Series of National School Books.

A TREATISE

ON

MENSURATION,

FOR THE USE OF SCHOOLS.

AUTHORISED BY THE COUNCIL OF PUBLIC INSTRUCTION FOR UPPER CANADA.

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PREFACE TO SECOND EDITION.

To this Edition there is an Appendix printed in a separate form, for the use of teachers, containing the leading properties of the Conic Sections, and the Demonstrations of the Rules of Mensuration. These were in the First Edition, interspersed through the work, partly interwoven with the text, and partly in the shape of notes. It is hoped that the present arrangement will better suit the convenience of both teachers and pupils. Several other alterations have been made, which, it is hoped, will be found to be improvements.

Teachers should direct their pupils to learn only such portions of the work as may be necessary for their intended occupations : for most pupils, the first and second sections, and a few problems in the fourth and sixth will be quite sufficient.

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MENSURATION.

SECTION I.

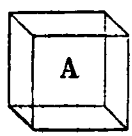
PRACTICAL GEOMETRY.

DEFINITIONS.

1. Geometry teaches and demonstrates the properties of all kinds of magnitude or extension; as solids, surfaces, lines, and angles.

2. Geometry is divided into two parts, theoretical and practical. Theoretical Geometry treats of the various properties of extension abstractedly; and Practical Geometry applies these theoretical properties to the various purposes of life. When length and breadth only are considered, the science which treats of them is called Plane Geometry; but when length, breadth, and thickness are considered, the science which treats of them is called Solid Geometry.

3. A Solid is a figure, or a body, having three dimensions, viz., length breath, and thickness; as A.

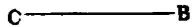


The boundaries of a solid are surface or superficies.

4. A *Superficies*, or surface, has length and breadth only; as B.

The boundaries of a superficies are lines.

5. A *Line* is length without breadth, and is formed by the motion of a point; as C B.

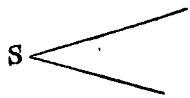


The extremities of a line are points.

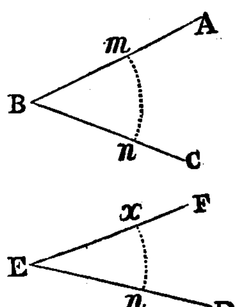
6. A *Straight or Right Line* is the shortest distance between two points, and lies evenly between these two points.

7. A *Point* is that which has no parts or magnitude; it is indivisible; it has no length, breadth or thickness. If it had length, it would then be a line; were it possessed of length and breadth, it would be a superficies; and had it length, breadth, and thickness, it would be a solid. Hence a point is void of length, breadth, and thickness, and only marks the position of their origin or termination in every instance, or of the direction of a line.

8. A *Plane rectilineal Angle* is the inclination of two right lines, which meet in a point, but are not in the same direction; as S.

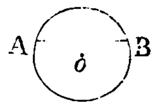


9. One angle is said to be less than another, when the lines which form that angle are nearer to each other than those which form the other, measuring at equal distances from the points in which the lines meet. Take Bn Bm, Ex, and En, equal to one another; then if $m\ n$ be greater than $x\ n$, the angle ABC is greater than the angle FED. By conceiving the point A to move towards C, till $m\ n$ becomes equal to $x\ n$, the angles at B and E would then be equal; or by conceiving the point F to recede from D, till $x\ n$ becomes equal to $m\ n$, then the angles at B and E would be equal.

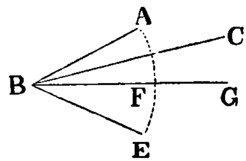


Hence it appears that the nearer the extremities of the lines forming an angle approach each other, while the point at which they meet remains fixed, the less the angle; and the farther the extreme points recede from each other, the vertical point remaining fixed, as before, the greater the angle.

10. A *Circle* is a plane figure contained by one line called the circumference, which is every where equally distant from a point within it, called its centre, as *o*; and an arc of a circle is any part of its circumference; as AB.



11. The magnitude of an angle does not consist in the length of the lines which form it: the angle CBG is less than the angle ABE, though the lines CB, GB are longer than AB, EB.

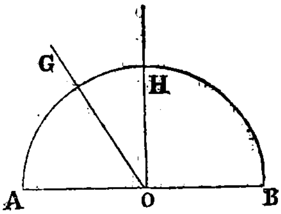


12. When an angle is expressed by three letters, as ABE, the middle letter always stands at the angular point, and the other two any where along the sides; thus the angle ABE is formed by AB and BE. The angle ABG by AB and GB.

13. In equal circles, angles have the same ratio to each other as the arcs on which they stand, (33. VI.) Hence also, in the same, or equal circles, the angles vary as the arcs on which they stand; and therefore the arcs may be assumed as proper measures of angles. Every angle then is measured by an arc of a circle, described about the angular point as a centre; thus the angle ABE is measured by the arc AE; the angle ABG by the arc AF.

