, the 6th part of 3 shillings is 6 pence, and the 6th

part of $1\frac{1}{2}$ is $\frac{1}{4}$. Thus you see I take the parts of the remainder as well as the parts of the whole.

Pray observe, *Tyro*, and look well at the example; for I shall now leave you to yourself for a while.

Yards

2d $\frac{1}{6}$ 1753 at 2d. per yd.

2|0 29|2 - 2d.

£14-12-2 Ans.

lb.

2d $\frac{1}{6}$ 1753 at $2\frac{1}{4}$

$\frac{1}{4}$ $\frac{1}{8}$ 292 - 2d.
36 - $6\frac{1}{4}$

2|0 32|8 - $8\frac{1}{4}$

£16-8- $8\frac{1}{4}$ Ans.

oz. d.

2d $\frac{1}{6}$ 1753 at $2\frac{1}{2}$

$\frac{1}{2}$ $\frac{1}{4}$ 292 - 2d.
73 - $0\frac{1}{2}$

20 36|5 - $2\frac{1}{2}$

£18-5- $2\frac{1}{2}$

		<i>Ells</i>
2d	$\frac{1}{6}$	1753 at 2d. $\frac{3}{4}$
$\frac{1}{2}$	$\frac{1}{4}$	292 - 2d
$\frac{1}{4}$	$\frac{1}{2}$	73 - $0\frac{1}{2}$
		36 - $6\frac{1}{4}$
2 0		40 1 - $8\frac{3}{4}$
		£20 - 1 - $8\frac{3}{4}$ Ans.
		<i>lb.</i>
3d	$\frac{1}{4}$	1753 at 3d.
2 0		43 8 - 3
		£21 - 18 - 3 Ans.
		<i>lb.</i>
3d	$\frac{1}{4}$	1753 at 3d. $\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{2}$	438 - 3
		36 - $6\frac{1}{4}$
2 0		47 4 - $9\frac{1}{4}$
		£23 - 14 - $9\frac{1}{4}$ Ans.
		<i>lb.</i> <i>d.</i>
3d	$\frac{1}{4}$	1753 at $3\frac{1}{2}$ lb.
$\frac{1}{2}$	$\frac{1}{6}$	438 - 3d.
		73 - $0\frac{1}{2}$
2 0		51 1 - $3\frac{1}{2}$
		£25 - 11 - $3\frac{1}{2}$

		<i>lb.</i> <i>d.</i>
3d	$\frac{1}{4}$	1753 at $3\frac{3}{4}$ lb.
$\frac{3}{4}$	$\frac{1}{4}$	438 - 3d.
		109 - $6\frac{3}{4}$
2 0		547 - $9\frac{3}{4}$
		£24 - 7 - $9\frac{3}{4}$ Ans.
		<i>Dozen</i>
4d	$\frac{1}{3}$	1753 at 4d.
2 0		58 4 - 4
		£29 - 4 - 4 Ans.
		<i>lb.</i> <i>d.</i>
3d	$\frac{1}{4}$	1753 at $4\frac{1}{4}$ lb.
1d	$\frac{1}{3}$	438 - 3
$\frac{1}{4}$	$\frac{1}{4}$	146 - 1
		36 - $6\frac{1}{4}$
2 0		62 0 - $10\frac{1}{4}$
		£31 - 0 - $10\frac{1}{4}$ Ans.
		<i>lb.</i> <i>d.</i>
4d	$\frac{1}{3}$	1753 at $4\frac{1}{2}$ lb.
$\frac{1}{2}$	$\frac{1}{8}$	584 - 4
		73 - $0\frac{1}{2}$
2 0		65 7 - $4\frac{1}{2}$
		£32 - 17 - $4\frac{1}{2}$

4d $\frac{1}{3}$ 1753 lb. at 4d $\frac{3}{4}$ lb.

$\frac{1}{2}$ $\frac{1}{8}$
 $\frac{1}{4}$ $\frac{1}{2}$
 584 - 4d
 73 - 0 $\frac{1}{2}$
 36 - 6 $\frac{1}{4}$

2|0 69|3 - 10 $\frac{3}{4}$

£34-13-10 $\frac{3}{4}$

4d $\frac{1}{3}$ 1753 lb. at 5d. lb.

1d $\frac{1}{4}$
 584 - 4
 146 - 1

2|0 73|0 - 5

£36-10-5 *Ans.*

Ells

4d $\frac{1}{3}$ 1753 at 5d. $\frac{1}{4}$

1d $\frac{1}{4}$ $\frac{1}{4}$
 584 - 4
 146 - 1
 36 - 6 $\frac{1}{4}$

2|0 76|6 - 11 $\frac{1}{4}$

£38-6-11 $\frac{1}{4}$ *Ans.*

4d $\frac{1}{3}$ 1553 lb. at 5d $\frac{1}{2}$ lb.

1d $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{2}$
 584 - 4
 146 - 1
 73 - 0 $\frac{1}{2}$

2|0 80|3 - 5 $\frac{1}{2}$

£40-3-5 $\frac{1}{2}$ *Ans.*

* Or you may take the parts thus, viz. 4d. is $\frac{1}{3}$ of a shilling, as before.

Or thus,

4d $\frac{1}{3}$ 1753 lb. at 5 $\frac{1}{2}$ lb.

$\frac{1}{2}$ $\frac{1}{8}$
 584 - 4d
 219 - 1 $\frac{1}{2}$

2|0 80|3 - 5 $\frac{1}{2}$

£40-3-5 $\frac{1}{2}$ *Ans.*

* Here I say, 1 $\frac{1}{2}$ is $\frac{1}{8}$ of a shilling, and take it out of 1753, the top-line, which answers the same end as the last work
 lb. d.

4d $\frac{1}{3}$ 1753 at 5 $\frac{3}{4}$

$\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{6}$
 584 - 4d
 219 - 1 $\frac{1}{2}$
 36 - 6 $\frac{1}{4}$

2|0 83|9 - 11 $\frac{3}{4}$

£41-19-11 $\frac{3}{4}$ *Ans.*

6d $\frac{1}{2}$ 1753 lb. at 6d. lb.

2|0 87|6 - 6

£43-16-6 *Ans.*

Yards

Yards	
3d $\frac{1}{4}$	1753 at 6d. $\frac{1}{4}$ yard
d.	
3* $\frac{1}{4}$	438 - 3
$\frac{1}{12}$	438 - 3
	36 - 6 $\frac{1}{4}$
2l 0	91 3 - 0 $\frac{1}{4}$

£45-13-0 $\frac{3}{4}$ Ans.

* Here I say 3 pence is the $\frac{3}{4}$ of a shilling in the second line as well as in the first, because then $\frac{3}{4}$ will be the $\frac{1}{12}$; whereas, if I was to say 6d. is the $\frac{1}{2}$ of a shilling, then I must say $\frac{1}{4}$ is the $\frac{1}{24}$ of 6 pence: now 24 is not so easy to divide by as 12 is; so by bringing sixpence into 2 parts, the work is a great deal more easy and natural.

Bushels d.	
6a $\frac{1}{2}$	1753 6 $\frac{1}{2}$ bushel
$\frac{1}{2}$ $\frac{1}{12}$	876 - 6
	73 - 0 $\frac{1}{2}$
2l 0	94 9 - 6 $\frac{1}{2}$
	£47-9-6 $\frac{1}{2}$ Ans.

oz.	
6d $\frac{1}{2}$	1753 at 6 $\frac{3}{4}$ oz.
$\frac{3}{4}$ $\frac{1}{8}$	876 - 6
	109 - 6 $\frac{3}{4}$
2l 0	98 6 - 0 $\frac{3}{4}$

£49-6-0 $\frac{3}{4}$ Ans.

lb. d.	
6d $\frac{1}{2}$	1753 at 7 lb.
1d $\frac{1}{6}$	876 - 6d.
	146 - 1d.
2l 0	102 2 - 7

£51-2-7 Ans.

Gallons	
6d $\frac{1}{2}$	1753 at 7 $\frac{1}{4}$ gallon
1d $\frac{1}{6}$ $\frac{1}{4}$	876 - 6d.
$\frac{1}{4}$ $\frac{1}{4}$	146 - 1
	36 - 6 $\frac{1}{4}$
2l 0	105 9 - 1 $\frac{1}{4}$

£52-19-1 $\frac{1}{4}$ Ans.

6d $\frac{1}{2}$	1753 lb. at 7 $\frac{1}{2}$ lb.
1 $\frac{1}{2}$ $\frac{1}{4}$	876 - 6
	219 - 1 $\frac{1}{2}$
2l 0	109 5 - 7 $\frac{1}{2}$

£54-15-7 $\frac{1}{2}$ Ans.
Ells

$$\begin{array}{r}
 4d \quad \frac{1}{3} \quad 1753 \text{ ells at } 7\frac{3}{4} \\
 \hline
 3d \quad \frac{1}{4} \quad 584 - 4 \\
 \quad \frac{3}{4} \quad 438 - 3 \\
 \quad \quad 109 - 6\frac{3}{4} \\
 \hline
 2|0 \quad 113|2 - 1\frac{3}{4}
 \end{array}$$

$$\text{£}56-12-1\frac{3}{4} \text{ Ans.}$$

Or you may say
 $6d.$ is $\frac{1}{2}$, and $1d.$ is $\frac{1}{6}$,
 and $\frac{3}{4}$ is the $\frac{1}{8}$ of
 6 pence

$$4d \quad \frac{1}{3} \quad 1753 \text{ yards at } 8d.$$

$$\begin{array}{r}
 4d \quad \frac{1}{3} \quad 584 - 4 \\
 \quad \quad 584 - 4
 \end{array}$$

$$2|0 \quad 116|8 - 8$$

$$\text{£}58-8-8 \text{ Ans.}$$

Or you may say
 $6d.$ is $\frac{1}{2}$, and $2d.$ is
 $\frac{1}{3}$ of 6 pence; or as
 $8d.$ is $\frac{2}{3}$ of a shil-
 ling, you may mul-
 tiply by 3, and di-
 vide by 2.

$$6d \quad \frac{1}{2} \quad 1753 \text{ lb. at } 8\frac{1}{4}$$

$$\begin{array}{r}
 2d \quad \frac{1}{3} \quad 876 - 6 \\
 \quad \frac{1}{4} \quad 292 - 2 \\
 \quad \quad 36 - 6\frac{1}{4}
 \end{array}$$

$$2|0 \quad 120|5 - 2\frac{1}{4}$$

$$\text{£}60-5-2\frac{1}{4}$$

$$4d \quad \frac{1}{3} \quad 1753 \text{ pair at } 8\frac{1}{2}$$

$$\begin{array}{r}
 4d \quad \frac{1}{3} \quad 584 - 4 \\
 \quad \frac{1}{2} \quad 584 - 4 \\
 \quad \quad 73 - 0\frac{1}{2}
 \end{array}$$

$$2|0 \quad 124|1 - 8\frac{1}{2} \text{ Ans.}$$

$$\text{£}62-1-8\frac{1}{2} \text{ Ans.}$$

$$4d \quad \frac{1}{3} \quad 1753 \text{ ells at } 8d. \frac{3}{4}$$

$$\begin{array}{r}
 4d \quad \frac{1}{3} \quad 584 - 4 \\
 \quad \frac{1}{2} \quad 584 - 4 \\
 \quad \quad 73 - 0\frac{1}{2} \\
 \quad \quad 36 - 6\frac{1}{4}
 \end{array}$$

$$2|0 \quad 127|8 - 2\frac{3}{4}$$

$$\text{£}63-18-2\frac{3}{4} \text{ Ans.}$$

$$6d \quad \frac{1}{2} \quad 1753 \text{ lb. at } 9d.$$

$$\begin{array}{r}
 3d \quad \frac{1}{2} \quad 876 - 6 \\
 \quad \quad 438 - 3
 \end{array}$$

$$2|0 \quad 131|4 - 9$$

$$\text{£}65-14-9 \text{ Ans.}$$

$$6d \quad \frac{1}{2} \quad 1753 \text{ yards at } 9\frac{1}{4}$$

$$\begin{array}{r}
 3d \quad \frac{1}{2} \quad 876 - 6 \\
 \quad \frac{1}{4} \quad 438 - 3 \\
 \quad \quad 36 - 6\frac{1}{4}
 \end{array}$$

$$2|0 \quad 135|1 - 3\frac{1}{4}$$

$$\text{£}67-11-3\frac{1}{4} \text{ Ans.}$$

Ells.

6d $\frac{1}{2}$ 1753 ells at $9\frac{1}{2}$

3d $\frac{1}{2}$ $\frac{1}{6}$ 876 - 6
438 - 3
73 - $0\frac{1}{2}$

2|0 138|7 - $9\frac{1}{2}$

£69-7 $9\frac{1}{2}$ Ans.

6d $\frac{1}{2}$ 1753 lb. at $9\frac{3}{4}$

3d $\frac{1}{2}$ $\frac{1}{4}$ 876 - 6
438 - 3
109 - $6\frac{1}{4}$

2|0 142|4 - $3\frac{3}{4}$

£71-4- $3\frac{3}{4}$ Ans.

6d $\frac{1}{2}$ 1753 ells at 10d.

4d $\frac{1}{3}$ 876 - 6
584 - 4

2|0 146|0 - 10

£73 0-10 Ans.

6d $\frac{1}{2}$ 1753 oz. at $10\frac{1}{4}$

3d $\frac{1}{2}$ $\frac{1}{4}$ 876 - 6
438 - 3
146 - 1
36 - $6\frac{1}{4}$

2|0 149|7 - $4\frac{1}{4}$

£74-17 $\frac{1}{4}$ Ans.

6d $\frac{1}{2}$ 1753 lb. at $10\frac{1}{2}$

3d $\frac{1}{2}$ $\frac{1}{2}$ 876 - 6
438 - 3
219 - $1\frac{1}{2}$

2|0 153|3 - $10\frac{1}{2}$

£76-13- $10\frac{1}{2}$ Ans.

6d $\frac{1}{2}$ 1753 oz. at $10\frac{3}{4}$

3d $\frac{1}{2}$ $\frac{1}{4}$ 876 - 6
438 - 3
219 - $1\frac{1}{2}$
36 - $6\frac{1}{4}$

2|0 157|0 - $4\frac{3}{4}$

£78 10- $4\frac{3}{4}$ Ans.

6d $\frac{1}{2}$ 1753 oz. at 11d.

4d $\frac{1}{3}$ $\frac{1}{4}$ 876 - 6
584 - 4
146 - 1

2|0 160|6-11

£80-6-11 Ans.

6d $\frac{1}{2}$ 1753 ells at $11\frac{1}{4}$

4d $\frac{1}{3}$ $\frac{1}{4}$ 876 - 6
584 - 4
146 - 1
36 - $6\frac{1}{4}$

2|0 164|3 - $5\frac{1}{4}$

£82-3- $5\frac{1}{4}$ Ans.

lb.

$$6d \quad \frac{1}{2} \quad 1753 \text{ lb. at } 11\frac{1}{2}$$

$$4d \quad \frac{1}{3} \quad 876 - 6$$

$$1\frac{1}{2} \quad \frac{1}{4} \quad 584 - 4$$

$$219 - 1\frac{1}{2}$$

$$2|0 \quad 167|9 - 11\frac{1}{2}$$

$$\text{£}83 - 19 - 11\frac{1}{2} \text{ Ans}$$

lb.

$$6d \quad \frac{1}{2} \quad 1753 \text{ at } 11\frac{3}{4}$$

$$4d \quad \frac{1}{3} \quad 876 - 6$$

$$1\frac{1}{2} \quad \frac{1}{4} \quad 584 - 4$$

$$1\frac{1}{4} \quad \frac{1}{6} \quad 219 - 1\frac{1}{2}$$

$$36 - 6\frac{1}{4}$$

$$2|0 \quad 171|6 - 5\frac{3}{4}$$

$$\text{£}85 - 16 - 5\frac{3}{4} \text{ Ans}$$

lb.

$$1s \quad 2|0 \quad 175|3 \text{ at } 12d.$$

$$\text{£}87 - 13 \text{ Ans}$$

And thus, *Tyro*, I have shewn you from one farthing to a shilling, how to take the parts, so as to do any sum for any number of *lbs. yards, &c.* You may by your own practice take the parts in what manner you please, only remember this *standing rule*, That when you take the parts of a shilling, you must divide the top number by such parts; but when you take the parts of any part, you must divide such a part by the number of parts, and not the top line. Do you understand it?

Tyro. I do, Sir, very well; but pray, suppose the price exceeds one shilling, how must I do then?

Philo. That you will soon learn by the following CASE.

CASE

CASE 3.

When the given Price exceeds one shilling, or 12 pence, but is less than two shillings, then let the top line or given number, stand as it is given, without drawing any line under it; for that is the price or value at 1 shilling; then take the parts, as before, out of the top line, or given number, and add the said parts and top number together, you have the answer in shillings, pence, and farthings, which divide by 20, as before.

EXAMPLES *above twelve pence, and under two shillings.*

$$3d \frac{1}{2} \quad 1753 \text{ lb. at } 15d. \\ 438 - 3d.$$

$$2|0 \quad 219|1 - 3$$

£109-11-3 *Ans.*

Here you see that I let 1753 stand without drawing a line, and that is of itself an answer for 12d. a lb. Then I say, as before, 3d. is $\frac{1}{4}$, and I add these two together, must be the price at 15d. for 12 and 3 is 15.

$$6d \frac{1}{2} \frac{1}{4} \quad 1753 \text{ ells at } 19d. \frac{1}{2} \\ 1\frac{1}{2} \frac{1}{4} \quad 876 - 6d. \\ 219 - 1\frac{1}{2}$$

$$2|0 \quad 284|8-7\frac{1}{2}$$

£142-8-7 $\frac{1}{2}$ *Ans.*

EXAMPLES *of other numbers.*

$$6d \frac{1}{2} \frac{1}{2} \frac{1}{6} \quad 47 \text{ yards, at } 21d. \frac{1}{2} \\ 3d \frac{1}{2} \frac{1}{2} \frac{1}{6} \quad 23 - 6d. \\ \frac{1}{2} \frac{1}{6} \quad 11 - 9 \\ 1 - 11\frac{1}{2}$$

$$2|0 \quad 8|4 - 2\frac{1}{2}$$

£4-4-2 $\frac{1}{2}$ *Ans.*

$$\begin{array}{r|l}
 6d & \frac{1}{2} \\
 3d & \frac{1}{2} \\
 1\frac{1}{2} & \frac{1}{2} \\
 \hline
 & *85 \text{ ells at } 22d. \frac{1}{2} \\
 & 42 - 6 \\
 & 21 - 3 \\
 & 10 - 7\frac{1}{2} \\
 & \hline
 2|0 & 15|9 - 4\frac{1}{2} \\
 & \hline
 & £7 - 19 - 4\frac{1}{2} \text{ Ans.}
 \end{array}$$

* There is a sort of *mechanical* method of doing sums or casting up the total of the price of any commodity, which is often used in business by such as are not acquainted with taking the parts according to the strict order of this rule.

A mechanical or customary Way of casting up any of the foregoing examples for those that cannot divide by the parts.

Let us take the last example, viz. 85 ells at 22d. $\frac{1}{2}$ per ell.

Rule. Count how many shillings, 6 pences, groats, pence, or any other denomination the given number makes, and set the products under one another, as in *Addition*, and add all together, you have the answer.

Thus, the last example 85 ells at 22d. $\frac{1}{2}$ per ell.

$$\begin{array}{l}
 \text{First, 85 shillings} \\
 85 \text{ six-pences} \\
 85 \text{ three-pences} \\
 85 \text{ pence} \\
 85 \text{ half-pence}
 \end{array}
 \left. \vphantom{\begin{array}{l} \text{First, 85 shillings} \\ 85 \text{ six-pences} \\ 85 \text{ three-pences} \\ 85 \text{ pence} \\ 85 \text{ half-pence} \end{array}} \right\} \text{ is } \left\{ \begin{array}{l}
 £ 4 - 5 - \\
 2 - 2 - 6 \\
 1 - 1 - 3 \\
 - 7 - 1 \\
 - 3 - 6\frac{1}{2}
 \end{array} \right.$$

£ 7 - 19 - 4 $\frac{1}{2}$ as before

Tyro. I heartily thank you, Sir, for these observations.

Philo. I do it to serve you, take which way you please; for that is best which is soonest understood, tho' it be a little deviating from the stated rule itself. And now,

now, *Tyro*, we will proceed to the other part of this rule.

CASE 4.

Rule 1. When the given Price is above two shillings, then (by *table 2*) take such shillings as even parts of a £. and divide the given quantity thereby; then take the pence as parts of a shilling, and the farthings as parts of a penny, divide each sum or line by the parts to which they belong. Or,

Rule 2. Multiply the given quantity by the shillings, then take the parts for the pence and farthings, as in the last cases; then add all together, and divide by 20, it is done

	Yards
2s $\frac{1}{10}$	125 at 2s. 8d. $\frac{1}{2}$ yd.
6d $\frac{1}{4}$	12 - 10s.
2d $\frac{1}{3}$	3 - 2 - 6d.
$\frac{1}{2}$	1 - 0 - 10
	- 5 - 2 $\frac{1}{2}$
	£16 - 18 - 6 $\frac{1}{2}$ Ans.

Here 2s. is the $\frac{1}{10}$ of a £. therefore I divide 125 by 10, and it gives £ 12-10s. the price at 2s. per yard. Then 6d. is the $\frac{1}{4}$ th of 2s. I therefore divide £ 12-10s. by 4, and it gives £ 3-2-6. Then 2d. is $\frac{1}{3}$ of 6d. I therefore divide £ 3-2-6 by 3, and it gives £ 1 and 10 pence, the price at 2d. per yard. Lastly, I say 1 half penny is $\frac{1}{4}$ of 2d. and take the $\frac{1}{4}$ of £ 1-10d. is 5s. 2d. $\frac{1}{2}$, the price at a halfpenny a yard. The sum of all these is £ 16 18-6 $\frac{1}{2}$.
Ans.

PROOF

PROOF by the second RULE.

		Yards
d		125 at 2s. 8d. $\frac{1}{2}$
6*	$\frac{1}{2}$	2

		250
		62 - 6d.
2d	$\frac{1}{3}$	20 - 10
$\frac{1}{2}$	$\frac{1}{4}$	5 - 2 $\frac{1}{2}$

		33 8 - 6 $\frac{1}{2}$

		£16-18-6 $\frac{1}{2}$ as before

* Here I multiply 125 by 2, and it gives 250 shillings, at 2s. a yard. Then I say 6d is $\frac{1}{2}$ of a shilling, therefore I divide the top line 125 by 2, (and not 250) and it gives 62s. 6d. and so for any other sum; then I take the other parts, as before directed, as by the work appears.

Philo. Do you understand it?

Tyro. I do very well; but I like the last way the best.

Philo. Take which you please, 'tis only use that makes both easy.

		Ells
5s	$\frac{1}{4}$	417 at 5s. 10d.
d.		-----
10	$\frac{1}{6}$	104 - 5s.
		17 - 7 - 6d.

		£121 - 12 - 6 Ans.

First, 5s. is $\frac{1}{4}$ of a £. I divide 417 by 4, gives £104 - 5s. then 10 pence is $\frac{1}{6}$ of 5s. therefore I divide £104 - 5s. by 6, and find the quotient £17, and 2 over, this 2 I call 2 pounds, and the odd 5s. makes £2 - 5, or 45 shillings; then I say the 6's in 45 is 7 times, and 3 over; this I call 3 shillings, and divide that also by 6, it gives 6 pence.

PROOF.

P R O O F.

$$\begin{array}{r|l}
 4d \quad \frac{1}{3} & 417 \text{ ells at } 5s. 10d. \\
 & 5 \\
 \hline
 4d \quad \frac{1}{3} & 2085 \\
 & 139 \\
 2d \quad \frac{1}{2} & 139 \\
 & 69 - 6d. \\
 \hline
 2|0 & 243 | 2 - 6 \\
 \hline
 & \text{£ } 121 - 12 - 6 \text{ as before}
 \end{array}$$

Here I multiply by 5s and it makes shillings; then I say 4 pence is $\frac{1}{3}$ of a shilling, and take it out of the top line 417: I say again 4 pence is $\frac{1}{3}$, and set down the same; then 2 pence is $\frac{1}{2}$ of 4 pence, and having added all up, and divided by 20, I have the same answer.

And now, *Tyro*, I shall set you a few examples with words at length.

$$\begin{array}{r|l}
 10s \quad \frac{1}{2} & 57 \text{ pieces at } 12s 9d. \quad \frac{1}{2} \\
 s.d. & \\
 2-6 \quad \frac{1}{4} & 28 - 10 - \\
 3d \quad \frac{1}{10} & 7 - 2 - 6 \\
 \frac{1}{2} & - 14 - 3 \\
 & - 2 - 4\frac{1}{2} \\
 \hline
 & \text{£ } 36 - 9 - 1\frac{1}{2} \text{ Ans.}
 \end{array}$$

P R O O F.

$$\begin{array}{r|l}
 6d \quad \frac{1}{2} & 57 \text{ pieces at } 12s 9d \frac{1}{2} \\
 & 12 \\
 \hline
 & 684 \\
 3d \quad \frac{1}{2} & 28 - 6 \\
 \frac{1}{6} & 14 - 3 \\
 & 2 - 4\frac{1}{2} \\
 \hline
 2|0 & 72 | 9 - 1\frac{1}{2} \\
 \hline
 & \text{£ } 36 - 9 - 1\frac{1}{2} \text{ as before}
 \end{array}$$

$$\begin{array}{r|l}
 10s \quad \frac{1}{2} & 185 \text{ load at } 18s. 10d. \\
 & \\
 4s. \quad \frac{1}{5} & 92 - 10 \\
 4s. \quad \frac{1}{5} & 37 - \\
 8d \quad \frac{1}{6} & 37 - \\
 2d \quad \frac{1}{4} & 6 - 3 - 4d. \\
 & 1 - 10 - 10 \\
 \hline
 & \text{£ } 174 - 4 - 2 \text{ Ans.}
 \end{array}$$

P R O O F.

$$\begin{array}{r|l}
 6d \quad \frac{1}{2} & 185 \text{ load at } 18s 10d \\
 & 18 \text{ multiply} \\
 \hline
 4d \quad \frac{1}{3} & 3330 \\
 & 92 - 6 \\
 & 61 - 8 \\
 \hline
 2|0 & 348 | 4 - 2 \\
 \hline
 & \text{£ } 174 - 4 - 2 \text{ as above}
 \end{array}$$

And

And thus, *Tyro*, you may take which method you please; but in the next *case* you are confined to take the parts according to *table 2*.

CASE 5.

When the given price of any thing is more than 20 shillings, or 1 pound, and not so much as 2 pounds, then let the given quantity stand without drawing a line under it, and that is the price, at 1 pound; then take the parts for the shillings and pence, as in the last *case*, and the work is done.

EXAMPLES.

		Ells
10s.	$\frac{1}{2}$	45 at £ 1-12-6
2s 6d	$\frac{1}{4}$	22 - 10
		5 - 12 - 6 <i>Ans.</i>
		£ 73 - 2 - 6 <i>Ans.</i>

Here I let 45 stand without drawing any line under it, which is the price at 1 pound *per ell*; then I take the parts according to the rule in the last *case*, adding all together have £ 73-2-6 *Ans.*

		Tons
10s.	$\frac{1}{2}$	75 at £ 1-17-11 $\frac{1}{2}$
5	$\frac{1}{2}$	37 - 10
2s 6d	$\frac{1}{2}$	18 - 15
5d	$\frac{1}{6}$	9 - 7 - 6
$\frac{1}{2}$	$\frac{1}{10}$	1 - 11 - 3
		- 3 - 1 $\frac{1}{2}$
		£ 142 - 6 - 10 $\frac{1}{2}$ <i>Ans.</i>

Here you see 75 tons at £ 1 *per ton* is £ 75, which I let stand without drawing any line under it, and then I take the parts for 17s. 11d. $\frac{1}{2}$ as you find in the margin.

CASE 6.

When the given price of the commodity is above two pounds, then multiply by the pounds gives the answer, for the number of pounds, and the shillings and pence and farthings must be taken out of the top, or given number, as before directed.

EXAMPLE.

10s	$\frac{1}{2}$	37 loads at £ 3-16-9
		3
		111
5s	$\frac{1}{2}$	18 - 10
1s	$\frac{1}{5}$	9 - 5
6d	$\frac{1}{2}$	1 - 17
3d	$\frac{1}{2}$	18 - 6
		9 - 3
		£141 - 19 - 9 Ans.

Here I multiply 37 by £3, which gives £111, and then I say, 10s. is $\frac{1}{2}$ of a £. and take it out of the top £37, which is £18 - 10s. then I say 5 shillings is $\frac{1}{5}$ of 10, and take the $\frac{1}{5}$ of £18 - 10s. which is £9 - 5s. and so I go on, taking the parts as the work plainly shews.

One EXAMPLE at large.

		Hhds.
10s	$\frac{1}{2}$	15 at £12-18-0 $\frac{3}{4}$
		12
		180
5s	$\frac{1}{2}$	7 - 10
2-6	$\frac{1}{2}$	3 - 15
6d	$\frac{1}{5}$	1 - 17 - 6
$\frac{3}{4}$	$\frac{1}{8}$	7 - 6
		11 $\frac{1}{4}$
		£193 - 10 - 11 $\frac{1}{4}$ Ans.

Here 15 hhds. at £12 per hhd. is £180; then I say 10s. is $\frac{1}{2}$, and take the $\frac{1}{2}$ of £15, the top number, which is £7 - 10s. and so proceed, by taking the rest of the parts, saying, 5 shillings is the $\frac{1}{5}$ of 10 shillings, and take the $\frac{1}{5}$ of £7 - 10s.

Ec.

And

And now, *Tyro*, I think you have had sufficient instructions in this rule, if you carefully observe them. However, that I may omit nothing that may be useful to you in business, there are some shorter methods of casting up things than the rule itself teaches; but this is according to the given price, that is, when the given price is just two shillings, or when it is any even number of shillings, as in the two following *cases*.

C A S E 7.

When the given price is any even number of shillings, as 2, 4, 8, 14, 16, &c. then multiply the given quantity by half the even number of shillings, and cut off the first figure in the units place with a dash of your pen, or pencil: then all the figures to the left-hand are pounds, and the figure you cut off in the units place being doubled, will be shillings, and you have the answer required.

EXAMPLES.

48 yards at 4*s.* per yard.

2
 ———
 £9|6
Ans. £9-12*s.*

Here the price is 4 shillings per yard, therefore I multiply the given quantity 48 by half the price, (*viz.* 2) and it is 96; then I cut off the first figure (*viz.* 6) and double it, which is 12 for the shillings, and the figure 9 is the pounds; so is the answer £ 9-12*s.* Again,

328 ells at 12*s.* per ell.

6
 ———
 £196|8
Ans. £196-16*s.*

Here I multiply by 6, which is $\frac{1}{2}$ the given price, and cut off the first figure, and double it for the shillings, which is 16, and the figures on the left hand of the dash are pounds.

More EXAMPLES.

417 at 14s.

7£291|9 *Ans.* £291-18s.

Again, 695 ells at 18s.

9£625|5 *Ans.* 625-10

CASE 8.

When the given price is any even number of shillings, and you would know what quantity of any thing may be bought for any even number of pounds sterling, then only add a cypher to the given pounds, and divide that sum by half the number of the given price, gives the answer in pounds sterling.

EXAMPLES.

How many *lb.* of tea, at 12s. *per lb.* may I have for £36. *Ans.* 60 *lb.*

£36 add a cypher is 360 $\frac{1}{2}$ of 12s is 6) 60 *lb.* *Ans.*

How many gallons of rum, at 8s. *per* gallon, may I have for £250.

£250 add a cypher is 2500 $\frac{1}{2}$ of 8s. is 4) 625 gallons. *Ans.*

Tyro. I like this very well, sir; and it is not only easier, but much shorter than taking the parts: but suppose there be fractions after the given number, how then?

Philo. Take such *fractional* parts out of the given price and add it to the rest of the work, as by the following *case*.

CASE

CASE 9.

When the given quantity, or number, has *fractional* parts after it, then take such parts out of the given price, and place the sum under the rest, and add all together. Or thus, (which may be easier) multiply the price by the *numerator*, (or top figure) of the fraction, and divide the product by the denominator (or lower-figure) and you have the *fractional* value at once.

$$\begin{array}{r|l}
 10s. & \begin{array}{l} \frac{1}{2} \\ \hline \frac{5}{8} \end{array} \left| \begin{array}{l} 15 \frac{5}{8} \text{ yards at } 10s \\ \hline 7 - 10 \\ 6 - 3 \text{ for } \frac{5}{8} \text{ths} \\ \hline \pounds 7 - 16 - 3 \text{ Ans.} \end{array} \right.
 \end{array}$$

Here I say $\frac{1}{8}$ th of 10s. is 1s. 3d. therefore, $\frac{5}{8}$ th is 5 times as much, *viz.* 6s. 3d. Or thus, I multiply 10s. the price, by 5, the numerator, which

is 50, and divide by 8, the denominator, I have 6s. 3d. the price at $\frac{5}{8}$ ths.

$$\begin{array}{r|l}
 & \begin{array}{l} 3 \frac{7}{8} \text{ yards at } 18s. \\ 9^* \end{array} \left| \begin{array}{l} \hline 2 - 14 \\ 15 - 9 \\ \hline \pounds 3 - 9 - 9 \text{ Ans.} \end{array} \right.
 \end{array}$$

* Here the price being an even number of shillings, I multiply by $\frac{1}{2}$ of it, *viz.* 9, (according to *case* 7) and double the first figure, which is $\pounds 2 - 14s$. Then for $\frac{7}{8}$ I multiply the price, *viz.* 18s. by 7, and divide by 8, and it is 15s. 9d. And thus

may many sums in this *case*, as well as in others, be done in a very short and easy manner, by care and observation.

Tyro. I see it plainly, sir; and now, if you please, I would be glad to have a notion of working *weight*, *viz.* tons, cwt.s. qrs. lbs. &c.

Philo. I am as ready to instruct you.

SECTION II.

Of WEIGHT.

Tyro. **W**HAT is the nature of working this rule?
Philo. By taking the parts, as before directed; as you will plainly see by and by; but first of all the two following tables ought to be learned perfectly.

TABLE 1.

Even parts of a ton.

C. qrs.		
10 - 0	} is {	$\frac{1}{2}$ a ton.
5 - 0		$\frac{1}{4}$
4 - 0		$\frac{1}{5}$
2 - 2		$\frac{1}{8}$
2 - 0		$\frac{1}{10}$
1 - 1		$\frac{1}{16}$
1 - 0		$\frac{1}{20}$

TABLE 2.

Even parts of an Cwt.

qrs. lbs.		
2 - 0	} is {	$\frac{1}{2}$ an Cwt.
1 - 0		$\frac{1}{4}$
16		$\frac{1}{7}$
14		$\frac{1}{8}$
8		$\frac{1}{14}$
7		$\frac{1}{16}$

Having got the above *tables* pretty readily by heart, the rule is,

R U L E.

Multiply the given tons, or hundreds, by the given price, and if there be any parts of shillings and pence, work with them as in the foregoing examples; this being done, take the parts for the odd weight out of the given price, and place it under the rest, and the work is done.

But this is better understood by an example or two.

What

What comes 7 C. 3 qrs. to, at £2 - 6s. per Cwt.

	$\frac{1}{2}$	
5s.	$\frac{1}{4}$	14
1	$\frac{1}{5}$	1 - 15
2 qrs.	$\frac{1}{2}$	- 7
1	$\frac{1}{2}$	1 - 3
		- 11 - 6
		£ 17 - 16 - 6

Here I multiply 7 by 2, and it gives £ 14; then I say 5s. is $\frac{1}{4}$, and take the $\frac{1}{4}$ of £7, the top number, then I say 1s. is $\frac{1}{5}$ of 5s. and take the $\frac{1}{5}$ of £1-15s. This being done, I begin with the odd weight, saying, 2 qrs. is $\frac{1}{2}$ of

an Cwt. and I take the $\frac{1}{2}$ of the price £2-6s. and it is £1-3s. then 1 qr. is $\frac{1}{2}$ of 2 qrs. I take the $\frac{1}{2}$ of £1-3s. which is 11s. 6d.

What cost 14 ton, 6 C. 3 qrs. 14 lb. at £12-10 per ton?

	$\frac{1}{2}$	168	—	The value at £12 per ton.
10s.	$\frac{1}{4}$	7	—	Ditto at 10s. per ton.
5 C.	$\frac{1}{5}$	3 - 2 - 6	—	The $\frac{1}{4}$ th of £12-10s.
1 £.	$\frac{1}{2}$	12 - 6	—	The price of 1 C.
2 qrs.	$\frac{1}{2}$	6 - 3	—	Ditto of 2 qrs.
1 qr.	$\frac{1}{2}$	3 - 1 $\frac{1}{2}$	—	Ditto of 1 qr.
14 lb.	$\frac{1}{2}$	1 - 6 $\frac{3}{4}$	—	Ditto of 14 lb.
		£179 - 5 - 11 $\frac{1}{4}$	Ans.	

I have here, Tyro, set against every line what the value is, and where it is taken out of, that you may be the better grounded in the work: for remember the money is taken out of the given weight, and the odd weight is taken out of the given price, as you were told in the rule.

Tyro. I see the nature of it, sir, plainly.

Philo. Then I shall leave you a few questions for you to work, or prove at leisure.

QUESTIONS to exercise the learner.

1. What cost 125 lb. of indigo, at 14d. $\frac{3}{4}$ per lb?
Ans. £7 - 13 - 7 $\frac{3}{4}$.
2. What is gained in £476, at 4s. 10d. $\frac{1}{2}$ per £.
Ans. £116 and 6d.
3. What is the value of 47 load, at £3 - 15 - 6 $\frac{1}{2}$ per load?
Ans. £177 - 10 - 5 $\frac{1}{2}$.
4. What cost 9 ton, 15 £. 3 qrs. 27 lb. of iron, at £14 per ton?
Ans. 137 - 3 - 10 $\frac{1}{2}$.

Rice Price 56 Reg DIALOGUE X.

SECTION I.

SIMPLE INTEREST.

Tyro. **S**IR, you have given me great satisfaction in the rules of *practice*, and now I should be glad to be informed something concerning *interest*. Pray what is it?

Philo. *Interest* is money that arises from a certain sum lent out at so much *per cent. per annum*; and is a consideration allowed by the borrower to the lender.

Tyro. What do you mean by *per cent. per annum*?

Philo. *Per cent.* signifies 100 pounds, and *per annum* signifies a year, or 12 calendar months.

Tyro. How is the interest found?

Philo. *Interest* may be performed by the *Rule of Three Direct*, (as you may see by *example 1* following) but in some cases it is done easier and shorter by *Practice* and custom, and that is the reason that *Interest* has a place and title in all books of arithmetic by itself.

Tyro.

Tyro. Please to give an example by the *Rule of Three*, and prove it by the short customary way.

Philo. I will.

EXAMPLE.

What is the interest of £ 175 - 10s. for a year, at £5. per cent. per annum?

£.	£.	£.	s.
If 100—	be 5—	what is 175—	10
20		20	
2000		3510	
		5	

2000	) 17550 (8
	16000

1550
20

2000	) 31000	15 shillings
	2000	

11000
10000

Ans. £8 - 15 - 6

1000
12

2000	) 12000 (6
	12000

0

Here you see I am obliged to make my *first* and *second* numbers of one name, so that according as the sum is, it requires a great many figures; but by the other method you spare all this trouble; which pray observe.

Second METHOD. CASE I.

Multiply the principal, or given sum, by the rate *per cent.* and cut off two figures towards the right-hand, (which is the same, observe, as dividing by 100) and the figures towards the left are pounds interest: This done, multiply the figures cut off to the right-hand by 20, and take in the odd shillings, and cut off two figures, as before, and the figures on the left-hand are shillings: then multiply the remainder by 12, and cut off two figures, and the figures on the left-hand are pence. *Lastly*, multiply by 4, and cut off as before, you have the farthings.

Thus the above example is done, as under.

$$\begin{array}{r}
 \text{£.} \quad \text{s.} \\
 175 - 10 \\
 \hline
 5 \\
 \text{£}8)77 - 10 \\
 20 \\
 \hline
 \text{s.} 15)50 \\
 \phantom{\text{s.}}12 \\
 \hline
 \text{d} 6)00 \quad \text{Ans. £8 - 15 - 6 as above}
 \end{array}$$

Tyro. Indeed this is very short to what the first work is.

Philo. Then pray observe the rule; for all the following examples are performed after the same manner; and remember, that cutting off two figures towards the right-hand is the same as dividing by 100.

Quest.

Quest. 2. What is the interest of £515 - 10s. for 1 year, at £4 per cent. per annum?

£515 - 10

4 per cent. multiply

£20)62

20

s. 12)40

12

d. 4)80

4

grs. 3)20

Ans. £20 - 12 - 4 $\frac{3}{4}$ $\frac{20}{100}$, or $\frac{2}{10}$ th of a farthing

Quest. 3. What is the interest of £1050 for 1 year, at £3 per cent. per annum?

£1050

3 per cent.

£31)50

20

s. 10)00 *Ans.* £31 - 10s.

Tyro. You need not give me any more examples of this sort; but suppose the money be lent for more than one year, how then?

CASE

CASE 2.

When the given rate *per cent.* is an even part of £100, then divide the principal by that even part, and the quotient is the answer at once.

Take example 1, viz. £ 175 - 10s. at £ 5 *per cent.*

Here £5 being $\frac{1}{20}$ of £100, I only divide £175-10s. by 20, and it is done.

$$\begin{array}{r} \text{£.} \quad \text{s.} \\ 20 \overline{) 175} - 10^* \\ \hline \text{£ } 8 - 15 - 6 \text{ Ans.} \end{array}$$

* Note, After I divide 175 by 20, I say, the 20th part of 10s. is 6 pence.

Quest. 4. What is the interest of £345 - 17 - 6, at £25 *per cent.*

Here £25 being $\frac{1}{4}$ of £100, I divide the principal £345 - 17 - 6 by £4, and it is done.

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 4 \overline{) 345} - 17 - 6 \\ \hline \text{£ } 86 - 9 - 4\frac{1}{2} \text{ Ans.} \end{array}$$

CASE

CASE 3.

When the principal is put out for years and months, find the interest for the years first, and take the months as even parts of a year, according to the rules of *Practice*.

Quest 5. What is the Interest of £175-10s. for 3 years 9 months, at £5 per cent. per annum?

$$\begin{array}{r} \text{£ } 175 - 10 \\ \hline \text{5 per cent.} \end{array}$$

$$\begin{array}{r} \text{£ } 8)77 - 10 \\ \hline 20 \end{array} \quad \text{interest for 1 year is £8-15-6}$$

$$\begin{array}{r} 3.15)50 \\ \hline 12 \end{array} \quad \begin{array}{r} 26 - 6 - 6 \text{ for 3 yr.} \\ 4 - 7 - 9 \\ 2 - 3 - 10\frac{1}{2} \end{array}$$

$$\begin{array}{r} 4.6)00 \\ \hline \end{array}$$

6 mths. $\frac{1}{2}$ a year
3 mths. $\frac{1}{4}$

$$\text{Ans. £ } 32 - 18 - 1\frac{1}{2}$$

A prac-

A practical QUESTION.

Quest. 6. A Gentleman left his niece by will £558 - 15 shillings, to be paid her when she came to age, with interest, at £4 *per cent.* Now she came to age in 5 years, 9 months, and 3 weeks: what has she got to receive in all, principal and interest?

£558 - 15s.

4 *per cent.*

£22)35

20

15.7)00

Interest for 1 year £22 - 7s.

5 years

6 mths $\frac{1}{2}$ year

3 mths $\frac{1}{4}$

2 weeks $\frac{1}{10}$

1 week $\frac{1}{20}$

111 - 15

11 - 3 - 6d.

5 - 11 - 9

- 18 - 7 $\frac{1}{2}$

- 9 - 3 $\frac{3}{4}$

£ 129 - 18 - 2 $\frac{1}{4}$ interest

558 - 15 legacy

The legacy and interest is £688 - 13 - 2 $\frac{1}{4}$

Tyro. Sir, I thank you for these two useful examples: and now I would only ask this one favour, and that is, how am I to tell the interest of any sum for any time, at $3\frac{1}{2}$, $4\frac{1}{2}$, $4\frac{3}{4}$, or the like *per cent.*

Philo. It is something more difficult than the other, but an example or two, with your due observation, will make it very easy.