14. The circumference of every circle is generally divided into 360 equal parts, called degrees; and every degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds. The angles are measured by the number of degrees contained in the arcs which subtend them, thus, if the arc AE contain 40 degrees, or the ninth part of the circumference, the angle ABE is said to measure 40 degrees.

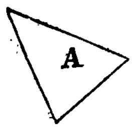
15. When a straight line HO, standing on another AB, makes the angle HOA equal to the angle HOB, each of these angles is called a right angle; and the line HO is said to be a perpendicular to AB. The measure of the angle HOA is 90 degrees, or the fourth part of 360 degrees. Hence a right angle is 90 degrees.



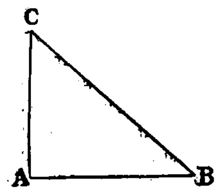
16. An acute angle is less than a right angle; as AOG, or GOH.

17. An obtuse angle is greater than a right angle; as GOB.

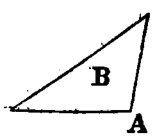
18. A *plane Triangle* is the space enclosed by three straight lines, and has three angles; as A.



19. A *right angled Triangle*, is that which has one of its angles right; as ABC. The side BC, opposite the right angle, is called the hypothenuse; the side AC is called the perpendicular; and the side AB is called the base.

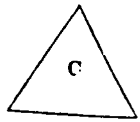


20. An *obtuse angled Triangle* has one of its angles obtuse; as the triangle B, which has the obtuse angle A.

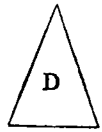


21. An *acute angled Triangle* has all its three angles acute, as in figure A, annexed to Definition 18.

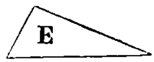
22. An *equilateral Triangle* has its three sides equal, and also its three angles; as C.



23. An *isosceles Triangle* is that which has two of its sides equal; as D.



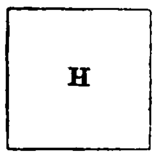
24. A *scalene Triangle* is that which has all its sides unequal; as E.



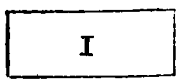
25. A *quodrilateral* figure is a space included by four staight lines. If its four angles be right, it is called a rectangular parallelogram.

26. A *Parallelogram* is a plane figure bounded by four straight lines, the opposite ones being parallel; that is, if produced ever so far, would never meet.

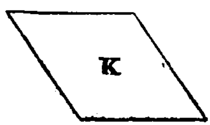
27. A *Square* is a four-sided figure, having all its sides equal, and all its angles right angles; as H.

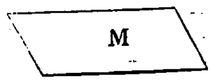


28. An *Oblong*, or rectangle, is a right-angled parallelogram, whose length exceeds its breadth; as I.

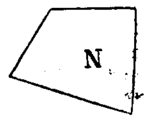


29. A *Rhombus* is a parallelogram having all its sides equal, but its angles not right angles; as K.





30. A *Rhomboid* is a parallelogram having its opposite sides equal, but its angles are not right angles, and its length exceeds its breadth; as M.



31. A *Trapezium* is a figure included by four straight lines, no two of which are parallel to each other; as N.

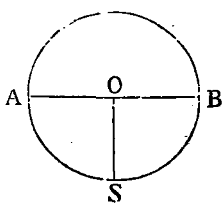
A line connecting any two of its opposite angles, is called a diagonal.

32. A *Trapezoid* is a four-sided figure having two of its opposite sides parallel; as F.

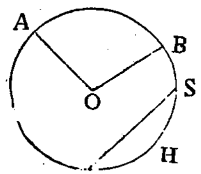
33. Multilateral Figures, or Polygons, are those which have more than four sides. They receive particular names from the number of their sides. Thus, a *Pentagon* has five sides; a *Hexagon* has six sides; a *Heptagon*, seven; an *Octagon*, eight; a *Nonagon*, nine; a *Decagon*, ten; an *Undecagon*, eleven; and a *Dodecagon* has twelve sides.

If all the sides of each figure be equal, it is called a regular polygon; but if unequal, an irregular polygon.

34. The *Diameter* of a circle is a straight line passing through the centre, and terminated both ways by the circumference; thus AB is the diameter of the circle. The diameter divides the circle into two equal parts, each of which is called a semicircle; the diameter also divides the circumference into two equal parts, each containing 180 degrees. Any line drawn from the centre to the circumference is called the radius, as AO, OB, or OS. If OS be drawn from the centre perpendicular to AB, it divides the semicircle into two equal parts. AOS and BOS, each of which is called a quadrant, or one fourth of the circle; and the arcs AS and BS contain each 90 degrees, and they are said to be the measure of the angles AOS and BOS.



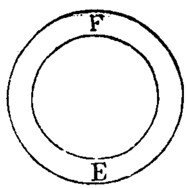
35. A *Sector* of a circle, is a part of the circle comprehended under two *Radii*, not forming one line, and the part of the circumference between them. From this definition it appears that a sector may be either greater or less than a semicircle; thus A O B is a sector, and is less than a semicircle; and the remaining part of the circle is a sector also, but is greater than a semicircle.



36. A *Chord* of an arc is a straight line joining its extremities, and is less than the diameter; T S is the chord of the arc T H S, or of the arc T A B S.

37. A *Segment* of a circle is that part of the circle contained between the chord and the circumference, and may be either greater or less than a semicircle; thus T S H T and T A B S T are segments, the latter being greater than a semicircle and the former less.

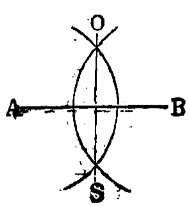
38. *Concentric Circles* are those having the same centre and the space included between their circumferences is called a ring; as F E.



PROBLEM I.

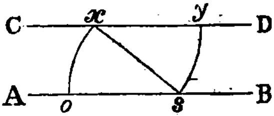
To bisect a given straight line A B; *that is, to divide it into two equal parts.*

From the centres A and B, with any radius, greater than half the given line A B, describe two arcs intersecting each other at O and S, then the line joining O S will bisect A B.



PROBLEM II.

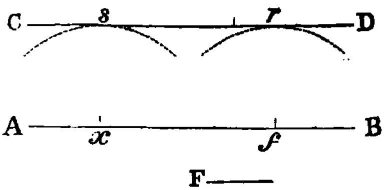
Through a given point x to draw a straight line C D *parallel to a given straight line* A B.



In A B take any point *s* and with the centre *s* and radius *s x* describe the arc *o x*; with *x* as a centre and the same radius *s x*, describe the arc *s y*. Lay the extent *o x* taken with the compasses from *s* to *y*; through *x y* draw C D, which will be parallel to A B.

PROBLEM III.

To draw a straight line C D *parallel to* A B *and at a given distance* F *from it.*

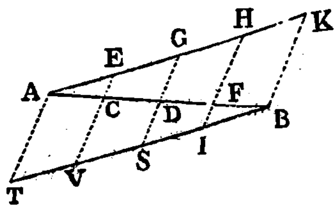


In A B take any two points *x f*; and from the two points as centres with the extent F taken with the compasses, describe two arcs, *s*, *r*; then draw a line C D touching these arcs at *r* and *s*, and it will be at the given distance from A B, and parallel to it.

PROBLEM IV.

To divide a straight tine A B *into any number of equal parts.*

Draw A K making any angle with A B; and through B draw B T parallel to A K; take any part A E and repeat it as often as there are parts to be in A B, and from the point B on the line B T, take B I, I S, S V, and V T equal to the parts taken on the line A K; then join A T, E V, G S, H I, and K B, which will divide the line A B into the number of equal parts required, as A C, C D, D F, F B.

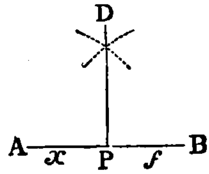


PROBLEM V.

From a given point P *in a straight line* A B *to erect a perpendicular.*

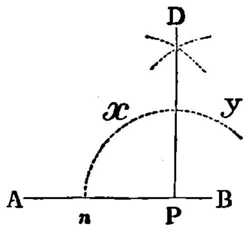
1. *When the given point is in, or near the middle of the line.*

On each side of the point P take equal portions P x, *P* f; and from the centres, x, f, with any radius greater than P x, describe two arcs, cutting each other at D; then the line joining D P will be perpendicular to A B.



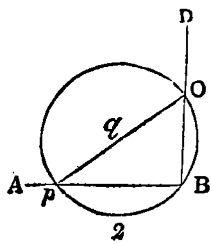
Or thus:

From the centre P, with any radius P n describe an arc n x y; set off the distance P n from n to x, and from x to y; then from the points x and y with the same or any other radius, describe two arcs intersecting each other at D; then the line joining the points D and P will be perpendicular to A B.



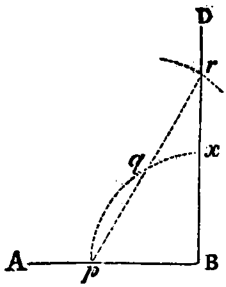
2. *When the point* P *is at the end of the line.*

From any centre q out of the line, and with the distance q B as radius, describe a circle, cutting A B in p draw p q O; and the line joining the points O, B, will be perpendicular to A B.



Or thus:

Set one leg of the compasses on B, and with any extent B *p* describe an arc *p x*; set off the same extent from *p* to *q*; join *p q*; from *q* as a centre, with the extent *p q* as a radius, describe an arc *r*; produce *p q* to *r*, and the line joining *r* B will be perpendicular to A B.