CASE

CASE 4.

When the given rate *per cent.* is not even pounds, but is $2\frac{1}{2}$, $3\frac{3}{4}$, $4\frac{1}{2}$, or the like, then multiply the principal or given sum, by the even pounds, and take the $\frac{1}{2}$ or $\frac{1}{4}$, or $\frac{3}{4}$ of the said principal, or given sum, and add to that product; then cut off two figures, and proceed in all respects as before.

Quest. 7. What comes the interest of a bond of £685 10s. at £4 $\frac{1}{2}$ (that is, £4 - 10s.) *per cent. per annum?*

Here I multiply by 4, then for $\frac{1}{2}$ *per cent.* which is 10 shillings. I say 10s. is $\frac{1}{2}$ as in *Practice*, and take $\frac{1}{2}$ of 685, and add to the other, it is £3082 - 10s. then I cut off two figures, thus, 30|82 and proceed as before, I find it £30 - 15 - 6.

685	
4	
2740	$\frac{1}{2}$
342 - 10	
£ 30 82 - 10	
20	
s. 16)50	
12	
£ 6)00	

Quest.

Quest. 9. What is the interest of £175 - 10s. for 8 months, at £5 per cent. per annum?

£ 175 - 10	Interest for 1 year is £8 - 15 - 6						
$\begin{array}{r} \text{---} 5 \\ \text{£8) } 77 - 10 \\ \text{20} \\ \text{---} \end{array}$	Then say 6 mths 2 mths $\left \begin{array}{l} \frac{1}{2} \\ \frac{1}{3} \end{array} \right$ <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 0 10px;">4 - 7 - 9</td> <td>for 6 m</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 10px;">1 - 9 - 3</td> <td>for 2 m</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 10px;"> $\text{---} \\ \text{£5 - 17 -}$ </td> <td><i>Ans.</i></td> </tr> </table>	4 - 7 - 9	for 6 m	1 - 9 - 3	for 2 m	$\text{---} \\ \text{£5 - 17 -}$	<i>Ans.</i>
4 - 7 - 9	for 6 m						
1 - 9 - 3	for 2 m						
$\text{---} \\ \text{£5 - 17 -}$	<i>Ans.</i>						

J. 15) 50

$$\begin{array}{r} 12 \\ \text{---} \end{array}$$

d. 6) 00

Here you see I take the half of 1 year's interest, which is £4-7-9 for 6 months; then I say, 2 months is $\frac{1}{3}$ of 6 months, and take the $\frac{1}{3}$ of £4-7-9, and add both together, is £5-17 for 8 months.

Tyro. I am satisfied, Sir.

Philo. This is the best way for calculating interest in general, and will be near enough for common practice, should there be weeks or days; for it is only taking the parts for such weeks and days, as was said before.

Tyro. But suppose the interest be put out for such a number of days as cannot well be taken in their even parts, how then?

R U L E I.

Philo. First, find the interest for one year as before; then say, if 365 days give so much, what will so many days give? Thus, suppose the interest for one year be £17-10-10, and it was required to find the interest for 73 days, I say,

If 365 days give £17 - 10 - 10, what will 73 days give? *Ans.* £3-0-2.

R U L E 2.

Bring the principal, or money lent out, into pence, and multiply those pence by the number of days the sum is put out at, or continues out; then if the rate *per cent.* it be put out at be £ 5, divide by 7300; and if it be £ 6 *per cent.* divide by 6083, you have the answer in pence. There are many other ways to find the interest for days; but I would recommend the first rule of these two to the learner, as it serves for all rates *per cent.* and is most certain.

S E C T I O N II.

Of Assurance, Brokerage, or Commission.

Tyro. **W**HAT is *Assurance, Brokerage, or Commission?*

Philo. *Assurances, or Insurances,* are of various sorts. An *Assurance* is when any person agrees with another at a certain rate *per cent.* to insure his life for such a time, or his ship upon a voyage, from the dangers of the seas, or his house or goods from fire. *Brokerage* is an allowance of so much *per cent.* given to *Brokers*, or persons employed in buying and selling stocks, or transacting business between buyer and seller: And *commission* is also an allowance of so much *per cent.* for buying or selling any sort of commodity, by the order of any other person, &c.

Tyro. Then I perceive these are all cast up the same as interest of money is.

Philo. They are so, only interest is so much *per cent. per annum*; but *brokerage* is cast up only at so much *per cent.* ready money, without any regard to time. Of these in their order.

I. Of ASSURANCES.

Quest. 1. What comes an assurance of £580 to, at £10 $\frac{3}{4}$ per cent.

		£ 580	Here I multiply by 10, then I
		10	take the parts out of the top, or
s.		—	£580, and add all together, and
10	$\frac{1}{2}$	5800	cut off two figures, (which is di-
5	$\frac{1}{2}$	290	viding by £100) and proceed as in
		145	interest, by multiplying the re-
		—	mainder by 20, &c.
		£62)35	
		20	
		—	
		s. 7)00	

Ans. 62 - 7

Quest. 2. Shipped for Jamaica goods to the value of £1200, upon which I made an assurance, a £7 $\frac{5}{8}$ ths per cent. what does it come to?

		£.	Here I multiply by 7. Then I
		1200	say for the fraction 4 eighths is $\frac{1}{2}$,
		7	that is, $\frac{1}{2}$ of $\frac{8}{8}$ th (for 8 eighths is
		—	equal to 1 whole integer) and take
$\frac{4}{8}$ is	$\frac{1}{2}$	8400	the $\frac{1}{2}$ of the top number; then I say
$\frac{1}{8}$ is	$\frac{1}{4}$	600	$\frac{1}{8}$ is $\frac{1}{4}$ of $\frac{4}{8}$ ths; then I add all to-
		150	gether, and cut off two figures, as
		—	before, &c. and find the answer
		£91)50	£91 - 10s.
		20	
		—	
		s. 10)00	

2. *Of BROKERAGE and COMMISSION.*

Tyro. You need not give me any more examples; for I see the work is the same, though under different titles, or names.

Philo. It is so; but still when the *per cent.* is shillings it may be a little difficult to you, therefore, I will give you an example or two.

Quest. 3. What is the commission of a *Broker* for buying or selling £520 stock, at half a crown *per cent.*

Rule. Multiply the money by the shillings, and take the parts for the pence as in *Practice*; then add all together, and cut off two figures to the right-hand, and those towards the left are your answer in shillings.

$$\begin{array}{r}
 \text{£ } 520 \\
 \quad 2 \\
 \hline
 6d \mid \quad \frac{1}{2} \mid \quad 1040 \\
 \quad \mid \quad \quad \mid \quad 260 \\
 \hline
 \text{s. } 13)00
 \end{array}$$

Quest. 4. What is my commission on £1500 at 7s. *per cent.*

$$\begin{array}{r}
 7 \\
 \hline
 \text{shill. } 105)00
 \end{array}$$

Answer £5 - 5s.

Ans. 13 shillings.

Another way.

Cut off two figures of the given sum, and multiply the remainder by 20, 12, and 4, and gives the answer, at £1 *per cent.* then take the parts of the sum of £1 *per cent.* as in *Practice*, you have your answer.

Quest.

Quest. 5. What is my commission on £1252 - 10s. at 7s. 6d. per cent.

£12)52 - 10s.

20
—
s. 10)50
12
—
d. 6)00

At £1 per cent. it is £12 - 10 - 6

Then, 5s. $\left| \frac{1}{4} \right|$ 3 - 2 - 7 $\frac{1}{2}$
2-6 $\left| \frac{1}{2} \right|$ 1 - 11 - 3 $\frac{3}{4}$

Ans. £4 - 13 - 11 $\frac{1}{4}$

3. Compound INTEREST.

What is *Compound Interest*?

Philo. *Compound Interest* is called interest upon interest; that is, the interest of the principal, and the interest of the interest added together, is the *Compound Interest*. But it being seldom used without it be in purchasing of annuities, &c. and being very tedious to calculate, is done with more ease by *Decimal Fractions*.

Tyro. But I should be glad you would give me a little better notion of it.

Philo. Suppose I put out £100 for 3 years, at £5 per cent. *compound interest*, what does it come to?

First, I find the simple interest for one year is £5; then the second year I am to count what the interest of £105 is, and find it £5 - 5s. This I add to the last principal £105, and it makes £110 - 5s. Then I see what is the interest of £110 - 5s. and find it £5 - 10 - 3 for the third year. Then I add these different interests together, *viz.* £5, £5 - 5s. and £5 - 10 - 3, and it makes £15 - 15 - 3, the *compound interest* for three years on £100, at £5 per cent. And thus for any number of years, at any rate per cent.

SECTION III.

Of REBATE, or DISCOUNT.

Tyro. **W**HAT do you mean by *Rebate* or *Discount*?

Philo. Discount is when a sum of money, which is to be paid at any time to come, is satisfied by paying of it down in present money, which present money being put out after the same rate *per cent.* and for the same time, will be increased to the debt or sum that was first to be paid.

Tyro. Explain this a little fuller in other words.

Philo. Suppose a person owes me 100 guineas, *viz.* £ 105 to be paid in a year, or 12 months to come, how much present money will satisfy the debt? *Ans.* £ 100 present money will satisfy a debt of £ 105 due 12 months hence; because £ 100 put out to interest 12 months will be £ 5.

There are several methods of performing this rule of *Discount*; but as it is best done by *Decimal Fractions*, I shall only give this one general rule.

A general RULE.

1. Add the interest of £ 100 for the given time to £ 100 and make this your *first number*; then place £ 100 only for your *second number*, and the given debt your *third number*, and the answer will be the present money that will satisfy the debt due 12 months to come; then take this present money from the debt, and the remainder is the *rebate* for 12 months.

2. Having thus found the *rebate* for one year, if the time be more or less than 12 months, take the parts by the rule of *Practice* accordingly.

3. Thus

3. Thus it appears that the *first number* is always known according to the rate *per cent.* for one year, as by the following table.

For at £ 3 *per cent.* the *first number* will be 103, &c.

£ 3 <i>per cent.</i>	103	These observe are <i>first numbers</i> according to the <i>rebate per cent.</i> for one year, £100, is your <i>second</i> , and the debt your <i>third</i> , and the answer is the present money to be paid down.
4	104	
5	105	
6	106	
7	107	
8	108	

Quest. 1. What present money will pay a draught or note of hand of £ 368 - 4s. due 3 months hence, allowing rebate at £ 5 *per cent.* *per annum*?

If 105 — 100 — 368 - 4. *Ans.* 350 - 13 - 4, which taken out of the debt £ 368 - 4s. leaves £ 17 - 10 - 8 the discount for one year. See the work.

Rule. Bring the *first* and *third* number into shillings; then multiply the *third* by your *second*, and divide by your *first*, you have £ 350, and 14 remains, which also multiply by 20, and divide by your *first number*, you have 13 shillings, and 7 remains, which multiply by 12, and dividing by the same *first number* you have 4d.

Note of hand	£ 368 - 4 -
Present money	350 - 13 - 4

Discount is	17 - 10 - 8 for 1 year
-------------	------------------------

3 months	$\frac{1}{4}$	4 - 7 - 8 <i>Ans.</i> for 3 months.
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Which taken out of £ 368 - 4s. remains £ 363 - 13 - 4 present money to be paid down.

Here

Here you see for one year the *discount* is £ 17-10-8, and as 3 months is $\frac{1}{4}$ th of a year, I take the $\frac{1}{4}$ th of that sum, which is £ 4-7-8 *Ans.*

Note. Had the note been drawn for 15 months, that is, 1 year and 3 months; then add £ 4-17-8. the *discount* of 3 months to £ 17-10-8, which is £ 21-18-4, the *discount* of £ 368 - 4s. for 15 months.

An EXAMPLE for practice.

What present money will pay a debt of £ 500, due 3 years, 4 months hence, allowing *rebate* at £ 6 per cent. per annum? *Ans.* £ 416 - 13 - 4. *Rebate* £ 83 - 6 - 8. For by the table your *first* number will be £ 106 for one year, your *second* £ 100, and the *third* the debt. And, for proof, find the interest of £ 416 - 13 - 4 for three years and four months, at £ 6 per cent. and add to it £ 416 - 13 - 4, you will have £ 500 the debt itself.

And by these examples you may find the *discount* at any rate per cent. and for any time, by observing the foregoing directions.

Tyro. I see the nature of working this rule very plainly.

Philo. Then I will give you a notion of another useful rule, *viz.*

SECTION IV.

EQUATION of PAYMENTS.

Tyro. **W**HAT do you mean by *Equation of Payments*?

Philo. *Equation of Payments* is when a certain sum of money becomes due at different times, or is to be paid

paid at different payments, and it is agreed between the debtor and creditor to set, or pitch upon a certain time for the payment of the whole, without any damage or loss to either.

Tyro. This is very pretty, as well as useful: pray give me the rule for working questions of this sort.

R U L E.

Multiply the sum of each particular payment by the time it is to be paid in; then add the several products together, and divide the sum by the whole debt, and the quotient is the *equated*, or just time for the payment of the whole.

Quest. 1. A merchant buys goods to the value of £ 140, £ 30 of which is to be paid in 1 month, £ 40 in 3 months, £ 20 in 4 months, and £ 50 in 6 months; but it is agreed upon an *equated* time to pay the whole at once; I demand the *equated*, or just time of payment?

I set down each payment, and the time right against it, and multiply the money by the time, and set the products of each right against them, as follows:

£ 30	multiplied by 1 mth	is £ 30
40	by 3 mths	is 120
20	by 4 mths	is 80
50	by 6 mths	is 300

Debt 140) 530 (3 months

420

110

30 * *

140) 3300 (23 days

280

500

420

80

M

Ans. 3 months, 23 days

* * Note

* * *Note*, I multiply by 30, the days in a calendar month, according to the custom of business, because 12 such months are reckoned to the year.

Quest. 2. A merchant bought goods to the value of £500 upon the following conditions, *viz.* $\frac{1}{4}$ th to be paid in 2 months, and the remainder in 6 months; I demand the equated time to pay the whole at one payment?

First, $\frac{1}{4}$ of £500 is £125 multiplied by 2 mths £250
The remainder is 375 multiplied by 6 mths 2250

Debt	£500	5 00)25 00
------	------	------------

Ans. 5 mths

Now £2500 divided by the debt £500 is 5 months for the answer, and thus for any other question. Do you understand it?

Tyr. There can be nothing easier to apprehend than this.

QUESTIONS to exercise this RULE.

3. A father left his son £600, to be paid by his executor, as follows, $\frac{1}{3}$ in 2 months, $\frac{1}{3}$ in 3 months, and $\frac{1}{3}$ in 8 months; but he being very troublesome to the executor, and he as willing to be rid of him, they agree upon a time for the payment of the whole at once. The *equated* time is demanded; *Ans.* 4 $\frac{1}{3}$ months, *viz.* 4 months 10 days.

4. A owes B £2000 to be paid 3 months hence; but A agrees to pay him £1200 down, provided he will give him a longer time for the payment of the remainder. The question is, how long time B ought to give him to pay the other £800 in? *Ans.* 7 $\frac{1}{2}$ months.

Tyro.

Tyro. What other rules are there in common arithmetic?

Philo. Several others, as *Allegation Medial*, and *Alternate*, *Single* and *Double Position*, *Composition of Medicines*, *Arithmetical* and *Geometrical Progression*, &c. but these being not so much required in business, (or at least may be done by the *Rule of Three Direct*) I shall give a little notion of the two last, because it may be of service to you in case your fancy should turn to any part of the mathematical studies.

DIALOGUE IX.

SECTION I.

Of ARITHMETICAL PROGRESSION.

Tyro. WHAT do you mean by *Arithmetical Progression*?

Philo. *Arithmetical Progression* is by some called *Comparative Arithmetic*, because in a series of numbers one is compared with another, in order to know what ratio or proportion one bears to the other; that is, in what manner they differ, and how much, and that difference is always equal in numbers that are continued.

OBSERVATION.

1. Let us take any numbers that differ alike from one another: thus, 3, 6, 9, 12, 15, &c. differ by 3, called the common excess, difference, or ratio. Also, 1, 9, 17, 25, 33, are numbers in *Arithmetical Progression*, whose ratio, or common difference is 8.

M 2

2. In

2. In any series of figures in *Arithmetical Progression*, when the number of places is odd, *viz* 3, 7, 11, 15, 19, or the like, the double of the middle figure, or place, is equal always to the sums of the first and last figure; thus, 2, 6, (10) 14, 18. The double of the middle term 10 is equal to the sum of the first and last number, *viz.* 20. So 3, 8, 13, (18) 23, 28, 33. The middle term 18 doubled is equal for the sum of 3, and 33, *viz.* 36.

Tyro. How many cases are there in this rule?

Philo. Six; but two of them will be sufficient for your purpose in working questions in general, or to give you a true notion of the nature of the rule itself.

CASE I.

The number of places or terms, and the *ratio*, or common excess, being given to find that last number,

Multiply the number of places less by 1, by the *ratio*, or common excess, and to that product add the first number, and that sum will be equal to the last number.

1. Let the first number be 3, number of places 7, and common excess 5, what is the last number?

Here the number of places is 7, therefore I count them for 6; (which is less 1) and the *ratio*, or common excess is 5, now 5 times 6 is 30; then I add the first number, *viz.* 3, and it is 38, the last number, and so on if the places were ever so many.

2. Let the number of terms be 121, the first number 5, and the common excess, or *ratio* 9; I demand the last number?

Here

Here the number of places less by 1 is 120, which multiplied by 9, the *ratio* gives 1080, to which add the first number 5, gives 1085, the last number. And this you may prove by setting them down thus, 5, 14, 23, 32, 41, till you have 121 places, and your last number will be 1085. But how tedious is this! since you see the last number is so easily found.

A QUESTION for Exercise.

A traveller sets out, and the first day went 6 miles, the second day 9, increasing every day's journey 3 miles, and travelled 61 days; how many miles did he go the last day? *Ans.* 186 miles. See the next Case.

C A S E 2.

The extremes, that is, the first and last number, and the number of terms being given to find the aggregate, or total sum of all the series of numbers,

Add the first and last numbers together, and multiply that sum by half the number of places, and the product is the sum of all the series. Or, in case the number of terms be odd, then add the first and last numbers together, and multiply that sum by the number of the terms, and divide that product by 2, and you have the total of all the places.

Thus, in the last question concerning the travels, the first number is 6, the last number 186 miles, and the places 61. Therefore, by the rule, add the first number 6, and the last number 186 together, and it is 192, which I multiply by 61, and it is 11712, and dividing by 2, I have 5856, the miles he went in all in 61 days.

A QUESTION for Exercise.

Quest. 2. Suppose 100 stones be placed in a strait line, at a yard distant from each other, and a person undertakes to pick up one at a time, and bring it back to the place where he first set out; how far will he have gone when he has picked up the whole?

Here the number of places are 100, and as the stones are but one yard from each other, the common excess is but 1, and therefore the last number is also 100. But here you are to consider, the man is to come back with every stone; therefore, when he fetches the first stone in coming back, he goes 2 yards to put in the first stone, therefore 2 is your first number, and by the same rule 200 will be your last number.

Last number 200

First number 2

202

Half of the number of places 50

He goes in all 10100 yards, which reduced into miles, is $5\frac{1}{2}$ miles, and 420 yards, which wants but 20 yards of $5\frac{3}{4}$ miles.

Quest. 3. Suppose Stones were laid one yard asunder, in a right Line, for one Mile in Length, and to be picked up one by one, coming back with one at a time; how far would the person go that performs it? *Ans.* 1761 miles.

SECTION II.

Of GEOMETRICAL PROGRESSION.

Tyro. I Am mightily pleased with what you have shewn me; but pray what is *Geometrical Progression*, and wherein does it differ from the other?

Philo. *Geometrical Progression*, or *Proportion*, is when numbers differ from one another, by the like *ratio*, or reason, as in *Arithmetical Progression*, only with this difference, that the former is the effect of *Addition*, and here of *Multiplication*, all the numbers having one common multiplier. As 2, 4, 8, 16, differ by double reason, or the *ratio* is 2; for twice 2 is 4, twice 4 is 8, &c. So also, 1, 7, 49, 343, &c. Here the *ratio* is 7, for 7 times 1 is 7, and 7 times 7 is 49, &c. So that you see it is *Multiplication*.

Tyro. I thank you Sir; What else pray?

NOTE 1.

Philo. Any three numbers differ in *Geometrical Progression*; the product of the *extremes*, that is, the product of the *first* and *last* number, is equal to the square of the *mean*, that is, equal to the middle number multiplied by itself.

Let 4, 12, and 36 be the 3 numbers, whose *ratio* is 3, that is, each is 3 times more than the foregoing number; then will the product of the extremes 4 and 36 be equal to the square of 12, the *mean*, viz. equal to 144. So also 2, 16, and 128. Here 128 multiplied by 2 is equal to 16 multiplied by 16 equal to 256.

NOTE 2.

Any 4 numbers that differ in *Geometrical Proportion*, either continued or interrupted (provided the interruption or breaking off be between the *second* and *third* number)

number) the product of the *means* is equal to the product of the *extremes*; that is, the product of the *first* and *fourth* is equal to the product of the *second* and *third* number.

Let the 4 numbers be 3, 12, 48, and 192, whose *ratio* is 4. Here 192 multiplied by 3, the *first* number, is equal to 48 by 12, *viz.* equal to 576.

Again, Suppose the numbers be interrupted, as 5, 30, 448, and 2688. Here the *second* term is 6 times more than the *first* term, and the *fourth* term 6 times more than the *third*: therefore, 2688 multiplied by 5 is equal to 448 multiplied by 30.

Note. This mark ($\div\div$) stands for *Geometrical Progression*.

NOTE 3.

The *ratio* of any numbers in $\div\div$ is found by dividing any consequent by its antecedent, that is dividing any number by the foregoing number. Thus the *ratio* of the last numbers is found by dividing 30 by 5, or 2688 by 448, which gives 6, the *ratio*.

Tyro. I understand this very well; but in a long series of numbers, how am I to find the last number?

Philo. This is very easily done, and as you have not the rule in many, or, at least, have it but for certain *ratio*, or difference of numbers, I shall give you one standing rule, be the first number and the *ratio* what it will.

CASE 1.

To find the last number in any series in $\div\div$ having the *first* number and *ratio* given, set down your *first* number, and multiply it by the *ratio*, and that product again by the *ratio*, or common difference, and thus go on for 5 or 6 terms, at pleasure: then multiply any
of

of those places by itself, and divide the product thereof by the *first* number, and it will be the double of that number, wanting one place. Thus, suppose you multiply the 6th place by itself, and divide by the *first* number, the quotient will be the 11th place; then, if you multiply the 11th place by itself, and divide by the *first* number, the quotient will be the 21st place, which is the double of 11, wanting 1.

☞ N. B. When the first term is unity, or 1, there is no occasion to divide; for having multiplied by the *ratio* a few times, as before directed (suppose as far as the 6th place) this term multiplied by itself will be the product of the double of terms wanting 1, as was said before.

Tyro. I am obliged to you, Sir; but how shall I find the sum of all these series?

Philo. By the next case.

C A S E 2.

The ratio, (or common difference) and the last number being given to find the aggregate, or total sum of all the series,

1. Multiply the last term, or number, by the common difference, or *ratio*, and from that product take the *first* number; then divide the remainder by the common difference less 1, and the quotient will be the sum of all the series. Or,

2. Multiply the *second* and last term together; then multiply the *first* term into itself, and take that product from the product of the *second* and last term then divide the remainder the *second* term, less 1, the *first* term will give the sum of all the series.

Tyro. Please to give me one example.

Philo. I will give you a practical question to exercise both cases.

Quest. A cunning jockey had a fine gelding, to which a Gentleman took a particular fancy, and after many words had passed between them, the Jockey agreed to sell him to the Gentleman for the price his shoes would come to at 1 farthing for the *first* nail, and to double the price every nail. Now the number of nails in the gelding's shoes, were 28, I demand then what he was sold for at this rate?

Now, observe, *Tyro*, here the common excess is 2, (that is, a farthing a nail doubled) Now to find the last number by *Case* 1, double a few places, suppose as far as the 7th, which will be 64; then multiply 64 by itself, *viz.* 64, you have 4096, the 13th place, which doubled, gives the 14th place, *viz.* 8192. This multiplied by itself gives the 27th place, which doubled, gives the 28th, or last place, *viz.* 134217728 farthings; then proceed according to *Case* 2, you will find the sum of all the series to be 268435455 farthings, which is £ 279620 5s. 3d. $\frac{3}{4}$. a sum too large for the ignorant purchaser. And had there been but 4 nails more in his shoes (*viz.* 32) he would have come to £ 4473924 5s.*

* It is no easy task to make some persons believe the truth of the increase of figures in Geometrical Progression in these questions; but suppose a servant was to agree with his master to serve him thirty years, for 1 single wheat corn the first year, for the second year ten, for the third 20, and so on. The produce of the wheat would be more than all the ships in England could carry away at once; and the money for his wages would be more than all the land could pay, if sold at 90 years purchase for ever.

SECTION III.

Of PERMUTATION, or variety of changes.

Tyro. **I** Imagine this is much after the same manner as *Progression*; is it not?

Philo. This shews the increase of numbers as well as the other; but does not teach you to find the sum of all, only their different variations or changes, as follows:

Multiply every number together, that is, the *first* by the *second*, and that product by the *third* figure, and so on, till you have gone through all the given numbers; so is the last product the variety of changes.

EXAMPLE.

I demand the changes that may be rung on 12 bells, or the different position 12 persons may sit at table.
Ans. 479001600.

 *Note*, This sign ($\times$) signifies, that all numbers between which it stands are to be multiplied continually one into the other, thus: take the 12 bells, and multiply the *first* by the *second*, you have two changes upon two bells; multiply this by the *third* bell, you have 6 on three bells; multiply this product by 4, or the *fourth* bell, you have 24 changes, &c. as follow.

Thus, $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12$ will produce 479001600 changes, or different positions which you may prove very easily at leisure by *Multiplication* only*.

* Notwithstanding the changes on 8 bells may be rung out in a few hours; yet the changes on 12 (allowing 10 changes in a minute) will take 91 years, 26 days; and add to the incredibility of this increase, two bells more, viz. 14, would take up 16575 years to ring the changes through. See Mr. Ward, p. 85.

Tyro.

Tyro. I thank you for giving me a short hint of these things, which I perceive the nature of very plainly : And now, pray what will you instruct me in next ?

Philo. There are two or three rules, such as the *Rule of False Allegation*, *Alternate*, *Composition of Medicines*, which are not material in business, and therefore for want of room must be omitted, and the more necessary part, *viz. Vulgar Fractions*, be treated of in their stead. And here I beg you would take care and make yourself master of *Vulgar Fractions*, they being very necessary in almost every branch of life, and the very foundation of *Decimals*.

Rice Price 5d Reg

D I A L O G U E XII.

S E C T I O N I.

Of Notation of VULGAR FRACTIONS.

Tyro. **W**HAT do you mean by *Notation* ?

Philo. *Notation*, like *Numeration*, shews you how to note, write down, or express any *Fraction*.

Tyro. What is a *Vulgar Fraction* ?

Philo. It is a broken number, or a part or parts of an integer, or whole number, and consists of two parts, *viz. the Numerator* which always stands a-top, and the *Denominator* which always stands under it, or below. Thus, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{11}{12}$. Here 4, 3, and 11 are the *Numerators*, and 5, 4, and 12 the *Denominators*.

Tyro. How many sorts of *Vulgar Fractions* are there ?

Philo. Three, *viz. First, Simple, Single, or Proper Fractions* (for these are all one.) *Secondly, Improper*; and *thirdly, Compound Fractions*.

Tyro. What is a *Simple Fraction*, or how is it known ?

Philo.

Philo. Simple Fractions, have their Numerators less than their respective Denominators. Thus $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{14}{87}$, are all Simple Fractions.

Tyro. What is an Improper Fraction?

Philo. Improper Fractions have their Numerators larger than their Denominators. Thus, $\frac{4}{3}$, $\frac{9}{5}$, $\frac{87}{14}$, $1\frac{2}{7}$, &c. are Improper Fractions.

Tyro. What are Compound Fractions?

Philo. Compound Fractions are Fractions of Fractions, compounded, or joined together by the word of. Thus, $\frac{2}{3}$ of $\frac{3}{5}$, or $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{11}{12}$ are Compound Fractions, and are thus read, 2 thirds of 3 fifths, or 3 fourths of 5 sixths of 11 twelfths of an integer or whole number.

Tyro. I understand you, Sir; but this is very difficult to tell surely.

Philo. You have nothing to do with this at present; you will know it by and by.

Tyro. What do you mean by a mixt number?

Philo. A mixt number, is a whole number with a Fraction after it. Thus, $14\frac{2}{5}$, and $147\frac{5}{8}$ are mixt numbers.

Tyro. I understand you; but how am I to find the value of different Fractions.

Philo. By Reduction, which is always learned first, because you can't add, subtract, multiply or divide, till the fractions are first reduced to their proper order: but before you proceed any farther, it will be proper to learn the following signs, or characters, by heart, so as to know what they signify, which will be a great help to you.

Of the Signs used in Vulgar Fractions.

This (=) is the sign of Equality, and signifies, that the numbers before it are equal to those after it. Thus, 5 and $2=7$ is thus read, 5 and 2 is equal to 7.

This mark (+) is the sign of addition, and signifies, all the numbers are to be added together, Thus, $2+6+7=15$: That is, 2 more, 6 more, 7 more, or 2 added to 6 and 7 is equal to 15.

This

This (—) is the sign of *subtraction*, and signifies the number after it is taken, or is to be taken out of the number before it. Thus, $15 - 7 = 8$, that 7 taken, or subtracted from 15, is equal to 8.

This ($\times$) is the sign of continual multiplication.

This ($\div$) is the sign of *division*, and signifies the number before it is to be divided by the number that follows it. Thus, $56 \div 8 = 7$, that is, 56 divided by 8 is equal to 7.

SECTION II.

Reduction of VULGAR FRACTIONS.

CASE I.

To reduce a mixt number to an improper fraction.

MULTIPLY the whole number by the denominator of the fraction, and take in the numerator besides; then place the denominator under the product, and it is done.

Reduce $57 \frac{3}{5}$ to an improper fraction?

$$\begin{array}{r} 57 \frac{3}{5} \\ \underline{5} \\ 288 \\ \text{---} \text{Ans. } 288 \frac{3}{5} \end{array}$$

Reduce $1753 \frac{1}{12}$ to an improper fraction. *Ans.* $21047 \frac{1}{12}$.

CASE

C A S E 2.

To reduce an improper fraction to a whole or mixt number.

Divide the *numerator* by the *denominator*, and if any thing remains, place it for a new *numerator* over the *denominator*.

Reduce $28\frac{8}{5}$ to a mixt number?

$$5 \overline{) 288}$$

57 $\frac{3}{5}$ Ans.

Reduce $2104\frac{7}{12}$ to a mixt number? Ans. $1753\frac{11}{12}$.

☞ This is only a proof to the foregoing case, and is the same question backwards.

C A S E 3.

To reduce or make a whole number into an improper fraction.

Multiply the whole number by any figure at pleasure, and place the product over the figure you multiplied by, and you have an *improper fraction* equal to the given *whole number*.

Reduce 14 to an *improper fraction*, whose denominator must be 9? Ans. $12\frac{6}{9}$.

Here because it says 9 must be the denominator, I multiply 14 by 9, and it is 126, which I place over the 9, and it is done. Ans. $12\frac{6}{9}$.

NOTE

NOTE 1.

To express any *whole number* fraction-wise, it is only putting unity, or 1 under it. Thus, 14, 26, 490, &c. will be $\frac{14}{1}$, $\frac{26}{1}$, $\frac{490}{1}$, &c. Remember this, for it is very useful.

NOTE 2.

Every *improper fraction* is more than an unit or 1, and every *simple fraction* is less than unity or 1. Thus, the *simple fraction* $\frac{3}{4}$ of a £. sterling, is but 15 shillings: but $\frac{4}{3}$ of a £. sterling is £ 1, and $\frac{1}{3}$ over; for if you divide 4 by 3, it is 1, and $\frac{1}{3}$, that is, £ 1-6-8.

NOTE 3.

When the numerator and denominator are alike, the fraction is equal to a whole number. Thus, $\frac{4}{4}$ is 1, or $\frac{25}{25}$ is equal to 1; because the numerator divided by the denominator produces 1.

CASE 4.

To reduce a compound fraction to a simple one, of the same value.

Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

N. B. N. N. signifies *new numerator*, and N. D. *new denominator*, and C. D. *common denominator*, which pray remember.

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{8}$ to a simple fraction? *Ans.* $\frac{5}{160}$.

For $2 \times 3 \times 5 = 30$ N. N. and $5 \times 4 \times 8 = 160$ N. D.

That

That is, 2 multiplied by 3, and that product by 5, is equal to 30 N. N; and 5 multiplied by 4, and that product by 8, is equal to 160 N. D. *Ans.* $\frac{30}{160}$.

* * *Note*, When there are cyphers in the numerator and denominator, then cut them off, and the *fraction* is still the same. Thus, in the foregoing answer the *fraction* is $\frac{30}{160}$ viz. $\frac{3}{16}$. So $\frac{4000}{7000}$ is only $\frac{4}{7}$.

2. Reduce $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{11}{12}$ to a *simple fraction*.
Ans. $\frac{11}{288}$.

C A S E 5.

To reduce a fraction to its lowest term, that shall be equal to the original given fraction.

Divide the numerator and denominator by any figure that will divide them without any remainder; so shall the last quotients be a new numerator, and a new denominator equal to the given fraction.

1. Reduce $\frac{72}{120}$ into its lowest terms? *Ans.* $\frac{3}{5}$.

Divisors, $\frac{2}{120} | \frac{6}{36} | \frac{2}{18} | \frac{3}{9}$ *Ans.* $\frac{3}{5} = \frac{72}{120}$. So also $\frac{168}{448}$ is $= \frac{3}{5}$, which you may prove by dividing the numerator and denominator by any figure that will divide them both, as was said before.

Note, There is another way to reduce a fraction to its lowest terms at once, and that is by finding a common measurer.

To find a common measure.

Divide the denominator by the numerator, and if any thing remains, divide the last divisor by such remainder, and if any thing again remains, divide the last

last divisor thereby : thus go on till nothing remains, and your last divisor is the *common measure*, or number, that will divide the numerator and denominator, at once into their lowest terms.

1. Reduce $\frac{168}{448}$ into its lowest terms. *Ans.* $\frac{3}{8}$.

$$\begin{array}{r} 168)448(2 \\ \underline{336} \\ 112)168(1 \\ \underline{112} \\ 0 \end{array}$$

Common Measure $\begin{array}{r} 56)112(2 \\ \underline{112} \\ 0 \end{array}$

Here I divide the *denominator* 448 by the *numerator* 168, and 112 remains; then I divide 168, the last divisor, by that 112 and 56 remains; *lastly*, I divide 112 by 56, and 0 remains: so is 56 the *common measure*, that will divide both at one work.

$$\begin{array}{r} 56)168(3 \text{ N. numerator} \\ \underline{168} \\ 0 \end{array} \quad \begin{array}{r} 56)448(8 \text{ N. denominator} \\ \underline{448} \\ 0 \end{array} \quad \text{Ans. } \frac{3}{8} = \frac{168}{448}$$

2. Reduce $\frac{5\frac{1}{2}}{2\frac{1}{8}}$ to its lowest terms. *Ans.* $\frac{2}{5}$.

C A S E 6.

To reduce fractions of different denominators to fractions of the same value, having one common denominator.

First, Multiply all the denominators together for a common denominator; then begin with the first numerator, and multiply it into all the denominators, except its own denominator: do thus with all the other numerators, multiplying them into all the denominators except their own; so will these products be new numerators, which must be placed over the common denominator.

numerator, and each *fraction* will be equal to *that* fraction whose numerator you multiplied into the denominators.

1. Reduce $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{5}{8}$ to fractions, having one common denominator.

First, I multiply all the denominators together, and it makes 120 for a C denominator. That is, $3 \times 5 \times 8 = 120$ C. D. Then I begin with the numerator 2, that is, $2 \times 5 \times 8 = 80$ NN. Then the numerator 3 $\times 3 \times 8 = 72$ N. And *lastly*, $5 \times 5 \times 3 = 75$ NN. These new numerators I place over the C denominator, and the work is done. Thus, $\frac{80}{120} = \frac{2}{3}$ for 80 is $\frac{2}{3}$ of 120. Also, $\frac{72}{120} = \frac{3}{5}$ and $\frac{75}{120} = \frac{5}{8}$.

2. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{4}{5}$ to fractions having one C. denominator. *Ans.* $\frac{720}{960} = \frac{3}{4}$, $\frac{800}{960} = \frac{5}{6}$, $\frac{840}{960} = \frac{7}{8}$ and $\frac{768}{960} = \frac{4}{5}$.

CASE 7.

To reduce fractions of one name or denomination to another.

I. ASCENDING.

When a *fraction* is to be reduced from a less to a greater denomination, then make the given *fraction* into a compound *fraction*, by considering how many of the *less* make one of the *greater* denomination; then reduce this *compound fraction* to a simple one, and it is done.

1. Reduce $\frac{1}{4}$ of a penny to the fraction of £. sterling. *Ans.* $\frac{1}{960}\text{£}$.

Here, because 12 pence make 1 shilling, and 20 shillings a £. I say, $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$, which reduced to a *simple fraction* by multiplying the numerators together

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ther for a N.N. and all the denominators for a N. D.
gives $\frac{1}{960}$ of a $\text{£} = \frac{1}{4}$ of a penny.

2. Reduce $\frac{3}{4}$ of a farthing to the fraction of a guinea? *Ans.* $\frac{3}{4032}$, viz. $\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{21} = \frac{3}{4032}$

3. Reduce $\frac{1}{4}$ of a lb. to the fraction of a ton. *Ans.* $\frac{1}{8960}$.

DESCENDING.

When the *fraction* is to be brought from a *greater* to a *less* denomination, then make of it a *compound fraction*, as before; descending from the *great* to *lesser* denomination: then multiply every denominator by the numerator of the given *fraction*, (except its own denominator) and place the product over the denominator to the said given fraction, you have the answer.

N. B. This case is a proof to the last.

1. Reduce $\frac{1}{960}$ of a £ . sterling to the fraction of a penny? *Ans.* $\frac{240}{960}$, or $\frac{1}{4}$. For $\frac{1}{960}$ of $\frac{1}{20}$ of $\frac{1}{12}$. Now the numerator 1 multiplied into the denominator 20, and 12 is = 240, which I place over the denominator 960, thus, $\frac{240}{960}$ *Ans.*

2. Reduce $\frac{3}{4032}$ of a guinea to the fraction of a farthing? *Ans.* $\frac{3024}{4032}$. For it is $\frac{3}{4032}$ of $\frac{1}{21}$ of $\frac{1}{12}$ of $\frac{1}{4}$. Now $3 \times 21 \times 12 \times 4 = 3024$ N.N.

CASE

C A S E 8.

To find the value of a fraction in money, weight, or measure.

Multiply the numerator of the fraction by the next nearest part belonging to the integer itself, and divide by the denominator; then multiply the remainder by the next nearest part of the integer, and divide by the same divisor, or denominator, and thus go on till you have gone through all the parts of the integer, and the quotients will be your answer.

1. What is the value of $\frac{26}{30}$ of a £. sterling? *Ans.* 17s. 4d.

26 numerator
20

3|0)52|0

17s. 10d.

12

3|0)12|0

4d.

Here I multiply the numerator 26 by 20, the shillings in a £. and divide by the denominator 30, gives £ 17, and 10 over, which I multiply by 12, and divide again by 30, gives 4 pence; therefore I find $\frac{26}{30}$ of a £. to be 17s. 4d.

2. What is the $\frac{15}{206}$ of a moidore? *Ans.* 1s. 10d. $\frac{1}{2}$, as by the following work.

15 numerator
27 multiply

$$\begin{array}{r} 216 \overline{) 405} (1s. \\ 216 \end{array}$$

$$\begin{array}{r} 189 \\ 12 \end{array}$$

$$\begin{array}{r} 216 \overline{) 2268} (10d. \\ 216 \end{array}$$

$$\begin{array}{r} 108 \\ 4 \end{array}$$

$$\begin{array}{r} 216 \overline{) 432} (2qr. \\ 432 \end{array}$$

0

Here I multiply the numerator 15 by 27, the shillings in a moidore, and divide by the denominator, gives 1 shilling, and 189 over, which I multiply by 12, and divide by 216, as before, and so I go on, as appears by the work, and find $\frac{15}{27}$ of a moidore to be 1s. 10d. $\frac{1}{2}$.

Ans. 1s. 10d. $\frac{1}{2}$.

Tyro. I see the nature of it plainly; and like this case very well; for it is diverting as well as improving.
Philo. You are to proceed the same in weight and measure; therefore I shall only set you a few questions for practice to try at leisure.

3. What is the $\frac{324}{972}$ of a ton? *Ans.* 6 C. 2 qrs.
18 lbs. 10 oz. 10 dr. $\frac{648}{972}$.
4. What is the $\frac{14}{112}$ of a barrel? *Ans.* $4\frac{1}{2}$ gallons.
5. What is the $\frac{150}{225}$ of a mile? *Ans.* 1173 yards,
1 foot.
6. What is the $\frac{14}{12}$ of an acre? *Ans.* 3 roods, 20 rods.
7. What is the $\frac{5}{12}$ of a year (allowing 365 days)
Ans. 152 days, 2 hours.

SECTION III.

ADDITION of VULGAR FRACTIONS.

Tyro. **H**OW are *Vulgar Fractions* added together?
Pbilo. Very easily, by this one rule, *viz.*
 All *compound fractions* must be reduced to *simple ones*,
 and all fractions of different denominations, to a *common denominator*; then add all the numerators together, and place their sum over the *common denominator*, is the answer; and if it be an improper fraction at last, then reduce it to a *mixt* number, and the work is done.

1. Add $\frac{1}{11}$ $\frac{2}{11}$ $\frac{3}{11}$ and $\frac{4}{11}$ together. *Ans.* $\frac{10}{11}$.

Here being one *common denominator* to all the fractions, I only add the numerators together and the work is done. *Ans.* $\frac{10}{11}$.

2. Add $\frac{25}{27}$ $\frac{14}{27}$ and $\frac{19}{27}$ together. *Ans.* $\frac{58}{27} = 2 \frac{4}{27}$

Here I find the sum of all the numerators to be 58, which I place over the denominator 27; but being an *improper fraction*, I divide 58 by 27, and have for *Answer* 2 and $\frac{4}{27}$.

3. Add $\frac{2}{3}$ $\frac{3}{5}$ and $\frac{5}{8}$ together. *Ans.* $\frac{227}{120}$.

Here the *fractions* having different denominators, I reduce them (by *case 5*) to a common denominator, and find them to be $\frac{80}{120}$, $\frac{72}{120}$, and $\frac{75}{120}$. Then I add these numerators together, and find them 227: so the answer is $\frac{227}{120} = 1 \frac{127}{120}$.

4. Add

4. Add $\frac{2}{3}$ of $\frac{3}{5}$ and $\frac{1}{2}$ of $\frac{3}{8}$ together. *Ans.* $\frac{14}{40}$.

First, $\frac{2}{3}$ of $\frac{3}{5}$ is $= \frac{6}{15}$, and $\frac{1}{2}$ of $\frac{3}{8} = \frac{3}{16}$. Now $\frac{6}{15}$ and $\frac{3}{16}$ reduced to a *C D* is $\frac{48}{40}$ and $\frac{9}{40}$, which added together is $\frac{57}{40}$. *Ans.*

5. Add. £ 14 $\frac{3}{5}$ £ 19 $\frac{2}{3}$ £ 47 $\frac{5}{8}$ and £ 100 together. *Ans.* £ 181 $\frac{107}{240}$, or 17s. 10d.

6. What must I add to £ 54 $\frac{2}{3}$, and 19 $\frac{3}{5}$ to make it £ 100? *Ans.* 25 $\frac{11}{15}$.

SECTION IV.

SUBTRACTION of VULGAR FRACTIONS.

Tyro. **H**OW is *Subtraction of Fractions* performed? *Philo.* By the same Rule as in *Addition*, first reducing all *compound* to *simple*, and all to a *common denominator*; thus, *subtract* the numerator or the *less* fraction from the numerator of the other, and place the difference over the denominator, is the answer.