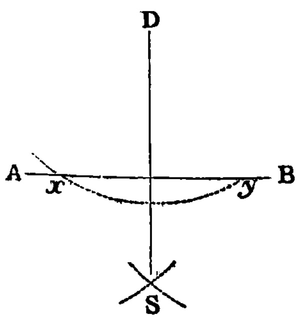


PROBLEM VI.

From a given point D *to let fall a perpendicular upon a given line* A B.

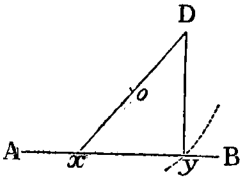
1. *When the point is nearly opposite the middle of the given line.*

From the centre D, with any radius, describe an arc *x y*, cutting A B in *x* and *y*, from *x* and *y* as centres, and with the same distance as radius, describe two arcs cutting each other at S; then the line joining D and S will be perpendicular to A B.



2. *When the point is nearly opposite the end of the given line, and when the given line cannot be conveniently produced.*

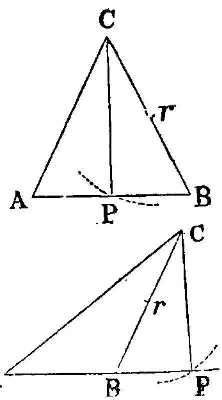
Draw any line D x, which bisect in o; from o as a centre with the radius $o\ x$ describe an arc cutting A B in y; then the line joining D y will be perpendicular to A B.



PROBLEM VII.

To draw a perpendicular from any angle of a triangle A B C, *to its opposite side.*

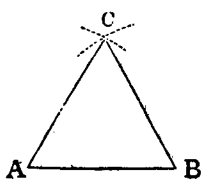
Bisect either of the sides containing the angle from which the perpendicular is to be drawn, as B C in the point r; then with the radius r C, and from the centre r, describe an arc cutting A B, (or A B produced if necessary, as in the second figure,) in the point P; the line joining C P will be perpendicular to A B, or to A B produced.



PROBLEM VIII.

Upon a given right line to A B *to describe an equilateral triangle.*

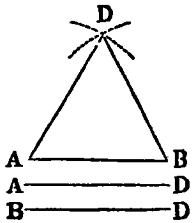
From the centres A and B, with the given line A B as radius, describe two arcs cutting each other at C; then the lines drawn from the point C to the points A and B will form, with the given line A B, an equilateral triangle, as A B C.



PROBLEM IX.

To make a triangle whose sides shall be equal to three given right lines A B, A D, *and* B D.

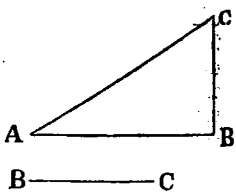
From the centre A with the extent A D, describe an arc, and from the centre B with the radius B D describe another arc cutting the former at D; then join D A, D B, and the sides of the triangle A B D will be respectively equal to the three given right lines.



PROBLEM X.

Two sides A B *and* B C *of a right-angled triangle being given, to find the hypothenuse.*

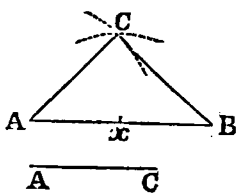
Place B C at right angles to A B; draw A C, and it will be the hypothenuse required.



PROBLEM XI.

The hypothenuse A B, *and one side* A C, *of a right angled triangle being given, to find the other side.*

Bisect A B in x; with the centre x, and x A as radius, describe an arc; and with A as a centre, and A C as radius, describe another arc cutting the former at C; then join A C and C B; and A B C will be a right-angled triangle, and B C the required side.

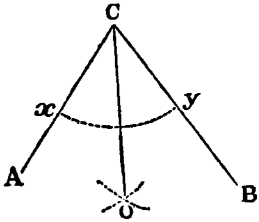


PROBLEM XII.

To bisect a given angle; that is, to divide it into two equal parts.

Let A C B be the angle to be bisected.

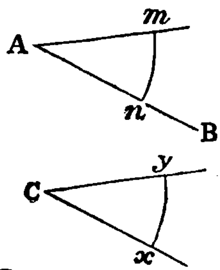
From C as a centre, with any radius C x, describe the arc x y; from the points x and y as centres, with the same radius, describe two arcs cutting each other at O; join O C, and it will bisect the angle A C B.



PROBLEM XIII.

At a given point A *in a given right line* A B *to make a angle equal to the given angle* C.

From the centre C with any radius C y, describe an arc x y; and from the centre A, with the same radius describe another arc, on which take the distance m n equal to x y; then a line drawn from A through m will make the angle m A n equal to the angle x *C* y.

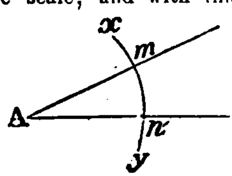


PROBLEM XIV.

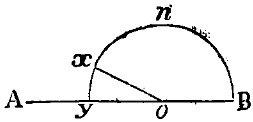
To make an angle containing any proposed number of degrees.

1. *When the required angle is less than a quadrant, as* 40 *degrees.*

Take in the compasses the extent of 60 degrees from the line of chords, marked cho. on the scale; and with this chord of 60 degrees as radius, and the centre A, describe an arc x y; take from the line of chords 40 degrees, which set off from n to m; from A draw a line through m; and the angle m A n will contain 40 degrees.



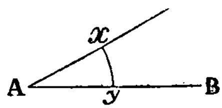
2. *When the required angle is greater than a quadrant, as 120 degrees.*



From the centre *o*, with the chord of 60 degrees as radius, describe the semicircle *y x n* B; set off the chord of 90 degrees, from B to *n*, and the remaining 30 degrees from *n* to *x*; join *o x*; and the angle B *o x* will contain 120 degrees; or subtract 120 from 180 degrees, and set off the remainder (60 degrees) taken from the line of chords from *y* to *x*; then join *x o*, and B *o x* will contain 120 degrees as before.

PROBLEM XV.

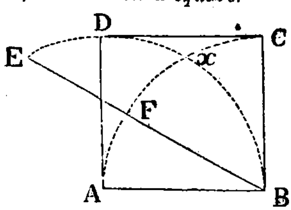
An angle being given, to find, by a scale of chords, how many degrees it contains.



From the vertex A as centre, with the chord of 60 degrees as radius, describe an arc *x y*; take the extent *x y* with the compasses, and setting one foot at the beginning of the line of chords, the other leg will reach to the number of degrees which the angle contains: but if the extent *x y* should reach beyond the scale, find the number of degrees in *x y*, which deducted from 180, will leave the degrees in the angle *B o x*. See figure to the second case of the last Problem.

PROBLEM XVI.

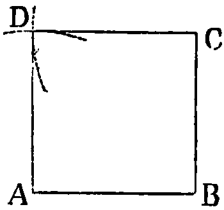
Upon a given right line A B, *to construct a square.*



With the distance A B as radius, and A as a centre, describe the arc E D B; and with the distance A B as radius, and B as a centre, describe the arc A F C, cutting the former in *x*; make *x* E equal to *x* B; join E B; make *x* C and *x* D each equal to A F or F *x*; then join A D, D C, C B, and A D C B will be the required square.

Or thus :

Draw B C at right angles to A B, and equal to it ; then from the centres A and C, with the radius A B and C B, describe two arcs cutting each other at D ; join D A and D C, which will complete the square.

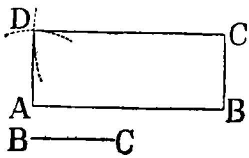


PROBLEM XVII.

To make a rectangular parallelogram of a given length and breadth.

Let A B be the length, and B C the breadth.

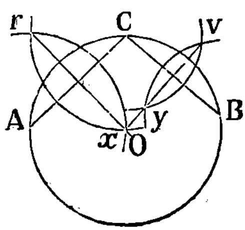
Erect B C at right angles to A B; through C and A draw C D and A D, parallel to A B and B C.



PROBLEM XVIII.

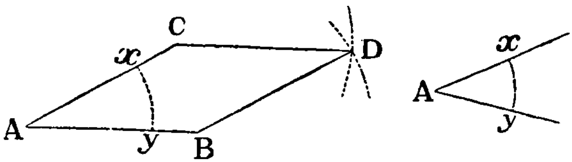
To find the centre of a given circle.

Draw any two chords A C, C B; from the points A, C, B, as centres, with any radius greater than half the lines, describe four arcs cutting in r x, and y v, draw r x and y v, and produce them till they meet in O, which will be the centre.



PROBLEM XIX.

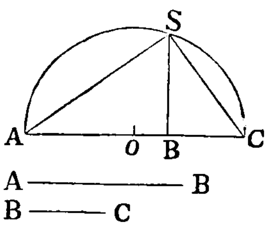
Upon a given right line A B, *to describe a rhombus having an angle equal to a given angle* A.



Make the angle C A B equal to the angle at A ; make A C equal to A B; then from C and B as centres, with the radius A B describe two arcs crossing each other at D; join D C and D B, which will complete the rhombus.

PROBLEM XX.

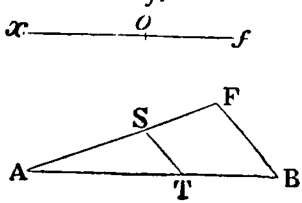
To find a mean proportional between two given right lines A B *and* C D.



Place A B and B C in one straight line ; bisect A C in *o* ; from *o* as a centre, with A *o* or *o* C as radius, describe a semicircle A S C; erect the perpendicular B S, and it will be a mean proportional between A B and B C ; that is A B : B S :: B S : B C.