From $\frac{14}{20}$
Take $\frac{1}{9}$

Difference $\frac{23}{180}$ *Ans.*
Proof $\frac{14}{20}$

From £ 67 $\frac{2}{7}$
Take 18 $\frac{3}{4}$

Ans. 49 $\frac{6}{7}$
Proof 67 $\frac{2}{7}$

Here in both these examples I take the numerator of the *less* fraction from the numerator of the top, or *greater* fraction, and place the difference over the denominator. Then I prove the work, as in common *Subtraction* by adding the numerator of the difference to the numerator of the *less* fraction, and it gives the numerator

tor of the top fraction. And thus for all examples of this sort.

Tyro. I understand you very well, sir, but suppose the numerator of the fraction to be subtracted be larger than the numerator of the fraction I am to take it out of, how am I to do then?

Philo. This is easy enough: for when you cannot take the lower numerator out of the top numerator; then take it out of the common denominator, and to that difference add the top numerator, and place it over the denominator for the true difference; only pray remember this is called *borrowing*, as in common subtraction, and you must carry one to the next figure for so doing.

2. Lent	$134\frac{14}{24}$	is = £ 134-11-8	<i>The PROOF by</i>
Received	$93\frac{18}{24}$	is = 95-15	<i>common Sub-</i>
			<i>traction.</i>
Due	$38\frac{20}{24}$	<i>Ans.</i> is = £ 38-16-8	
Proof	$134\frac{14}{24}$	Proof £ 134-11-8	

3. From $\frac{2}{3}$ of $\frac{3}{5}$ take $\frac{1}{2}$ of $\frac{3}{8}$. That is, from $\frac{6}{15}$ take $\frac{3}{16}$. Now $\frac{6}{15}$ is $\frac{96}{240}$, and $\frac{3}{16}$ is $\frac{45}{240}$. Then I take $\frac{45}{240}$ from $\frac{96}{240}$ and there remains $\frac{51}{240}$. *Ans.*

4. From 1 take $\frac{4}{11}$. *Ans.* $\frac{7}{11}$. For I make an *improper* fraction equal to 1, which has a denominator equal to the fraction to be subtracted, *viz.* $\frac{11}{11}$ is = 1. Then $\frac{7}{11}$ from $\frac{11}{11}$ remains $\frac{4}{11}$. And thus for any other example.

SECTION V.

MULTIPLICATION of VULGAR FRACTIONS.

Tyro. **H**OW is Multiplication performed?

Philo. The fractions must be reduced to simple ones, as before directed, and mixt numbers to improper fractions. Then the rule is, multiply the numerators together for a new numerator, and the denominators together for a new denominator, and the work is done.

Multiply $\frac{4}{7}$ by $\frac{3}{6}$. *Ans.* $\frac{12}{42}$. For $4 \times 3 = 12$ N. N.
 $7 \times 6 = 42$ N. D.

2. Multiply $2\frac{2}{6}$ by $\frac{4}{5}$. *Ans.* $9\frac{8}{54} = 18\frac{1}{54}$.

3. Multiply $4d. \frac{1}{2}$ by $4d. \frac{1}{2}$. *Ans.* $20d. \frac{1}{4}$.

First, I reduce $4\frac{1}{2}$ to an improper fraction, which is $\frac{9}{2}$; therefore I multiply $\frac{9}{2}$ by $\frac{2}{2}$. *Ans.* $8\frac{1}{4} = 20\frac{1}{4}$, as before.

4. Multiply $\pounds 4\frac{3}{8}$ by $\pounds 2$. Here $\pounds 4\frac{3}{8} = \frac{35}{8}$, and $\pounds 2$ is expressed in fractions $\frac{2}{1}$; therefore multiply $\frac{35}{8}$ by $\frac{2}{1}$. *Ans.* $\frac{70}{8} = \pounds 8\frac{6}{8}$, viz. $\pounds 8-15$.

5. Multiply $\frac{2}{3}$ of $\frac{5}{8}$ by $\frac{3}{7}$ of $\frac{1}{5}$, that is, $\frac{10}{24}$ by $\frac{3}{35}$, *Ans.* $\frac{30}{840} = \frac{1}{28}$. And thus may any sum be multiplied by another, with more exactness than by any other rule, though not so easy as decimal fractions.

SECTION VI.

DIVISION OF VULGAR FRACTIONS.

Tyro. **H**OW is division of fractions performed?
Philo. First reduce all compound to simple fractions; and all mixt numbers to improper fractions, as before directed: then the rule is, multiply the numerator of the dividend into the denominator of the divisor, for a new numerator, and the numerator of the divisor (or fraction you divide by) into the denominator of the dividend for a new denominator, and you have the answer.

1. Divide $\frac{8}{19}$ by $\frac{5}{12}$. *Ans.* $\frac{96}{95} = \frac{1}{95}$. Here I multiply 8 into 12, which is 96, for a new numerator, and 5 into 19, which is 95, for a new denominator, which is $\frac{96}{95}$. Here you see the answer is an improper fraction; but if you change the fractions, that is, if $\frac{5}{12}$ were to be divided by $\frac{8}{19}$, then the answer would be $\frac{95}{96}$, which is a simple fraction.

Note. The character ($\div$) signifies division, or that the number is divided by what follows it.

2. Divide £ 47 $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{2}{5}$ of a £. *Ans.* $42\frac{60}{36}$. For $\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6}$, and $\frac{2}{3}$ of $\frac{2}{5} = \frac{4}{15}$; therefore, divide £ 47 $\frac{2}{6}$ by $\frac{4}{15}$, that is, divide $284\frac{4}{6}$ by $\frac{4}{15}$. First, $284 \div 4 = 71$ N. N. and $6 \times 6 = 36$ N. D. *Ans.* $42\frac{60}{36}$, or £ 118 $\frac{12}{36} = 6s. 8d.$

Tyro. Then I perceive that division of fractions makes more of a sum after divided than before.

Philo. Yes certainly, when the divisor is a simple fraction, or less than unity, as in this case; and it answers the same end as common multiplication, viz. increases the value.

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Tyro. Explain this a little more pray.

Philo. Observe then the following example.

3. It is required to bring 42 guineas into farthings by division only?

Here by case 4, I reduce a farthing to the fraction of a guinea, and find it $\frac{1}{16008}$ for a divisor. Then I make 42 guineas a dividend, thus, $\frac{42}{1}$. Now, $\frac{42}{1} \div \frac{1}{16008} = 42336$ farthings.

From hence, *Tyro*, arises this observation: that where any whole number is divided by a simple fraction, the quotient will be so much larger than the dividend, as the divisor is less than the unity, or 1; but on the contrary, when a simple fraction is to be divided by a whole number, then the quotient will be so many times less than the dividend, as the divisor exceeds unity.

Thus, £ 4 divided by $\frac{1}{4}$ of a £. is the same to multiply it by £ 4, viz. £ 16: but $\frac{1}{4}$ divided by 4 is $= \frac{1}{16}$ only, viz. but 1s. 3d.*

And now, *Tyro*, we are come to that rule wherein all the others are exercised, viz.

SECTION VII.

The Rule of Three in Vulgar Fractions.

Tyro. **H**OW is this rule performed?

Philo. After having reduced compound fractions and mixt numbers as before directed, first, (as in the common rule of three) make your first and third

* See more of the nature of vulgar fractions in my young algebraist's companion, dialogue 3, case 3, 4. num-

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number of one name, then multiply your second by your third, and divide by your first.

Or rather thus :

Having stated the question, and reduced the fractions, multiply the denominator of the first number into the numerators of the second and third numbers, for a new numerator, and the numerator of the first number into the denominator of the second and third, for a new denominator, and that is your answer required.

1. If $\frac{3}{4}$ of a yard cost $\frac{5}{6}$ of a £. what cost $51 \frac{1}{4}$ yards?

$$\begin{array}{r} 4 \\ \hline 205 \\ \hline 4 \end{array}$$

Thus, $\frac{3}{4}, \frac{5}{6}, 10 \frac{1}{4}$. Now I begin with the denominator of the first fraction, thus, $4 \times 5 \times 205 = 4100$ N. N. And $3 \times 56 \times 4 = 72$ N. D. So is the answer $4100 \div 72$ of a £. = £ 6-18-10. 29 $\frac{48}{72}$ or $\frac{2}{3}$.

2. If a load of wheat cost £ 15 $\frac{1}{2}$ what cost one bushel?

Here as 40 bushels make 1 load my first number will be 40 , the second number reduced to an improper fraction is $18 \frac{1}{2}$, and 1 bushel will be $\frac{1}{40}$. Then if 40 be $18 \frac{1}{2}$ what $\frac{1}{40}$. Now $1 \times 182 \times 1 = 182$ N. N. and $40 \times 12 \times 1 = 480$ N. D. Ans. $\frac{182}{480}$, which is $= 7s. 7d.$

Tyro. This is easy enough, if this be a general rule.

Philo. It is, and if you look over what has been done, you may with ease do any thing in this rule; therefore, I shall only set you a few questions for practice.

3. What is the interest of £ 219 $\frac{2}{3}$ for one year, at £ 5 $\frac{3}{8}$ per cent. *Ans.* $28\frac{337}{4000}$, viz. £ 11-16-1 2 q. $\frac{1200}{4000} = \frac{3}{100}$.

4. Bought South-Sea stock to the value of £ 420 13s. 4d. and gave £ 95 $\frac{4}{5}$ per cent. what comes it to? *Ans.* £ 402-19-11 2 q. $\frac{108}{1500}$ or $\frac{18}{250}$.

5. A person left 40 shillings to 4 poor widows, A, B, C, and D. To A he left $\frac{1}{3}$, to B $\frac{1}{4}$, to C $\frac{1}{5}$, and to D $\frac{1}{6}$, desiring the whole might be distributed accordingly; I demand the proper share of each?

Take $\frac{1}{3}$, $\frac{1}{4}$, &c. of 40 shillings, and add them together, it makes but 38 shillings: then say,

If 38s. be $\frac{1}{3}$, viz. 13s. 4d. what will 40 be? Thus proceed with all their shares, you will find A must have 14s. $\frac{16}{38}$, B 10s. 6d. $\frac{12}{38}$, C 8s. 5d. $\frac{2}{38}$, and D 7s. 0d. $\frac{8}{38}$. And now, *Tyro*, we will proceed to

DIALOGUE XIII.

SECTION I.

NOTATION OF DECIMAL FRACTIONS.

Tyro. **W**HAT do you mean by a decimal?

Philo. Any number, whether cyphers before it or not, having a dot before them, thus, .2 .005 or .4715, &c. are decimals?

Tyro. How are these decimals formed?

Philo. Every decimal is a vulgar fraction, having as many cyphers for its denominator and an unit besides

ides. Thus the foregoing decimals expressed in vulgar fractions are $\frac{2}{10}$, $\frac{005}{1000}$, and $\frac{4715}{10000}$, &c. but the denominators are cast away in decimals, because you see they are always known, and we work only with the numerator, or the decimal itself; and this is the reason that decimal fractions are so very easy to what vulgar are.

Tyro. I understand you, but you have no mixt numbers here as well as in vulgar fractions?

Philo. Yes: a whole number, and a decimal after it is a mixt number. Thus, 47.5 or 5.25 are mixt numbers, signifying 47 whole numbers, and $\frac{5}{10}$ parts or $\frac{1}{2}$; and the other is 5 whole numbers, and 25 hundredth parts, or 1 quarter, for 25 is $\frac{1}{4}$ of 100.

Note 1. Cyphers after figures are of no signification in decimals, thus, .5000 is but .5 and .750000 but .75, for .500 is $\frac{500}{1000}$, which is the same as $\frac{5}{10}$: but cyphers before decimals are necessary, for they decrease their value. Thus, .5 is 5 tenth parts, or $\frac{1}{2}$, but .05 is but 5 one hundredth parts, viz. $\frac{5}{100}$ in vulgar fractions.

Note 2. Pray remember this: that .5 signifies the half of any whole number, thing, or integer. .25 is one quarter, and .75 signifies three quarters. For 5 is $\frac{1}{2}$ of 10, 25 is $\frac{1}{4}$ of 100, and 75 is $\frac{3}{4}$ of 100.

This being well understood, you may proceed to addition.

SECTION II.

ADDITION OF DECIMALS.

Tyro. **H**OW is addition of decimals performed?

Philo. The same as common addition, only with this difference, that you must set tenths under tenths, and hundred parts under hundred parts, not regarding the number of decimal places, but set them all even towards the first left hand row, that stands next the whole numbers, and then add them all together as in whole numbers.

One example well explained will be sufficient, for you will see the reason more plainly in reduction.

1. Add £47.45 7259⁵ .25 .000545 and 15.5 together.

47.45

.7259

5.25

.000545

15.5

Ans. 68.926445

Here you see I place the whole numbers under one another, as in Addition and the decimals I place even next the whole numbers, not regarding how far they extend to the right-hand: then I add them as they stand, like whole numbers.

2. Add .575 .005 .0005 .95 and .3 and 675 together
 Ans. 2.5055. If you set these under one another, and add them as before, you will have 2 for a whole number and .5055 a decimal.

Note, Always remember to part the decimals from the whole numbers by a dot. Thus, I find in casting up this last sum it amounts to 5 figures; but as there are but 4 figures in the largest decimal, therefore, I make a dot between the 4th and 5th figure, always keeping an equal number of decimal places.

SECTION III.

SUBTRACTION OF DECIMALS.

Tyro. **I**F Addition of Decimals be performed like whole numbers, I imagine Subtraction is the same.

Philo. It is so, only place the figures to the left-hand, always under one another, let them be cyphers or figures; but don't regard cyphers to the right. An example will make it quite easy.

From 47.0075
Take 15.51

Ans. 31.4975

Proof 47.0075

From .7
Take .275

Ans. .425

Proof .7

From 5.25
Take .98407

Ans. 4.26593

Proof 5.25

Here you see I do by tens, as in whole numbers, and borrow and carry the same, only I never set down cyphers to the right hand of the figures, but entirely discard them.

2. Borrowed 100 guineas, and paid at three several times each £ 27.275 what is to pay? *Ans.* £ 23.175, or £ 23-3-6.

SECTION IV.

MULTIPLICATION of DECIMALS.

Tyro. **H**OW is Multiplication performed?

Philo. Like common Multiplication, having no regard at all to the Decimals till the work is done, and added up: then count how many decimal places you have both in the Multiplicand and Multiplier, and as many decimals as you find in both these, so many figures you are to prick, or point off from the right-hand to the left: so will the figures on the left-hand of the dot be whole numbers, and those towards the left, the decimal parts.

EXAMPLES.

Multiply 7.256
by .762

14512
43536
50792

Ans. 5.529072

Multiply .5759
by .0375

28795
40313
17277

Ans. .02159625*

Here in example 1, I have 6 decimals in the Multiplicand and Multiplier; therefore I prick or dot off 6 towards the left-hand, and have 5 for a whole number, and the rest are decimals.

* *Note,* In example 2, I have 8 decimals, in the Multiplicand and Multiplier; I find upon casting them up, that there are but 7 places of decimals, therefore, I place a cypher before the first figure, and make a dot before the cypher so I have 8 decimal places.

This

This is a standing rule, for had the product been less in the number by 2, 3, or more places, so many cyphers must have been added to supply the deficiency.

Tyro. This is easy enough, as you observed, to what Vulgar Fractions are: and I perceive that money, weight, or measure may be easily multiplied by this rule.

Philo. You say right, for suppose I was to multiply £4 15s. by £3 10s. it would produce £16-12-6 For £4 15s. is 4.75 (because 15s. is $\frac{3}{4}$ of a £.) and £3 10s. is £3.5. Therefore $£4.75 \times 3.5 = £16.625$. or 12s. 6d.

3. Multiply .000075 by .0015. *Ans.* .0000001125.

4. Multiply $4\frac{1}{2}$, viz. 4. 5. by 4. 5. *Ans.* 20.25, or, $20\frac{1}{4}$.

5. Multiply $\frac{1}{2}$ a crown by $\frac{1}{2}$ a crown, at a shilling the integer. *Ans.* 6s. .25, viz. 6s—3d.

See Multiplication of Vulgar Fractions; and more of the nature of this, with the way and manner of valuing any decimal, in Reduction of Decimals.

SECTION V.

DIVISION of DECIMALS.

Tyro. **H**OW is Division performed?

Philo. The same as common Division; but being a little more difficult than the other rules, you must be the more careful: but if you observe the three following notes, they will help you to work any sum.

Note 1. When there are more decimal places in the Dividend than in the Divisor, then (after the work is done) prick off as many in the quotient to supply that defect; that is, the decimal places of the divisor, and the quotient will always be equal in number to those in the dividend.

Note,

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Note 2. If there be more decimal places in the divisor than in the dividend, (before you begin to work) add as many cyphers to the dividend as make the number of decimal places equal to the divisor, and proceed as before directed.

Note 3. When it happens that the decimal places in the divisor and quotient (after the work is done) are not so many as those of the dividend; then you must place as many cyphers before the quotient, or answer, as will make up that deficiency.

$$\begin{array}{r} .5)47.3476 \\ \hline \end{array}$$

Ans. 946952
See Note 1.

$$\begin{array}{r} .12)63.00 \\ \hline \end{array}$$

Ans. 525.
See Note 2.

$$\begin{array}{r} 12).063 \\ \hline \end{array}$$

Ans. .00525
See Note 3.

Remember that all remainders in decimals after division are of no signification, and therefore are taken no notice of.

The same is to be observed in long division as in the three foregoing examples in relation of pointing off.

$$47.15)3.747565(0794. \quad \text{Ans.}$$

$$\begin{array}{r} 33005 \\ \hline \end{array}$$

But if thus,

$$\begin{array}{r} 44706 \\ \hline \end{array}$$

$$\begin{array}{r} 42435 \\ \hline \end{array}$$

$$.4715)3.747565(7.94. \quad \text{Ans.}$$

$$\begin{array}{r} 22715 \\ \hline \end{array}$$

$$\begin{array}{r} 18860 \\ \hline \end{array}$$

Here you see the same sum is proposed, but the answer is different, as you may prove at large; for the figures will be the same. And thus you see, let them be whole or mixt numbers the work is the same.

Tyro. I understand it: and I perceive it is very easy to divide money into any number of parts by this rule.

Philo.

Philo. Very easy indeed.

6. Divide £42 19s. 6d. by £4 15s. 6d. That is, divide £42.975 by £4.775. *Ans.*

7. Divide £4.5 (viz. £4 10s.) by £3.5 (viz. £3. 10s.) *Ans* £1.285, or £1 5s. 8d $\frac{1}{4}$.

8. Divide 1.753 by £1753. *Ans* .001.

This example, though it appears trifling at first, may be of service. For suppose it was required to divide £1.753 among 1753 persons, the answer would be .001, viz. near 1 farthing each. Thus you may vary or alter Decimals at pleasure; but as for money and the manner of finding its value in any Decimal Fraction, that you will see in the next Section, which if you observe well, you will see the whole order of Decimals, and their relation and harmony, compared with Vulgar Fractions.

SECTION VI.

REDUCTION OF DECIMALS.

Tyro. **W**HAT does Reduction of Decimals teach?
Philo. It teaches to reduce a Vulgar Fraction to a Decimal of the same value, and also shews you how to find that value either in money, weight, or measure, &c.

CASE I.

To reduce a Vulgar Fraction to a Decimal.
 Add as many cyphers as you please to the Numerator, (making a dot between the Numerator and the cyphers) then

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then divide by the Denominator as in common Division, and making a dot before the figures in the Quotient, you have the Decimal equal to the Vulgar Fraction.

1. Reduce $\frac{3}{4}$ to a Decimal. *Ans.* .75.

4)3.00 Here you see .75 is $\frac{3}{4}$ for 75 is $\frac{3}{4}$ of 100 its proper Denominator.

75

2. Reduce $\frac{5}{8}$ to a Decimal of the same value.

8)5.000 That is, 625 is $\frac{5}{8}$ of 1000, its Denominator.

.625 *Ans.*

3. Reduce $\frac{5}{39}$ to a Decimal. *Ans.* .01265.

4. Reduce $\frac{3}{175}$ to a Decimal. *Ans.* .00171.

Note, Sometimes in reducing a decimal there will be a remainder; but never regard that, if you have 5 places in a decimal it is sufficient.

C A S E 2.

To reduce the known parts of money, weight, or measure to a decimal.

Add cyphers to the lowest denomination (making a dot between the cyphers and the figure, and divide by the parts contained in the next higher denomination; then place the next higher denomination, before that Quotient (with a dot between) and divide by the parts contained in the next higher denomination; and so go on, and your last quotient will be the decimal required. But this must be illustrated by an example to make it plain.

1. What

1. What is the decimal of 14s. 6d. $\frac{3}{4}$?

$$\begin{array}{r} 4) 3.00000 \\ \hline 12) 6.75000 \\ \hline 2|0) 14.56250 \\ \hline 728125 \text{ Ans.} \end{array}$$

Here, according to the rule, I add cyphers to the lowest denomination, 3 farthings, and divide by 4; then I place the 6 pence before that quotient, and divide by 12. Lastly, I place 14 shillings before this last quotient, and divide by 20, (viz. by 2) without cutting any figure or cypher off to the right hand; for there is no occasion for that.

See the proof of this in example 1, next case.

2. What is the decimal of 18s. 9d. $\frac{1}{2}$. *Aus.* 93958.

Note. If you would know the decimal of any number of shillings, from 1 to 19, observe this general rule: if the shillings be even, take the $\frac{1}{2}$ of them is the decimal. Thus the decimal of 16s. is .8, and of 18s. is .9, &c. But if the shillings be odd, multiply them by 5, gives the decimal. Thus the decimal of 5s. is .25; for 5 shillings is $\frac{1}{4}$ of a £. and 25 is $\frac{1}{4}$ of 100. So also, the decimal of 17s. is .85, and 11s. is .55, and the decimal of 1s. is 05; for there must be 2 places when the shillings are odd.

A RULE to find the decimal of shillings, pence, and farthings, at a £. sterling the integer at once.

1. For farthings. Add cyphers to the shillings, and divide by 20.

2. For the pence. Add cyphers to the given pence, and divide by 240, the pence in a £. pricking off according to the rule of division. Thus you will find the decimal of 6d. .025 and of 3d. .0125.