PROBLEM XXI.

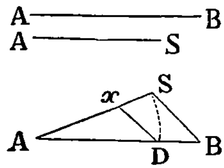
To divide a given right line A B *into two such parts, as shall be to each other as* x *o to o f*.



From the point A draw A S equal to x *o*, and produce it till F S becomes equal to *o f* ; join F B, and draw S T parallel to F B ; then will A T : T B :: x *o* : *o f*.

PROBLEM XXII.

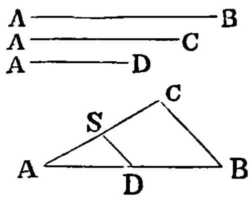
To find a third proportional to two given right lines A B, A S.



Place A B and A S so as to make any angle at A; from the centre A, with the distance A S describe the arc S D; then draw D x parallel to B S, and A x will be the third proportional required; that is, A B : A S :: A S : A x.

PROBLEM XXIII.

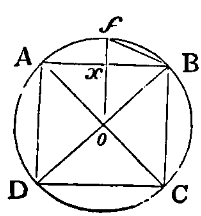
To find a fourth proportional to three given right lines, A B, A C, *and* A D.



Place the rignt lines A B and A C so as to make any angle at A; on A B set off A D; join B C; and draw D S parallel to it; then A S will be the fourth proportional required, viz. A B : A C :: A D : A S.

PROBLEM XXIV.

In a given circle to inscribe a square.

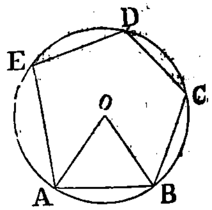


Draw any two diameters A C, D B at right angles to each other; then join their extremities, and the figure A B C D will be a square inscribed in the given circle.

If a line be drawn from the centre o to the middle of A B, and produced to f; the line joining f B will be the side of an octagon inscribed in the circle.

PROBLEM XXV.

To make a regular polygon on a given right line, A B.



Divide 360 degrees by the number of sides contained in the polygon; deduct the quotient from 180 degrees, and the remainder will be the number of degrees in each angle of the polygon. At the points A and B make the angles *o* A B and *o* B A each equal to half the angle of the polygon; then from *o* as a centre, and with *o* A or *o* B as radius, describe a circle, in which place A B continually.*

Or thus :

Take the given line A B from the scale of equal parts, and multiply the number of equal parts in it by the number in the third column of the following table, answering to the given number of sides; the product will give the number of equal parts in the radius A *o*, or *o* B, which taken from the scale of equal parts in the compasses, will give the radius, with which describe a circle, and place in it the line A B continually, *s* shown in the first method.†

* See Appendix, Demonstration 1. † See Appendix, Demonstration 2.

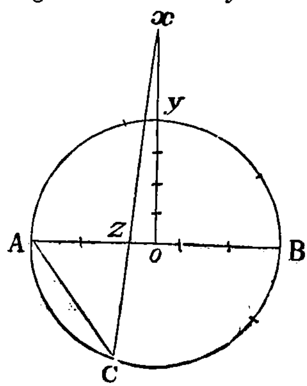
TABLE I.

When the side of the polygon is 1.

No of sides.	Name of the Polygon.	Radius of the circumscribing circle.	Angle O A B, or O B A.
3	Trigon	·5773503	30
4	Tetragon	·7071068	45
5	Pentagon	·8506508	54
6	Hexagon	1, *Side = radius.*	60
7	Heptagon	1·1523825	$64\frac{2}{7}$
8	Octagon	1·3065630	$67\frac{1}{2}$
9	Nonagon	1·4619022	70
10	Decagon	1·6186340	72
11	Undecagon	1·7747329	$73\frac{7}{11}$
12	Dodecagon	1·9318516	75

PROBLEM XXVI.

In a given circle to inscribe any regular polygon; or to divide the circumference of a given circle into any number of equal parts.

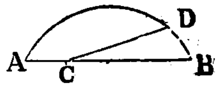


Divide the diameter A B into as many equal parts as the figure has sides; erect the perpendicular $o\ x$, from the centre o; divide the radius $o\ y$ into four equal parts, and set off three of these parts from y to x; draw a line from x to the second division z, of the diameter A B, and produce it to cut the circumference at C; join A C, and it will be the side of the required polygon.*

* See Appendix, Demonstration 8.

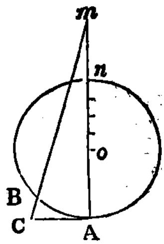
PROBLEM XXVII.

To draw a straight line equal to any arc of a circle A B.



Divide the chord A B into four equal parts; and set off one of these parts from B to D; then join D C, and it will be equal to the length of half the given arc nearly.*

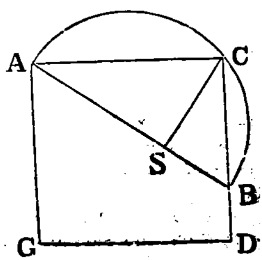
Or thus:



From the extremity of the arc A B, whose length is required to be found, draw A *o m*, passing through the centre; divide *o n*, into four equal parts, and set off three of those parts from *n* to *m*; draw *m* B, and produce it to meet A C drawn at right angles to A *m*; then will A C be nearly equal in length to the arc A B.†

PROBLEM XXVIII.

To make a square equal in area to a given circle.



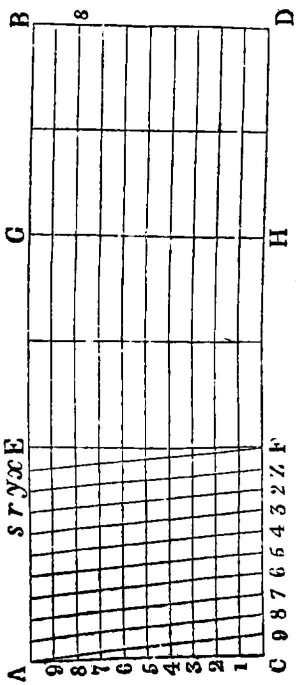
First divide the diameter A B into fourteen equal parts, and set off eleven of them from A to S; from S erect the perpendicular S C and join A C, the square of which will be very nearly equal to the area of the given circle.‡

* See Appendix, Demonstration 4. † See Appendix, Demonstration 5.
‡ See Appendix, Demonstration 6.

PROBLEM XXIX.

To construct a diagonal scale.

Draw an indefinite straight line; set off any distance A E according to the intended length of the scale; repeat A E any number of times, E G, G B, &c.; draw C D parallel to A B at any convenient distance; then draw the perpendiculars A C, E F, G H, B D, &c. Divide A E and A C each into ten equal parts; through 1, 2, 3, &c. draw lines parallel to A B and through *x* *y*, &c. draw *x* F *y* Z, &c. as in the annexed figure.



The principal use of this scale is, to lay down any line from a given measure; or to measure any line and compare it with others.—Whatever number C F represents, F Z will be the tenth of it, and the subdivisions in the vertical direction F E will be each one-hundredth part. Thus, if C F be a unit, the small divisions in C F, viz. F Z, &c. will be 10ths, and the divisions in the altitude will be the 100th parts of a unit. If C F be ten, the small divisions F Z, &c. will be units, and those in the vertical line, tenths; if C F be a hundred, the others will be tens and units.*

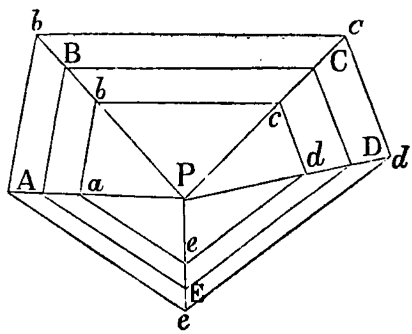
* See Appendix, Demonstration 7.

To take any number off the scale, as suppose 2 $\frac{38}{100}$ that is, 2·38 ; place one foot of the compasses at D, and extend the other to the division marked 3; then move the compasses upward, keeping one foot on the line D B, and the other on the line 3 *s*, till you arrive at the eighth interval, marked 88, and the extent on the compasses will be that required. This, however, may express 2·38, 23·8, or 238, according to the magnitude of the assumed unit.

NOTE. If C F were divided into 12 equal parts, each division would be 1 inch, and each vertical division 1-10th of an inch, by making C F one foot.

PROBLEM XXX.

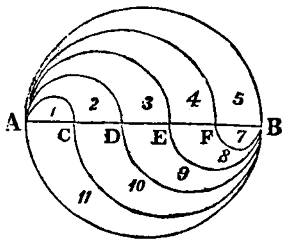
To reduce a rectilinear figure to a similar one upon either a smaller or a larger scale.



Take any point P in the figure A B C D E, and from this assumed point draw lines to all the angles of the figure; upon one of which P A take P *a* agreeably to the proposed scale; then draw *a b* parallel to A B, *b c* to B C, &c. then shall the figure *a b c d e* be similar to the original one, and upon the required scale. Or measure all the sides and diagonals of the figure by a scale, and lay down the same measures respectively from another scale, in the required proportion.

When the figure is complex, the reduction to a different scale is best accomplished by means of the Eidograph, an instrument invented by Professor Wallace, or by means of the improved Pentograph.

To divide a circle into any number of equal parts, having their perimeters equal also.



Divide the diameter A B into the required number of equal parts, at the points C, D, E, &c.; the non one side describe the semicircles 1, 2, 3, 4, &c. and on the other side of the diameter describe the semicircles 7, 8, 9 10, &c. on the diameters B F, B E, B D, B C, &c.; so shall the parts 1, 11, 2, 10, 3, 9, 4, 8, &c. be equal both in area and perimeter.—LESLIE'S GEOMETRY.