3. For

3. For farthings. Add cyphers and divide by 960: thus, the decimal of 3 farthings is 003125.

Note, The same is to be observed in finding the decimal of weight and measure, by adding cyphers to the given denomination, and dividing by the parts contained in the integer.

To tell the decimal of shillings, pence, and farthings, by inspection.

Note, If the shillings be even, take the $\frac{1}{2}$ of them, which will be the first decimal figure; then bring the pence and farthings into farthings, and if they be less than 25, join them to the first decimal figure, so have you a decimal of 3 places; but if the farthings be more than 25, set down 1 more than they really are; if they be above 40 set down 2 more than they amount to; so you have the decimal nearly.

1. Let it be required to find the decimal of 14s. 6d. $\frac{3}{4}$, and 18s. 9d. $\frac{1}{2}$, as before. Here I say the $\frac{1}{2}$ of 14 is 7, the first figure; then 6d. $\frac{3}{4}$ is 27 farthings, but being above 5, I set down 28 farthings by the side of the 7, so is the decimal .728, as before; and so for the other.

2. What is the decimal of 6s. 10d. $\frac{1}{2}$? *Ans.* .344.
Here the $\frac{1}{2}$ of 6s. is 3, and 10d. $\frac{1}{2}$ is 42 farthings; but being above 40, I, by the rule, set down 2 more, viz. 44; so is the decimal .344 nearly.

Note, When the shillings are odd, multiply them by 5, and bring the pence and farthings into farthings, as before, and set the first figure under the second figure of the decimal belonging to the shillings, encreasing them by 1, or 2, as before you have the decimal.

2. What

3. What is the decimal of 17s. and 6d. $\frac{3}{4}$: and 11s. 10d. $\frac{3}{4}$?

$$\begin{array}{r}
 17 \text{ multiplied by } 5 \text{ is } 85 \\
 6\frac{3}{4} \text{ more } 1 \text{ farthing } 28 \\
 \hline
 \text{Ans. } .878
 \end{array}
 \qquad
 \begin{array}{r}
 11 \times 5 = 55 \\
 10d. \frac{3}{4} \times 4 \times 2 = .45 \\
 \hline
 11s. 10d. \frac{3}{4} = 595
 \end{array}$$

The next case is a proof to this, and more useful.

C A S E 3.

To find the value of a decimal in money, weight, and measure.

Multiply the Decimal by the parts contained in the integer, and prick off as many figures as there are places in the given decimal, and the figures towards the left-hand will be whole numbers, and those that are pricked off are decimals, which decimals only must be multiplied by the next denomination: thus go on, multiplying and pricking off the same number of decimals; so will the figures towards the left-hand be the value required.

1. What is the .728125 of a £. sterling? 20
2. What is the .7615 of a guinea? 21

$$\begin{array}{r}
 \text{£ } 14.562500 \\
 12 \\
 \hline
 d. 67500 \\
 4 \\
 \hline
 \text{qrs. } 3.00
 \end{array}$$

$$\begin{array}{r}
 7615 \\
 15230 \\
 \hline
 s. 15.9915 \\
 12 \\
 \hline
 d. 11.8980 \\
 4 \\
 \hline
 \text{qrs. } 3.5920
 \end{array}$$

Ans. 14s. 6d. $\frac{3}{4}$. See Ex. 1. of last case.

Ans. 15s. 11d. $\frac{3}{4}$

3. What

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3. What is the .1756 of a ton?

20

C. .3.5120

4

qrs. 2.0480

28

3840

0960

lb. 1.3440

16

oz. 5.5040

4. What is the .09715 of a bar.

36

58290

29145

gallons 3.49740

8

pints 3 97920

Ans. 3 gall. 3 pints, and $\frac{1}{10}$, or very near $3\frac{1}{2}$ gallons

Ans. 3 C. 2 qrs. 1 lb. 5 oz.

To tell the value of any decimal in shillings, pence, and farthings (by inspection only) at a £. sterling the integer.

Note. If there be ever so many places in the decimal, the first 3 figures are sufficient and all that are required in business.

Note 2. Double the first decimal place towards the left-hand, and if the second figure be under 5, then the first figure only doubled will be the shillings; but if the second figure be 5, or above 5, then you must add 1 shilling more to those you doubled; and what remains from the second figure above 5, carry to the next figure, and reckon them as so many farthings; and if above 25, account them still as farthings, only abate 1; but if above 40, then abate 2 farthings less than they really are, and you have the value required.

1. What

1. What is the value of $.728125$ of a £. *Ans.* 14s. 6d. $\frac{3}{4}$.

Here I double the first figure 7, which is 14, for the shillings, and then I say, 28 farthings is 7d. but I abate 1, because it is above .25; so it is 14s. and 6d. $\frac{3}{4}$.

2. What is the $.39525$ of a £ *Ans.* 7s. 10d. $\frac{3}{4}$.

Here I say, twice 3 is 6, and the next figure being above 5, I count 1 more, which is 7 shillings; then there is 4 remains from the 9, which I carry to the 5, which is 45 farthings; but being above 40, I abate 2, and call it 43 farthings, which is 10d. $\frac{3}{4}$.

3. What is the $.0672$ of a £. sterling? *Ans.* 1s. 4d.

Here the Cyphers doubled is 0; but the second figure being 6, that is 1 shilling and 1 over, which I carry to the 7, is 17 farthings, and abating 1 farthing is 16, or 4 pence; which you may prove by multiplying the decimal by 20, 12, and 4, pricking off as before directed.

EXAMPLES for Exercise.

4. What is $.8145$ of a ton. *Ans.* 16 C. 1 qr. 4 lb. 7 oz. 10 dwts. .88.

5. What is the $.275$ of a lb. troy? *Ans.* 3 oz. 6 dwts.

6. What is the $.0729$ of a year, at 365 days the integer? *Ans.* 26 days, 14 hours, 36 minutes, 14 seconds.

These examples are sufficient for any diligent learner, therefore I will proceed to put them all in practice.

SECTION VII.

The Rule of Three Direct in DECIMAL FRACTIONS.

Tyro. **H**OW is the Rule of Three in decimals performed?

Philo. The same as in the common Rule of three direct, by multiplying the second number by the third, and dividing by the first.

1. If $\frac{3}{4}$ of a yard cost $\frac{5}{6}$ of a £. what cost $51\frac{1}{4}$ yards? that is, if 3 qrs. cost 16s. 8d. what cost 51 yards, 1 qr.

Decimally thus,

If .75 be .8333, what is 51.25 yards?

Here multiply your second by the third, and divide by the first, you have the quotient 56.942 , viz. £ 56 18s. 10d.

See example 1, rule of three of vulgar fractions.

2. If a bushel of wheat cost $3s. 9d. \frac{1}{2}$, what cost 40 bushels, or 1 load? *Ans.* £ $7-11-8$.

Decimally thus,

If a bushel of wheat cost .18958, what cost 40?

40

£ 7.58320 *Ans.* £ 7 11s. 8d.

Tyro. This last example is very short indeed.

Philo. It will always be so when the first number is 1, or unity; for decimals are superior to vulgar fractions for ease and expedition; and though not always so near the truth itself, yet answer every thing near enough for any business or demand. See the next question in variations.

The Rule of Three in Decimal Fractions. 297

3. If $5 \frac{1}{4}$ ells cost £ $2 \frac{800}{800}$ of a £. what cost 282 $\frac{1}{2}$ ells? That is, if $5 \frac{1}{4}$ cost £ 2 16s. 8d. $\frac{3}{4}$, what cost 282 $\frac{1}{2}$ ells?

Decimally thus,

If 5.25 ells cost £ 2.8364, what cost 282.5 ells?
Ans. £ 152 625, viz. £ 152-12-6.

4. If a ton of sweet oil cost £ 24-17-4 $\frac{1}{2}$, what cost 14T. 17C. 2qr. 19lb. *Ans.* £ 370.1335, or 2s. 8d. $\frac{1}{2}$.

Now, *Tyro*, in all such sums as these, which either by the rule of three, practice, or vulgar fractions, will become very tedious; yet by finding the decimal of the odd weight, and the odd price, you have only a common multiplication sum.

That is, multiply 14 ton .88348 by £ 24.86875, you will have £ 370.1335, viz. 2s. 8d. *Ans.*

Note, You may find the decimal of 17C. 2qrs. 19lb. by bringing them into lbs. and dividing by the lbs. in a ton, viz. 2240. The same for any other weight, or measure, by reducing the parts, and dividing by the parts contained in the integer.

DIA.

DIALOGUE XIV.

SECTION I.

SIMPLE INTEREST.

Tyro. **H**OW is simple interest in decimals performed?
Philo. Very easy by the following rule.

Multiply the principal by the rate per cent and prick off the decimals gives the interest for one year; and if there be odd time, take the parts of the principal itself, and add to the work, it is done.

Note, For £ 4 per cent. multiply by .04; for 5 per cent. by .05; for 6 per cent. .06; for 7 per cent. .07.

1. What is the interest of £ 147-15s. for a year, at £ 4 per cent. per annum? *Ans.* £5-18-2 $\frac{1}{4}$.

£ 147.75 2. What is the interest of £275-10s for
 .04 1 year, at £ 6 per cent.

$$\begin{array}{r} \text{£} 5).9100 \\ \underline{20} \end{array}$$

$$\begin{array}{r} \text{£} .18)20 \\ \underline{12} \end{array}$$

$$\begin{array}{r} d.2)40 \\ \underline{4} \end{array}$$

$$\begin{array}{r} 7.1)60 \end{array}$$

$$\begin{array}{r} \text{£} 275.5 \\ \underline{.06} \end{array}$$

$$\begin{array}{r} \text{£} 16.530 \end{array}$$

Ans. £ 16-10-7 $\frac{1}{4}$

And

And the work will be as easy, if the per cent. be for odd time, as appears by the following examples.

3. What is the interest of £ 1250. 15s. for $3\frac{1}{2}$ years. at 6 per cent. per annum?

$\frac{1}{2}$	$\begin{array}{r} \text{£ } 1250.75 \\ \quad .06 \\ \hline 75.0450 \text{ for 1 year} \\ \quad 3 \\ \hline 225.1350 \text{ for 3 years} \\ 37.5225 \text{ add for the } \frac{1}{2} \text{ year} \\ \hline 262.6575 \text{ for } 3\frac{1}{2} \text{ years. } \textit{Ans.} \end{array}$
---------------	--

4. A person left his wife by will $\frac{1}{3}$ of his estate, which was £ 355-7-6. Now this lay in hand $3\frac{1}{2}$ years; I demand what interest is due upon it, at $\text{£ } 4\frac{1}{2}$ per cent. per annum.

Ans. £ 55-19-5.

Multiply £ 355.375 by $\text{£ } 4\frac{1}{2}$, that is by .045 is £ 15.991875 for 1 year: this multiplied by 35 years gives £ 55.9715625.

And thus you see, that if the principal, interest, or time, be ever so much, yet it is easily performed by decimals.

5. A merchant made an assurance upon goods in a ship bound to a certain port, to the value of £ 2530, upon condition, that in case of a total loss or damage, the assurers were to pay $\text{£ } 97\frac{1}{2}$ per cent. deducting $\frac{1}{2}$ per cent. (viz. 10s. per cent.) out of it: now the ship was cast away, but there was as many goods saved as amounted to £ 955; I demand what the merchant has to receive? *Ans.* £ 1527-18-11.

First, £955 taken from £2530, remains £1575 loss: this multiplied by 97.5 is 1535.625 : and deducting $\frac{1}{2}$ per cent. from this, is only cutting off 2 figures in the pounds, thus, 15|35, and take the $\frac{1}{2}$ of 15|35.625, which is 7.678125 placing it in the units place of the pounds, and the rest in order, (as follows in division) and subtract it from £1535.625, leaves £1527.946875, viz. 18s. 11d. *Ans.*

The proof of this is worthy your observation, *Tyro.*

For observe $97\frac{1}{2}$ per cent. wants but $2\frac{1}{2}$ per cent. of £100, that is, of being cent. per cent. Now if you multiply £1575 by 2.5 viz. ($2\frac{1}{2}$) it gives £39.375, which added to £1535.625, gives the original sum or loss, viz. £1575.

Tyro. I thank you kindly, sir; for this, as you have observed, is a proof to me very plainly.

6. What comes an assurance, or a commission, factorage, or brokerage to, upon £3500, at 18s. per cent,

$$\begin{array}{r} 35'00 \\ .9 \text{ the decimal of } 18s. \end{array}$$

$$\text{£}31.5 \quad \text{Ans. } \text{£}31.10$$

And thus decimals perform any thing with ease and pleasure.

Tyro. I see it, sir, and now be pleased to give me an example or two in compound interest.

Philo. I will.

SEC-

SECTION II

COMPOUND INTEREST.

Tyro. **H**OW is compound interest performed?
Philo. By continual multiplication and addition.

Quest. 1. What will £ 100 amount to, if it be forborne 5 years at 5 per cent. per annum, compound interest. *Ans.* 127-12-6 $\frac{3}{4}$.

1. The common method is to multiply the principal by the per cent. and cut off two figures, (which is the same as dividing by 100) this gives the interest for one year, which is £ 5; this add to £ 100, gives £ 105 for the amount of the first year. Then this multiplied by the per cent. gives the interest for the 2d year, which added to the principal of £ 105, gives £ 110.25 for the amount the 2d year, &c. &c. But this is tedious; therefore the best way is,

2. Set down the principal 100 l. and find the interest the 1st year and add it to it, and it makes as before 105. Set the 5 under the 100 l. and make two dots after t, thus 5 .. this saves dividing by 100, and will supply the decimal places; then multiply 150 l. by 5 (viz. 05 in decimals) keeping always 2 figures of the decimals under the 2 dots, and it produces 5—25, which added to 105 l. gives 110.25 l. as before, &c. &c. See the operation.

O

£ 100

£ 100 $\times$ viz by .05 is 5.00, viz. 5
5... added

Amount

1 year is $\begin{array}{r} 105 \dots \text{this } \times .05 \\ 5.25 \text{ add} \end{array}$

2 year is $\begin{array}{r} 110.25 \text{ this } \times .05 \\ 5.5125 \text{ add} \end{array}$

3 year is $\begin{array}{r} 115.7625 \text{ this } \times .05 \\ 5.7881 \text{ add} \end{array}$

4 year is $\begin{array}{r} 121.5506 \text{ this } \times .05 \\ 6.0775 \end{array}$

5 year $\begin{array}{r} 127.6281 \end{array}$ Ans. £ 127.12

Quest. 2. *A* lent *B* £ 136.775. which *B* promised to pay compound interest for, at £ 6 per cent. per annum, and bound his heirs, executors, &c. to that condition: Now *A* died, and *B* took no notice of payment till it was at last discovered by the executors of *A*, that *B* had had the money in hand 20 years: I demand what *B* has got to remit for the debt? Ans. £ 438.651.

But the shortest method of all to calculate compound interest is by the following tables.

T A-

TABLE I.

A table shewing how much 1 pound sterling will amount to any number of years under 21, at £ 5 per cent. per annum, compound interest.

<i>Year.</i>	<i>£5 per cent.</i>	<i>Years</i>	<i>£5 per cent.</i>
1	1.05	11	1.7103
2	1.1025	12	1.7958
3	1.15763	13	1.8856
4	1.2155	14	1.9799
5	1.27628	15	2.079
6	1.3401	16	2.1828
7	1.4071	17	2.292
8	1.4744	18	2.4066
9	1.5513	19	2.5269
10	1.6289	20	2.6533

TABLE II.

A table shewing how much 1 pound sterling will amount to any number of years under 21, at the rate of £ 6 per cent. per annum, compound interest.

<i>Year</i>	<i>£6 per cent.</i>	<i>Years</i>	<i>£6 per cent.</i>
1	1.06	11	1.8982
2	1.1236	12	2.0121
3	1.191	13	2.1329
4	1.2624	14	2.261
5	1.3382	15	2.3965
6	1.4185	16	2.5403
7	1.5036	17	2.6927
8	1.5938	18	2.8543
9	1.6894	19	3.0256
10	1.7908	20	3.2071

1. The explanation of the tables.

The amount of £ 1 for 1 year, at £ 5 per cent. is 1.05 this $\times 105 = 1.1025$ the amount of the 2d year; this, $\times 1.05$ is 1.1576, &c. &c. The same for £ 6 per cent. which is 1.06 this into 1.06 is equal to 1.1236 for the second year, &c. &c.

2. The use of the tables.

When any sum is given for any number of years, then multiply the given sum by the number given right against or answering to the number of years, and you have the answer at one operation.

Let us take example 1, viz. £ 100, for 5 years, at £ 5 per cent.

I look in the table of 5 per cent. and against 5 years I find 1.27628, the (amount of £ 1 for 5 years) this multiplied by £ 100 gives 127.628, viz. £ 127-12-6½ as before.

Tyro. This is short indeed!

Philo. 'Tis the same as if it were for 20 years. Thus in example 2, it is required to tell the amount of £ 136-15-6, for 20 years, at £ 6 per cent. compound interest.

I look in the table for £ 6 per cent. and against 20 years I find 3.2071 the amount of £ 1 for that time; this multiplied by the sum £ 136-15-6. (viz. 136.775) gives £ 438.651, viz. £ 438. 13 shillings and 1 farthing.

Tyro. I like this very well; but suppose the years are more than in the tables, how then?

Philo. Very easy; only add any two or more numbers together, as make the number, and multiply the sums belonging to each number of years in the table together, gives the right sum for that number of years.

Thus,

Thus, suppose I wanted to tell the amount of the last question for 30 years, at £ 6 per cent. Here I take any two numbers, which added, make 30. Suppose for example 10 and 20 : against 10 I find 1.7908, and against 20 I find 3.2071, this multiplied together gives 5.7432, the amount of £ 1 for 30 years, at £ 6 per cent. which multiplied into £ 136.775, gives 785.526, the amount of this sum for 30 years; and thus for any other numbers.

Tyro. I heartily thank you, Sir.

Philo. You see how easy it is, and this is the only method to calculate annuities, pensions, &c. except you can do it by algebraic calculation.

Note, These tables are easily made, the construction depends, as I told you, in simple interest, upon this; That let the per cent. be what it will, suppose £ 4 per cent. say, if £ 100 be 104, what will £ 1 be? *Ans.* £ 1.04 for 1 year. So also for $4\frac{1}{2}$ per cent. it is £ 1.045; for 5, it is 1.05, &c.

Tyro. I understand you very well, sir, and shall endeavour, as time offers itself conveniently, to look over these things, and make myself yet more perfect.

Philo. You will perform your promise I hope; but before I leave you, I will give you a notion of the extraction of the square and cube roots, being very necessary in many businesses, but especially in the art of mensuration, and several other branches of the mathematics.

DIALOGUE XV.

SECTION I.

The extraction of the SQUARE ROOT.

Tyro. **W**HAT do you mean by the words square and root?

Philo. A square number is a number multiplied by itself, viz. any figure or figures multiplied by the same figure or figures, the product is the square of that number: Thus, $2 \times 2 = 4$, the square of 2; and $9 \times 9 = 81$, the square of 9.

Tyro. This is plain enough. And what is the root then?

Philo. The root is that from which the square is formed: Thus, I told you before, the square of 2 is 4, and the square of 9 is 81; therefore vice versa 2 is the root of 4, and 9 is the root of 81, as appears by the following table which should be readily known.

T A B L E.

<i>Roots</i>	1	2	3	4	5	6	7	8	9	10	11	12
<i>Squares</i>	1	4	9	16	25	36	49	64	81	100	121	144

Tyro. How is the square root extracted?

Philo. I will shew you the whole process, which pray observe.

Suppose it were required to extract the square root of 3136, or any other figures.

First, I set down the figures thus, 3136 , and beginning at the units place, I make a dot or point over it, and also over every other figure towards the left hand, as you see in the margin; and pray observe, that as many dots as you have, so many figures the root will always consist of, which here are 2.

Secondly, I seek (by the table) the nearest root to the figures contained in the first point of figures, viz. in 31 , and find it to be 5, which I place in the quotient thus, $3136(5$, which figure 5 is called the root or part of the root.

Thirdly, Square the root, that is, $3136(5$
multiply it by itself, and place it under the said first points as in common division, and subtract it therefrom, 25
and bring down the next point, viz. 636 resolvend
 36 , and place it by the side of the remainder, it is 636 , which is called the resolvend, as in the margin.

Fourthly, Then I double the quotient figure, or root 5, which is 10, and making $3136(5$
another crooked line I place 25
it for a divisor right against the resolvend, thus, Divisor $10)636$ resolvend

Fifthly, I now ask (as in division) how many times 10 I can have in the resolvend (always rejecting the last figure) $3136(56$ root ans.
that is, how many times 10 are contained in 63, and find it 6 times, which 6 I put in the root by the side of the 5, and 25
also by the side of the divisor $106)636$
 636
10, which makes 106; then I multiply 106 by 6, which is 636, and nothing remains; thus I find the square root of 3136 to be 56.

P R O O F.

I square the root of 56, that is, I multiply it by itself, viz. 56 by 56, and it gives 3136.

2. What is the square root of 56169?

56169 (237 root *Ans.*

4

43)161 resolvend

129

467)3269 new resolvend

3269

0

Here I proceed the same as in example 1, by making a dot over every other figure, and find the nearest the root of the first point 5 to be 2, which I square, and place under 5, and remains 1, to which I bring down the next two figures 61, and it is 161

then I double the root 2, it is 4, which I place on the left-hand for a divisor. Then I ask how many times 4 are contained in 16, which, tho' it be 4, yet upon trial will be but 3 times (for you must observe, it will often be less than it looks to be) which 3 I place in the quotient, and also by the side of the divisor, 4, which makes it 43; then I multiply 43 by 3, and it is 129; and subtracting 129 from 161, I have 32 remains, to which I bring down the next point, or two figures 69, and it is 3269, which I call a new resolvend; then I double the root 23, which is 46 for a new divisor, and ask how many times 46 I can have in 326, and find it 7, which I place in the quotient, and also after 46, and it is 467, which multiplied by 7, gives 3269. Thus I find the square root of 56169 to be 237. And for a proof I find 237 multiplied by 237 = 56169. Do you understand it, *Tyro*?

Tyro. Very well, sir: then I perceive that after I have done with the first figure in the root, I am to double it, and take down the next point, and then dou-

double the two figures, and take down the next point, and so keep on in the same order: am I not?

Philo. Your notion is right, and if you consider well the manner of the working the last example, if you have ever so many figures, you may do it with ease.

Tyro. But suppose after the work there should be a remainder.

Philo. That matters not at all; only when you come to prove the work, after multiplying the root by itself, you must add the remainder to the product, and it will be equal to the given number if the work be right.

Sums for practice.

3. I demand the square root of 2996361. *Ans.* 1731.
4. I demand the square root of 3076516. *Ans.* 1754.
5. What is the square root of 43623. *Ans.* 208, and 359 remains over.

N. B. If you have a mind at any time to know what the remainder will produce, add an even number of cyphers to the sum, and double the root, and proceed as before: thus, the square root of 43623,000000, is 208,861.

2. To extract the square root of a vulgar fraction: extract the square root of the numerator for a new numerator, and of the denominator for a new denominator.

6. What is the square root of $\frac{25}{9}$? $\frac{5}{3}$

Note. When you can't extract the root of the numerator and denominator, then reduce the vulgar fraction to a decimal, and extract the square root, you have the answer.

7. What is the square root of 1 and $\frac{3}{5}$?
 First, $\frac{3}{5}$ is $=.6$ which is 1.6; then I add cyphers, thus, 1.600000, and find the root to be 1.264.

Note. There must always be an equal number of decimals.

8. What is the square root of 19.2? *Ans.* 4.3817.

9. What is the square root of .0003? *Ans.* .01732.

SECTION II.

The use of the square root applied to various branches of the mathematics.

10. **I** Demand what is the mean proportional number between 30 and 50. *Ans.* 38.7 tenths.

Multiply one number by the other, and add cyphers and extract the square root, you have 38.7.

11. There is a triangle, whose base is 30 inches, and the perpendicular 40; I demand the hypotenuse? *Ans.* 50.

Note. The perpendicular is that part which is right up; the base is that which lies next you, and the hypotenuse is the slanting side, called also the diagonal line.

A general rule to find the third side of any triangle, having two sides given.

1. Having the perpendicular 40, and base 30, as above, to find the hypotenuse, add the square of the base and perpendicular together, and extract the square root of them

them, you have the hypotenuse. Thus the square of $30=900$. The square of $40=1600$, their sum is 2500 , the square root of which is 50 , the hypotenuse required.

2. Having the hypotenuse and perpendicular, or base, given to find the other side.

From the square of the hypotenuse take the square of the base, or perpendicular, and the square root is the other side required.

EXAMPLES.

12. There is a steeple to whose vane or top a string is tied, which reaching to the ground, is in length 60 yards, and the distance from where it touches the ground to the middle of the steeple end is 25 yards; I demand the height of the steeple? *Ans.* 54.5 yards.

Square 60 is 3600 , and take the square of 25 , viz. 625 , from it, and extract the square root, you have 54.5 .

13. Suppose a ship sails from a certain port, has made 87 miles difference of latitude, and 71 miles departure, what is her distance on a regular course? This is only finding the hypotenuse; for add the square of 87 and 71 together, and extract the square root, you have 112.2 miles distance.

14. There is a circle or triangle, whose superficial content is 30800.25 , I demand the side of a square, whose superficial content shall be equal thereto? *Ans.* 175.5 .

15. Suppose a rope 5 inches round, I demand the compass of another rope, that is double the strength? Square the compass of the rope, it is 25 , which multiply by 2 , and extract the square root, it is 7.07 inches. If it were required to be 3 , 4 , 5 , or 6 times the strength, then multiply the square by 2 , 3 , 4 , 5 , or 6 , and extract the root.

16. There

16. There is a cable 10 inches round, which weigh
21 C. I demand the weight of one 8 inches round ? *Anf.*
13.44C.

As the square of the one, viz. 100, is to the square of the other, viz. 64, so is the weight of the one to the other, viz. 13.44C.

17. There is a circle whose content is 153.9385, demand its diameter ?

First, as 22 is to 28 :: 153: 9385 to the square of the diameter, viz, 195.9217, whose square root is 13.99.

And thus I think I have given you sufficient examples in this rule, that you may with a little more practice become quite master of it.

Tyro. I thank you for your care, sir, and I understand it very well ; I wish I understood the cube root as well.

Pbilo. That you may soon do by care, and a little more pains, tho' it be something more difficult than this rule.

DIALOGUE XVI.

SECTION I.

The Extraction of the CUBE ROOT.

Tyro. **W**HAT do you mean by a cube ?

Pbilo. A cube is that which has length, breadth, and thickness. Thus suppose a piece of wood to be cut into the form of a die which is equal every way in length, breadth, and thickness, such a figure is called a solid, and by name a cube.

Tyro.

Tyro. Give me a further Description of a cube in figures.

Philo. You know any number multiplied by itself is a square; so any number multiplied twice by itself is a cube number: thus, the cube of 2 is 8: For 2×2 is 4, and $4 \times 2 = 8$: So also, the cube of 5 is 125: for $5 \times 5 \times 5 = 125$. Thus you see 8 is the cube, and 2 the root of that cube. Also 125 is a cube number, whose root is 5, as appears by the following table of both squares and cubes.

T A B L E.

<i>Roots</i>	1.	2	3.	4.	5.	6.	7.	8.	9.
<i>Squares</i>	1.	4	9.	16.	25	36.	49	64	81.
<i>Cubes</i>	1.	8.	27.	64.	125.	216.	343.	512.	729.

Tyro. How is the Cube Root extracted?

Philo. To give you a rule for it (I look upon) would be too tiresome for your memory, as there are many parts contained in it: I shall therefore take an example or two, and proceed in the whole process, or order of the work.

1. I demand the cube root of 32768? *Ans.* 32.

R U L E 1.

First, I make a dot over every fourth figure, beginning at the units place, as in the margin, and as many dots as you have, so many places the root will contain, which here are two places, 32768.

Secondly, Seek the root, or nearest root, to the first point 32, which (by the table) is 3, and place it in the quotient, which is the first figure in the root, thus, 32768(3.

Thirdly,

Thirdly, Cube the figure which you put in the quotient (that is, $3 \times 3 \times 3 = 27$) and place it under the 32768(3 first point 32, and subtract it therefrom, thus 27

5

Fourthly, To this remainder (5) bring down all the figures of the next point (viz. 768) and place them by the side of the remainder, and call this the resolvend,

32768(3
27

Thus, 5768 resolvend

Fifthly, Triple the quotient, (that is, always multiply it by 3) and put the units place to it under the tens place of the resolvend; and call that the triple quotient.

32768(3
27

Thus, 5768 resolvend
9 triple quotient

Sixthly, Now square the quotient (that is 9) and triple that square, that is 27, and place the units place of it under the tens place of the triple quotient; that is, place it one figure more to the left-hand, and call it the triple square, thus,

32768(3
27

5768 resolvend

Seventhly, Add these two together, and call it the divisor,

27 9 triple quotient
triple square

Thus, 279 divisor.

Eighty

Eighthly, Ask how many times the divisor is contained in the resolvend, rejecting the last figure as you did in the square root; that is, ask how many times 279 you can have in 576 the resolvend, which here is 2, and place this also in the quotient, which now is 32.

$$\begin{array}{r}
 32768(32 \\
 27 \\
 \hline
 5768 \text{ resolvend} \\
 \hline
 9 \text{ triple quotient} \\
 27 \text{ triple square} \\
 \hline
 \end{array}$$

Thus, 279 divisor

Ninthly, Cube the figure last put in the quotient (viz. 2, whose cube is 8) and place the units place under the units place of the resolvend.

$$\begin{array}{r}
 32768(32 \text{ root} \\
 27 \\
 \hline
 5768 \text{ resolvend} \\
 \hline
 9 \text{ triple quotient} \\
 27 \text{ triple square} \\
 \hline
 279 \text{ divisor} \\
 \hline
 8 \text{ cube of } 2
 \end{array}$$

Tenthly, Multiply the square of the figure last put in the quotient (viz. 4 32768(32 root into the T. quotient, viz. $\times 9$. 27 which is 36) and place the product one figure more towards the left-hand.

$$\begin{array}{r}
 5768 \text{ resolvend} \\
 \hline
 9 \text{ triple quotient} \\
 27 \text{ triple square} \\
 \hline
 279 \text{ divisor} \\
 8 \text{ cube of } 2 \\
 36 \text{ the square of } 2 \text{ by triple quotient}
 \end{array}$$

Eleventhly,

Eleventhly, Multiply the triple square (viz. 27) by the last figure put in the quotient, and place also this one figure more towards the left-hand, which is

32768 (32 root. *Ans.*
27

5768 resolved

9 triple quotient

27 triple square

279 divisor

Twelfthly, Add these 3

last numbers together

as they stand, and call

it the subtrahend,

which is 5768 equal

to the resolvend

8 cube of 2

36 square of 2 by T. quotient

54 triple square by the root 2

5768 subtrahend

Thus is the work finished, and the cube root of 32768 is found to be 32.

P R O O F.

For the proof of this multiply 32 by 32, and it is 1024, which 1024 I multiply by 32 again, and have 32768.

Note 1. If the subtrahend had been larger than the resolvend, then I must put a less figure in the second place in the quotient, and proceed as before directed.

Note 2. When there is another point of figures to take down, first, subtract the subtrahend from the resolvend and to the remainder bring down the next point, calling it new resolvend, or second resolvend: Then proceed to work as after the first resolvend in every respect.

2. Another

2. Another method to extract the cube root, which is in many respects easier and shorter than the former.

Let us take the last example 32768.

First, Find the root of the first point as before, and subtract it therefrom, and to the remainder bring down the next point of figures, and call it resolvend, or dividend, which you please.

Secondly, Square the Root, and multiply it by 300 for a divisor; and, as in common division, see how many times it is contained in the dividend, and place it in the quotient, or root, accordingly.

Thirdly, Multiply the divisor by the last figure of the root, and place it under the dividend (units under units) drawing a line between them.

Fourthly, Square the last quotient figure, and multiply it by the first quotient figure, and that product multiply by 30, and set this under the last work, units under units, &c.

Fifthly, Cube the last figure and put the units of this under the units of the last, and add these three together in order, as they stand, which is the subtrahend; which if it be more than the resolvend, or dividend, you must put a less figure in the quotient; and proceed as before; but if it be less than the dividend, subtract it therefrom, and the work is done, for two figures in the root: But if there be more figures, bring them down to the remainder, and call it a new dividend; and square the whole root, and multiply by 300, for a new divisor, and put the figures in the quotient. Then square this last figure, and multiply it by the foregoing figures in the root, and then by 30. *Lastly*, Cube the last figure, and place it as before directed, and the work is done, for three places. The same to be observed for more figures.

Thus

Thus, the last example 32768(32 root. *Ans.*

27

Divisor 2700) 5768 dividend

5400

360

8

5768 subtrahend.

Tyro. I understand it very well, and think this last way the easiest.

Philo. Take your choice, as I observed before; I shall now only set you one sum at large, the first way, which you may prove by the second.

2. What

2. What is the cube root of 12812904? *Ans.* 234.

12812904 (234 root. *Ans.*
8

4812 resolvend

6 triple quotient
12 triple square

126 divisor

27 cube of 3
54 square of 3. by T. quotient
36 triple square by the root 3

4167 subtrahend

645904 new resolvend

69 triple quotient
1587 triple square

15939 new divisor

64 cube of 4
1104 square of 4 by T. quotient
6348 triple square by the root 4

645904 new subtrahend = new resolvend.

This you may easily prove the other way at leisure.
Also $234 \times 234 \times 234 = 12812904$.

QUESTIONS for Practice.

3. What is the cube root of 5396209064? *Ans.* 1754.
3. 1 de.

4. I demand the cube root of 9423479350146861 ?
Ans. 211221.

To extract the cube root of any vulgar or decimal fractions.

5. What is the cube root of $\frac{27}{1331}$? *Ans.* $\frac{3}{11}$.

6. What is the cube root of $\frac{2197}{9261}$? *Ans.* $\frac{13}{21}$.

Extract the cube root of the numerator and denominator for a new numerator and denominator.

7. Extract the cube root of the 32.768. *Ans.* 3.2.
 Proceed as in whole numbers, only prick off as many decimals in the root as you have dots over the decimals.

Note, As in the square root you add either 2, 4, 6, &c. cyphers to the decimal, so here you must add by three's that is, you must add either 3, 6, or 9, &c. cyphers.

8. What is the cube root of .002 ? *Ans.* .1259,
 and 4383021 remains.

SECTION II.

The Use of the CUBE ROOT.

9. **T**HERE is a cube whose solidity is 1372 feet,
 I demand the side of a cube, whose solidity is
 4 times less ? *Ans.* 7.

Divide 1372 by 4, and extract the cube root.

10. If a bullet 2 inches diameter weighs 3 lb. what
 will one of the same metal weigh which is 8 inches
 diameter ? *Ans.* 192 lb.

Solids being in the triple proportion to their sides or diameter, it is thus found.

As the cube of the given diameter is to the weight, so is the cube of the other diameter to the weight of the other required.

12. Suppose a shot of 4 inches diameter weighs 18 lb. I demand the diameter of another that weighs 144 lb. *Ans.* 8 inches. This is the reverse of the last question.

13. There is a sphere, or globe, whose solid content is 250047 inches: I demand the side of a cube, whose solidity shall be equal to the solidity of the globe: *Ans.* 63 inches.

14. A country farmer lent his neighbour out of his hay stack 20 feet of hay in length, breadth, and depth, and his neighbour brought it home 10 feet at one time, and 10 at another: how is the balance, and who debtor? *Ans.* 6000 feet due to him that lent it; he having received but $\frac{1}{4}$ th.

15. Suppose a ship 300 tons burthen, 75 feet by the keel, $29\frac{1}{2}$ feet by the beam, and 14 feet deep in the hold; I demand the dimensions of another ship of the same make, of 500 tons burthen?

Say, as 300 ton, is to 500 ton, so is the cube of the given keel to the cube of the ship's keel required, the cube root of which is 88.9 feet *Ans.*

And thus for the other two dimensions which I leave for your practice.

And now, *Tyro*, before I leave you, I will give you a little hint of measuring, gauging, &c. which may possibly be of service to you, and your acquaintance. You must expect me to be very short; but you may, by your care and diligence, make a better progress.

POST.

P O S T C R I P T.

D I A L O G U E XVII.

AS this little treatise may fall into the hands of such persons in the country, who would be glad of an opportunity of having a notion of measuring a piece of timber, a brick wall, a cistern of malt, or a common regular field, or piece of land, &c. I have (on purpose for their amusement, and instruction of those youths that have a fancy this way) added this postscript, which I make no doubt, will be very acceptable to all such as delight to be industriously employed at leisure times: and I persuade myself, it must be very agreeable to a parent, in either of these ways of life, to see his son diligent and ready at these things; which though he may not measure so exact for want of more learning, proper instruments or experience; yet may come near enough the truth to give satisfaction.

Of Flooring, Roofing, &c.

Quest. I. How many clinkers, 6 inches long, and 3 inches wide, will floor a stable 17 feet long, and 9 feet wide? *Ans.* 1224.

Multiply the length of the stable by the breadth, gives 153 feet; this multiply by 144, the square inches in a square foot, gives 22032 inches; this divide by 18, the inches in 1 clinker, gives 1224 *Ans.*

Quest.

Quest. 2. How many oaken planks will floor a barn 60 feet $\frac{1}{2}$ long, and 33 feet $\frac{1}{2}$ wide; when the planks are 15 feet long, and 15 inches wide? *Ans.* 108.

Multiply $60.5 \times 33.5 = 2026.75$. Then $15 \text{ feet} \times 1.25 \text{ feet}$ (viz. 15 inches) gives 18.75 feet for 1 plank; now $2026.75 \div 18.75$, gives 108 planks.

Quest. 3. A thatcher thatches a barn 60 feet long, and 25 feet wide, and the two porches are each 15 feet long, and 10 feet deep, I demand how many squares are contained in it? *Ans.* 33 squares. *N. B.* 100 feet is 1 square.

Multiply 60×25 gives 1500 feet for 1 side, which doubled, gives 3000 for both the sides; then the porch, viz. 15×10 , gives 150 for 1 side, which doubled, gives 300, which added to 3000 is 3300, which divided by 100 (that is, cutting off 2 figures) gives 33 squares.

2. Of Paving, Painting, Wainscotting, &c.

Tyro. How is paving, painting, and wainscotting measured?

Philo. By the square yard; 9 square feet being 1 yard.

Quest. 4. A gentleman has a walk 22 yards long, and 12 feet wide, which is paved of stone; how many yards does it contain? *Ans.* 88 yards.

First, multiply 22 yards, viz. 66 feet by 12 gives 792, which divide by 9, gives 88 yards.

Quest. 5. There is a room 64 feet round, and 9 feet high, in which are two windows, each 6 feet high, and 3 feet wide, and the fire-place contains 9 square feet; I demand how many yards of paper, half yard wide will hang it? *Ans.* 118 yards.

First, $64 \times 9 = 576$ yards, the content out of which take 18 feet each window, viz. 36 feet, and 9 the fire-place, is 45; and the remainder is 531 feet which divide by 9, gives 59 yards, the content of the room; but as the paper is $\frac{1}{2}$ yard wide only, it will take double this number, viz. 118 yards. *Ans.*

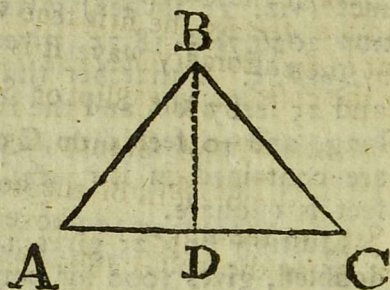
Painting,

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Painting, wainscoting, &c. are done by the yard square, and measured after this manner.

To measure the Peak End of a House or any Triangle

Quest. 6. Let ABC be the peak end of a roof, whose base AC measures 24 feet, and the perpendicular line BD from the top of the peak 16, I demand how many square yards it contains?



Multiply $\frac{1}{2}$ the perpendicular $B. D.$ by the whole base, or line $A. C.$; or else, multiply the whole perpendicular $B. D.$ by $\frac{1}{2}$ the base $A. C.$ viz. $A. D.$ or $C. D.$ gives the content in feet, which divide by 100, gives the squares, or by 9, gives square yards.

Thus $A. C.$ 24 feet, multiplied by $\frac{1}{2} B. D.$ 16 feet, (viz. 8 feet) gives 192 feet, viz. 1 square, 92 feet: or divide by 9, gives $21 \frac{2}{3}$ square yards of plaistering, and thus for any other triangle.

3. Of Board and Timber-measure

1. If the board be regular, multiply the length in inches, by the breadth in inches, and divide by 144, gives the answer, or, multiply the length in feet by the breadth in inches, and divide by 12 gives the answer.

Quest. 7. There is a board 9 feet long, and 10 inches wide; how many feet does it contain? *Ans.* $7 \frac{1}{2}$ feet.

If the board be wider at one end than another, the true way is, to multiply the ends together, and extract the square root for a mean breadth. But custom has brought up an easier way, though not so true, viz. add the breadth of both ends together, and take the $\frac{1}{2}$ for

a mean breadth, then multiply the length by this mean breadth into inches (as before directed) and divide by 144.

Quest. 8. There is a board 16 inches at one end, and 8 at the other, and 10 feet long; what is the content? *Ans.* By the first true way it is very near $9\frac{1}{2}$ feet? by the customary way, it is 10 feet.

By the Slip or Sliding R U L E.

The seventh QUESTION proved..

Set the breadth of the board, viz. 10 inches, on the slip, to the upper 12 above on the rule; then against 9 feet on the rule, you have $7\frac{1}{2}$ feet on the slip, the content required.

Again, suppose a board be 14 feet long, and 15 inches wide, what is the content? Set 15 on the slip, against the upper 12, then, again 14 feet on the rule, you have $17\frac{1}{2}$ feet on the slip, answer.

Tyro. I understand you, Sir; but pray how do you measure timber?

Philo. The customary way is this: with a small string or cord, take the circumference of the tree (which is done in any place, where the buyer and seller can agree) then double this string into 4 parts, and apply it to your rule, and that length is called the girt, or $\frac{1}{4}$ part of the circumference; and it is customary to abate one inch of the girt on account of the bark.

2. Having got the girt, multiply it by itself, that is, square it, and multiply that product by the length of the tree in feet, and divide by 144, gives the content; or multiply it by the length in inches, and divide by 1728, gives the content.

Note, Few persons mind less than $\frac{1}{2}$ a foot in the length of a tree, except it be very large.

Quest. 9. There is a tree 14 inches girt, and 9 feet long, I demand the content? *Ans.* $12\frac{1}{4}$ feet.

First, 14 multiplied by 14, is 196, this $\times 9$, the length, = 1764, which divided by 144, gives 12 feet, 36 inches, which is $\frac{3}{4}$ of 144, viz. $12\frac{1}{4}$ feet content.

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Quest. 10. There is a tree $10\frac{1}{2}$ inches girt, and 12 feet $\frac{1}{2}$ long, I demand the content? *Ans.* $9\frac{1}{2}$ feet. For $10.5 \times 10.5 = 110.25 \times 12.5 = 1376.125 \div 144 = 9.57$ feet or 9 feet, 82 inches. *Ans.*

By the Slip, or Sliding R U L E.

The Ninth QUESTION proved.

Set the length of the tree on the slip (viz. 9 feet) against 12 in the middle of the rule (wrote girt line) then against the girt (viz. 14) on the girt line, you have $12\frac{1}{2}$ feet on the slip.

The Tenth QUESTION proved.

Set $12\frac{1}{2}$, the length, against 12, the girt line; then again $10\frac{1}{2}$, you have better than $9\frac{1}{2}$ on the slip itself, viz. 9 feet, 82 inches.

Of tapering T I M B E R.

Some persons will take but one girt, though a tree be very long and tapering; but this is certainly very wrong, as it may do injustice to either the buyer or seller. The best way is, to measure such a tree, as if it were two or three distinct trees, by taking two or three several lengths and girts.

Some, indeed, take 2 girts, one at the great, and the other at the small end, and add them together, and take the $\frac{1}{2}$ of it for a mean girt (as in board measure) but this is a hurt to the buyer, and very erroneous; whereas they should multiply one girt by the other, and extract the square root for a mean girt.

Quest. 11. Suppose a tree 20 inches girt, at one end, and 40 at the other, and 9 feet long, I demand the content?

By the customary way the mean girt will be 30 inches, and the content will be 56 feet, 36 inches $= \frac{1}{4}$ of another foot. But, according to the true way the mean girt is but 28.28, and the content but 49.98 feet, viz. 49 feet, 141 inches, which is 6 feet 39 inches less than the other.

Tyro. This is a sensible difference, indeed, in many loads of timber.

Philo. Very true.

Note

Note 1. In some countries, 40 feet make a load, and in others, 50 feet make a load.

Note 2. When there are 50 feet to the load, then to cast up the content, at any given price, the rule is, multiply the content or number of feet by the price in shillings, and cut off the three first figures, from the right to the left-hand: so will the figures towards the left-hand be pounds sterling, and the other will be decimal parts of a £.

EXAMPLE.

Quest. 12. Suppose I measured 6 trees, and their content be 548 feet, at 1 l. 10s. per load?

I multiply 548 by 30, and it gives 16440, which I cut off thus, 16|440, and it is £ 16.440 viz. 16l. 8s. 9d. $\frac{1}{2}$.

N. B. Stone is measured the same, only observe 8 inches make 1 foot of stone.

4. Of BRICK - WORK.

Tyro. How is brick-work measured?

Philo. By the square rod, that is 16 feet $\frac{1}{2}$ in length, and 16 $\frac{1}{2}$ in breadth, make 272 $\frac{1}{4}$ feet, or one square rod; but for common practice, 272 feet only is sufficient.

Tyro. Is there not a certain standard for the thickness of brick-work?

Philo. Yes, all sorts of brick-work is reduced to the standard of 1 $\frac{1}{2}$ brick thick, of which I shall give you a farther notion by and by.

1. Of work at 1 $\frac{1}{2}$ brick thick the standard.

Multiply the length by the height in feet, and divide by 272, the quotient gives the square rods, and the remainder, the feet or parts of a rod.

Quest. 13. A Gentleman built a brick wall round his garden, which was 998 feet long, 9 feet high, and 1 $\frac{1}{2}$ brick in thickness: I demand how many rods it contains? *Ans.* 33 rods, 6 feet.

Here I multiply 998, the length by 9, the height, and it gives 8982 feet, which I divide by 272, (the feet in a rod) and it gives 33 rods, 6 feet. *Ans.*

Tyro. I understand it; this is easy enough: but suppose it was but 1 brick thick, or suppose it were 2, or 3 bricks thick, how then?

Philo. Having found the content at $1\frac{1}{2}$ brick thick, as before directed, say thus:

As 3 (the $\frac{1}{2}$ bricks in the standard-measure) is to the content in standard measure, at $1\frac{1}{2}$ brick thick, so is the number of half bricks in the wall to the content at that thickness.

Quest. 14. Let the wall be 998 feet round, and 9 feet high, as before, what is the content, at $2\frac{1}{2}$ bricks thick?

The content, at $1\frac{1}{2}$ thick, was found in the last question to be 33 rods, 6 feet. Say therefore,

As 3 to 33.6, so is 5 half bricks, viz. the thickness, at $2\frac{1}{2}$ bricks thick, to the content at that thickness, viz. 55 rods, 10 feet.

Tyro. I understand you well; but cannot any thickness be done at one operation?

Philo. Yes; for having multiplied the length by the height, divide by any of the following numbers, that are set against the given thickness, and you have the content in rods at once, and the remainder is feet.

Note. Though there be decimals in the divisors, you may divide by the whole numbers for common use.

For 1		[408.3]	<i>Quest.</i> 15. There is a
$1\frac{1}{2}$		[272.25]	wall 15.5 feet long, and
2	Brick thick divide by	[204.2]	9.5 feet high; what is the
$2\frac{1}{2}$		[163.3]	content, at $3\frac{1}{2}$ bricks
3		[136.12]	thick? <i>Ans.</i> 1 rod, 2
$3\frac{1}{2}$		[116.6]	tenths: for I multiply
4		[102.1]	$15.5 \times 9.5 = 147.25$, which
$4\frac{1}{2}$		[90.7]	divided by 116.6, the di-
5		[81.7]	visor for $3\frac{1}{2}$ bricks, gives
			1 rod, 2 tenths. And
			thus for any thickness; for at $4\frac{1}{2}$ thick, it is 1.5 rod,
			viz. $1\frac{1}{2}$

By

By the Slip or Sliding RULE.

There is a wall 9 feet high, and 76 feet long, and $1\frac{1}{2}$ brick thick; I demand the content? *Ans.* 2 rods, 140 feet, or better than $2\frac{1}{2}$ rods.

Set 272 on the slip to the height 9 above it; then again 76 the length on the slip, is $2\frac{1}{2}$ or better, on the rule.

A RULE for any Thickness.

Set any of the former divisors, answering any thickness on the slip, to the height; then against the length is the answer. Thus, the same wall at 3 bricks thick.

Set 136 to 9, then again 76 you have 5 rods, the content, at 3 bricks thick.

5. Of SURVEYING.

Tyro. How is land surveyed?

Philo. Land-Measure is a part of the mathematics, and to survey it true, and in a masterly manner, you should be provided, 1. With a chain called Gunter's chain. 2. A case of instruments. 3. A parallel ruler. 4. A plain table. 5. A platting-scale, or protractor. And to make it more complete, a theodolite.

Tyro. But cannot I measure a common regular field, or little piece of ground, without all these instruments?

Philo. Yes; by a chain only, or for want of that, a cord, a rod-pole, or any such thing; but this must not be depended upon for truth.

Tyro. Give me a description of the chain.

Philo. All land is now generally measured by a chain, containing 4 rods, or poles in length, (viz. 22 yards) according to a statute made in the 33d of Edward I. Anno 1235, which says, that a square acre shall contain 160 rods, viz. 40 rods in length, and 4 rods in breadth, make 160 rods, or 1 acre of ground.

Note. The chain is made of iron, containing 100 links, each in length is 7.92 inches, (or nearly 8 inches) 100 of which are 792 inches, or 22 yard, (viz. 4 rods) therefore, 1 chain in length, and 10 in breadth, or 10 in length and 1 in breadth, make an acre.

Note 2. For want of a chain you may take a cord 22 yards, or 4 rods long, or any number of rods long you please, dividing it into halves and quarters, with which you may measure any common field within a trifle of truth, or, at least, for common satisfaction.

Having provided yourself with a chain, or any convenient line, if the field or piece of ground, be regular, viz. a square, or the opposite sides alike; then measure the length and the breadth, in rods, or parts, and multiply the length by the breadth, and divide the product by 160 the rods in an acre, you have the content.

Quest. 5. There is a field in the form of a long square (called a parallelogram) whose length is 35 rods, and breadth 24 rods, I demand the content in acres? *Ans.* 5 acres 1 rood.

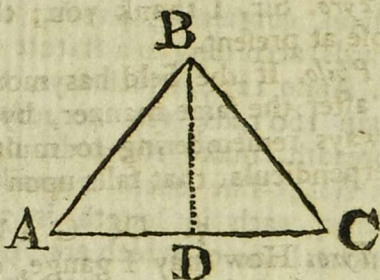
First, I multiply 35, the length, by 24 the breadth, and it gives 840 rods; which I divide by 160 gives 5 acres, and 40 remain, which I multiply by 4 (because 4 rods make 1 acre) and divide again by 160, gives 1 rood.

Quest. 16. There is a square piece of ground set out upon a high, or common, in order to form a camp for 1000 soldiers, each side contains 60 rods; how many acres does it contain? *Ans.* $22\frac{1}{2}$ acres.

For $60 \times 60 = 3600$, which divide by 160 = 22 acres, 80 rods, or $22\frac{1}{2}$ acres.

Quest. 17. There is a 3 sided or triangular field, A, B, C, the side A, C, is 51.5 rods, and the perpendicular B, D, is 34 rods; how many acres does it contain? *Answer* $5\frac{1}{2}$ acres nearly.

Note, You must first of all measure the side from A to C, called the base, which suppose $51\frac{1}{2}$ rods; then measure half way from A to C, and from D measure straight up to the point B, which is called the perpendicular, which

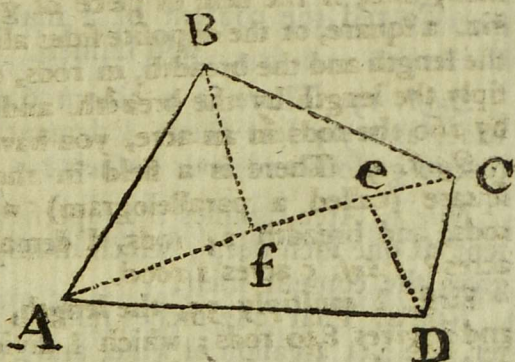


suppose to be 34 rods: now, I told you before, that the base multiplied by $\frac{1}{2}$ the perpendicular gives the content

content; that is, 51.5 multiplied by $\frac{1}{2}$ the perpendicular 17, gives 875.5 which divided by 160, gives 5.47 acres; that is, very near $5\frac{1}{2}$ acres.

To measure any four sided field, whose sides are unequal, called a Trapezium.

Quest. 18. There is a trapezium, or four sided field or piece of ground A, B, C, D, whose base AC is 64 rods, and the perpendicular Bf is 60, and the other perpendicular De is 40: I demand the content in acres? *Ans.* 20 acres.



First, to measure this field, go straight cross it from the corner A to the corner C, which here is called the base, and measures 64 rods: then measure right straight from the point B to f, which is 60 rods, and right straight from D to e, which is 40 rods. This done, the rule is,

Multiply the whole base A C 64 by $\frac{1}{2}$ B f 60, (viz. 30) and it gives 1920 rods, the content of the triangle A B C: then again multiply the base A C by $\frac{1}{2}$ D e 40 (viz. 20) and it gives 1280 rods, the content of the triangle A C D. Add those 2 together, viz. 1920, and 1280 rods, gives 3200 rods, which divide by 160, the rods in an acre, gives 20 for the answer.

Tyro. Sir, I thank you; this is enough for my purpose at present.

Philo. If the field has more sides, you may measure it after the same manner, by dividing it into triangles, always remembering to multiply the base by $\frac{1}{2}$ every perpendicular that falls upon it.

5. OF GAUGING.

Tyro. How may I gauge, or tell the content of any common cooler, or regular cask, or cistern, in gallons, or bushels.

Philo.

Phil. I shall give you some short instructions, by which you may tell the content of several things near enough truth, for your own satisfaction: but to be a practical gauger, you ought to understand several branches of the mathematics.

1. To tell the content of a malt cistern in gallons and bushels.

Quest. 19. There is a cistern 6.5 feet long, 4 feet wide, and 3.5 feet deep: I demand its area, and content in gallons and malt bushels?

Note 1. Area signifies the superficial content, or content at one inch deep; which multiplied by the depth, gives the content itself.

Note 2. That 282 inches make 1 gallon of ale, water, &c. 231 a gallon of wine, and 2150 inches 1 bushel, which are your divisors for all regular figures.

R U L E.

Multiply the length, 78 inches, by the breadth, 48, and it gives 3744 inches, which divide by 282, gives 13.276 the area, at 1 inch deep, or divide by 2150 gives 1.741, the area in bushels. The area multiplied by the depth, 42 inches, gives 557.592 gallons. The area for malt multiplied by 42, gives 73.122, the content in bushels.

Note, If the area be not required, or you do not understand decimals, you may more easily find the content at once, thus: multiply the length, breadth, and depth, in inches together, gives 157248, which divide by 282, gives 557 gallons $\frac{1}{2}$, 157248 divided by 2150, gives 73 bushels $\frac{1}{10}$ as before.

To find the area by the sliding rule.

Set 282 upon B to 48, the breadth on A; then against 78 the length on B, is 13.277, the area in gallons.

For malt, set 2150 on B to 48 on A; then against 78 the length on B, is 1.741 on A; the area in bushels: and thus for any regular figure.

To

To gauge a tub or cooler in the form of a cylinder, viz. whose top and bottom diameters are equal.

R U L E.

Square the diameter, viz. multiply it by itself, and this product by the depth, then divide by 359 for beer gallons, 294 for wine, and 2737, (or rather by 2737.47) for malt.

Quest. 20. There is a tub 4 feet 2 inches diameter, and 3 feet 4 inches deep: I demand the content in beer, wine, and malt?

I square the diameter 50, which is 2500, and multiply this by the depth, 40 inches, gives 100000; which divide by 359, gives 278 ale gallons; divide by 294, gives 340 wine gallons, and by 2737 only, gives 365 malt bushels.

Note. If the figure be the form of a triangle, or trapezium (as in *questions 17 and 18*) you must proceed to measure them as there directed, and after having multiplied by the depth, divide 282 for beer, 231 for wine, and 2150 for malt, gives the content.

Quest. 21. There is a tub, whose top diameter is 40 inches, bottom 30 inches, and the depth 60 inches: I demand the content in beer, wine, and malt?

There are several ways to do this. One is this, multiply the diameters together, and extract the square root for a mean diameter, which is here 34.64. This multiplied by itself, and divided by 359, gives the content in gallons, or by 2737, gives the malt bushels.

Or more easily thus, though not so true.

Add the diameters together, and take the $\frac{1}{2}$ for a mean, is 35. Now $35 \times 35 = 1225 \times 60 = 73500$, which divided by 359, gives about 205 gallons; and so for malt, viz. gives 26 bushels $\frac{8}{16}$.

For a couch of malt.

1. If it be a regular square only, multiply the length, breadth, and depth together, and divide by 2150, gives the bushels.

2. If it be a triangle, or trapezium, proceed as before directed, and divide still by 2150.

Tyro. But suppose the couch be uneven, how shall I tell where to take the depth?

Phila.

Philo. Take the depth at 4 or 5 places, add them all together, and divide by the number of places you took the depth at, for a mean depth.

Quest. 22. There is a bed or couch of malt, in the form of a long square, whose length is 35 feet, breadth 16 feet, and I find the mean depth to be 8.5 inches, viz. $8\frac{1}{2}$ inches; I demand the content?

Thus, $420 \text{ inches} \times 192 = 80640 + 8.5 = 685440$; this divided by 2150, gives 318.8 bushels.

Of cask gauging.

There is a great variety in gauging casks; but the following methods will be near enough truth for all common casks; such as barrels, butts, &c. that are pretty much bulged.

First, Having taken the bung and head diameters, the rule is, to the sum and $\frac{1}{2}$ the sum of the squares of the bung, and head diameters add $\frac{1}{2}$ the difference of the said squares: this sum multiply by the length, and divide by 1077 for beer, or 882 for wine gallons.

2. *Rule* which is as true, and much easier.

To the double square of the bung diameter add the square of the head diameter; then multiply this sum by the length of the cask, and divide by 1077 for beer, or 882 for wine.

Quest. 23. There is a cask, whose bung diameter is 28 inches, head diameter 25 inches, length 36: I demand the content in ale gallons?

First, The square of the bung diameter 28 is 784; which doubled is 1568. Then the square of the head, viz. $25 \times 25 = 625$, which added to 1568, is 2193; this $\times 36$, the length is 78948, which divided by 1077, gives 73 gallons, 2 pints for beer, and divided by 882, gives 89 gallons, and $\frac{1}{2}$ wine or brandy.

Note, If you find the area of the bung, and head diameters (by *question* 20,) and add twice the area of the bung, viz. 2.184 to the area of the head 1.741, it is 6.109. which multiplied by $\frac{1}{3}$ of the cask's length, viz. 12, gives 73.308 gallons as before.

These methods holding good for most casks, I shall give no more examples.

Note,

Note 2. If one of the head diameters be larger than the other, and the cask is straight in the sides, like some churns, then (by *question 21*) find a mean diameter throughout, and proceed as therein directed.

6. Of CROSS MULTIPLICATION.

There are two methods.

1. By multiplication only.

1. *Rule.* Feet multiplied by feet produce feet.
2. Inches multiplied by feet, or feet by inches, produce inches.
3. Inches multiplied by inches produce parts.

Note, 12 seconds make 1 part, 12 parts make 1 inch, and 12 inches 1 foot.

2. By multiplication and division.

Rule 2. Having placed the lesser sum for the multiplier, multiply the very last place of the multiplicand towards the right hand by the first place or name of the multiplier, and carry one for every 12, setting down what is over 12 under the part you multiplied, then take the parts of the multiplier as in practice, carrying as before 1 for every 12.

But an example will render it more easy, if I give it both ways.

	Feet	In.	Part:		Feet	In.
Multiply	4	3	6	by	4	3
by	4	3				

	17	2	-
	1	-	9
		1	6

Ansf.	18	2	10	6
	F.	In.	P.	Sec.

First, I begin and multiply the top 4 feet, 3 inches, and 6 parts, by 4 feet, (carrying 1 for every 12) saying 4 times 6 is 24 parts, that is, 0 and carry 2; then 4 times 3 is 12, and 2 I carried is 14 inches, that is 2 inches, and carry 1 to the feet; then 4 times 4 is 16 feet, and 1 is 17.

Secondly

Secondly, I multiply now 4 feet, 3 inches, 6 parts, by the lower 3 inches, saying 3 times 4 is 12 inches (because feet multiplied by inches are inches) then 3 times 3 inches is 9 parts (for inches by inches produce parts) and lastly, 3 times 6 parts is 18 parts, viz. 1 part, 6 seconds. The second method.

I first multiply the first or top line as before, and find it as before, 17-2-0; and now I take the parts as in practice; saying, 3 inches is $\frac{1}{4}$ of a foot, &c. See the work.

Feet In. P.

4 3 6

4 3

In.		17	2	-
3	$\frac{1}{4}$	1	-	10 6

18 2 10 6 as before.

I say 3 inches is $\frac{1}{4}$ of a foot, and take the parts from 4 feet, 3 inches, 6 parts, saying, the 4th of 4 is 1 foot; then the 4th of 3 inches 6 parts, (viz. 42 parts) is 10 times (4 is 40) and 2 parts over; lastly, I say the 4th of 2 parts (viz. 24 seconds) is 6 seconds; which is now done in 2 lines only.

And now, *Tyro*, I must bid you farewell, and I hope you will take care to improve yourself in them, rather than trifling away your time with idle fancies: for it is evident, that the knowledge of arithmetic is necessary in every station of life, since, almost all manner of business depends upon it: and not only this, *Tyro*, but it is a great help to protect us against the frowns of fortune, and keep us (by being qualified for some lawful post, or employment) from those common temptations and misfortunes, to which those, who know the want of it, so often fall into, and pay for so dearly.

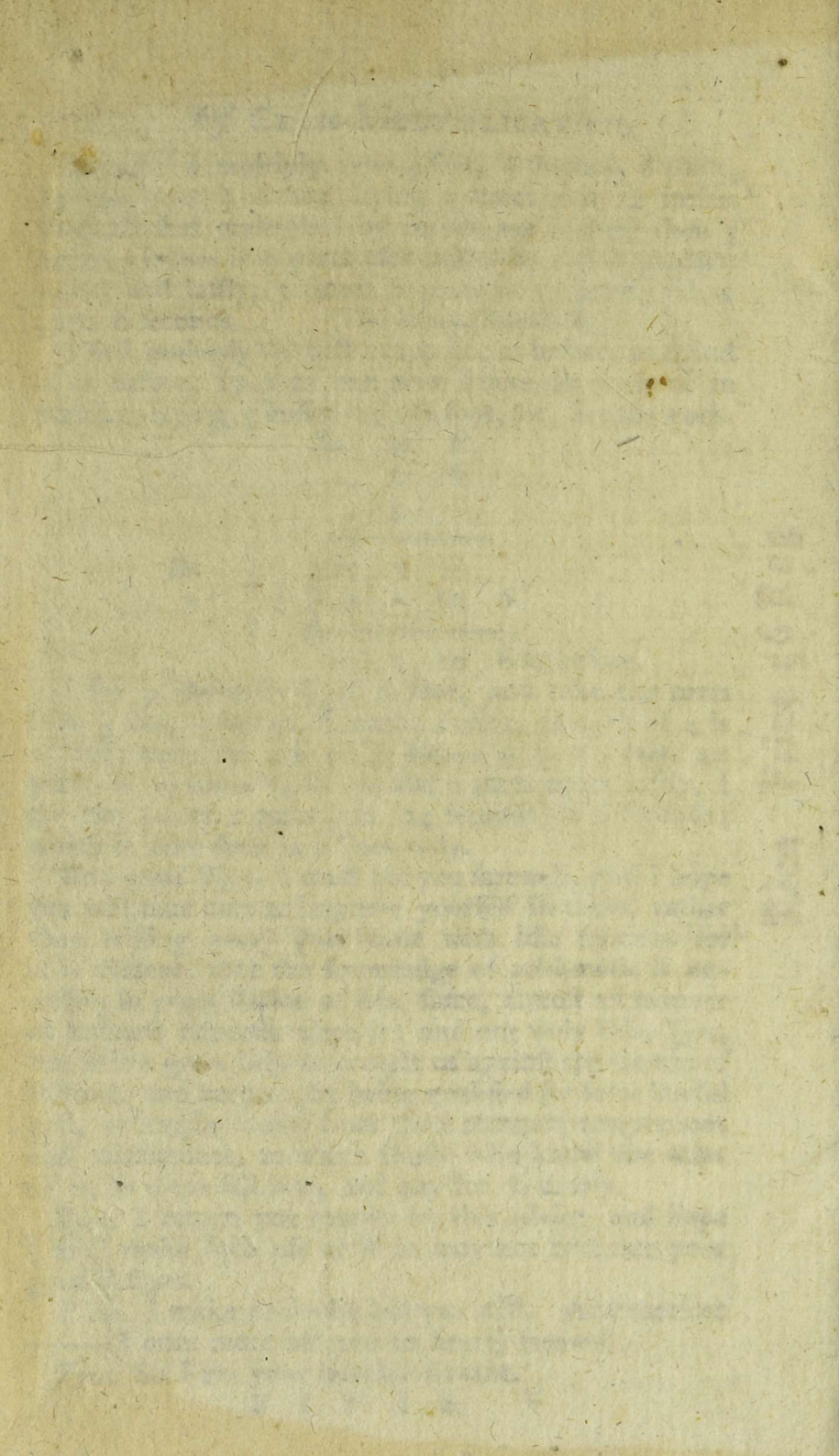
Tyro. I return you thanks for this advice, and hope I shall make such use of it as may not frustrate your good designs.

Philo. I make no doubt but you will. And therefore I once more bid you an hearty farewell.

Tyro. Sir I am your humble servant.

F I N I S.

Price 5 6 Res



BT 7/66