MENSURATION OF SUPERFICIES.

SECTION II.

The area of any plane figure is the space contained within its boundaries, and is estimated by the number of square miles, square yards, square feet, &c. which it contains.

Long Measure.

12	Inches . . .	1 Foot.
3	Feet . . .	1 Yard.
6	Feet . .	1 Fathom.
16½	Feet Eng. }	1 Pole or Perch.
5½	Yards }	
40	Perches . .	1 Furlong.
8	Furlongs . .	1 Mile.

II.

Square Measure.

144	Inches . .	1 Foot.
9	Feet . .	1 Yard.
36	Feet . .	1 Fathom.
272¼	Feet Eng. }	1 Pole or Perch.
30¼	Yards }	
1600	Perches .	1 Furlong.
64	Furlongs .	1 Mile.

In Ireland 21 feet make 1 pole or perch, and 7 yards therefore will make a pole or perch. There are other measures used, for which see *Arithmetical Tables.*

Land is generally measured by a *Chain* of 4 poles, or 22 yards; it consists of 100 links, each link being 22 of a yard. *See Section XI. Surveying.*

Duodecimals are calculations by feet, inches, and parts, which decrease by twelves: hence they take their name.

Multiplication of feet, inches, and parts, is sometimes called Cross Multiplication, from the factors being multiplied crosswise. It is used in finding the contents of work done by artificers, where the dimensions are taken in feet, inches, and parts.

Rule.

I. Write the multiplier under the multiplicand in such a manner, that feet shall be under feet, inches under inches, &c.

II. Multiply each term of the multiplicand by the number of feet in the multiplier, proceeding from right to left; carry 1 for every 12, in each product, and set down the remainder under the term multiplied.

III. Next multiply the terms of the multiplicand by the number under the denomination inches, in the multiplier; carry 1 for every 12, as before, but set down each remainder one place farther to the right than if multiplying by a number under the denomination feet.

IV. In like manner proceed with the number in the multiplier under the denomination parts or lines, remembering to set down each remainder one place farther to the right than if multiplying by a number under the denomination inches. And so on with numbers of inferior denominations.

V. Add the partial products thus placed, and their sum will be the whole product.

IN CROSS MULTIPLICATION IT IS USUAL TO SAY

Feet multiplied by feet, give feet.
Feet by inches, give inches.
Feet by parts, give parts.

Inches by inches, give parts.
Inches by parts, give thirds.
Inches by thirds, give fourths.

Parts by parts, give fourths.
Parts by thirds, give fifths.
Parts by fourths, give sixths, &c.*

---

* In multiplication, the multiplier must always be a number of times; to talk of multiplying feet by feet, &c. is absurd, for what notion can be formed of 7 feet taken 3 times? However, since the above easily suggests the correct meaning, and is a concise method of expressing the rule, it has been thought proper to retain it. See Appendix, Denomination 8.

1. Multiply 7 feet 9 inches by 3 feet 6 inches.

```
 F.  I.
 7 . 9
 3 . 6
--------
23 . 3
 3 . 10 . 6
-----------
27 . 1  . 6 Ans.
```

```
             F.     I.  P.     F.  I.  P.
2. Multiply 240 . 10 . 8  by  9 . 4 . 6
              9 .  4 . 6
           ---------------
           2168 .  0 . 0
             80 .  3 . 6 . 8
             10 .  0 . 5 . 4
           ---------------------
           2258 .  4 . 0 . 0 Ans.
```

	F. I. P.	F. I. P.	F. I. P.
3. Multiply	8 . 5 .	by 4 . 7.	*Ans.* 38 . 6. 11.‴″
4. Multiply	9 . 8 .	by 7 . 6.	— 72 . 6.
5. Multiply	7 . 6 .	by 5 . 9.	— 43 . 1. 6.
6. Multiply	4 . 7 .	by 3 . 10.	— 17 . 6. 10.
7. Multiply	7 . 5 . 9	by 3 . 5 . 3.	— 25 . 8 6.2.3.
8. Multiply	10 . 4 . 5	by 7 . 8 . 6.	— 79 . 11. 0.6.6
9. Multiply	75 . 7 . 0	by 9 . 8 . 0.	— 730 . 7. 8.
10. Multiply	57 . 9 . 0	by 9 . 5 . 0.	— 543 . 9. 9.
11. Multiply	75 . 9 . 0	by17 . 7 . 0.	—1331 . 11. 3.
12. Multiply	321 . 7 . 3	by 9 . 3 . 6	—2988 . 2. 10.4.6.
13. Multiply	4 . 7 . 8	by 9 . 6 .	— 44 . 0. 10.
14. Multiply	39 .10 . 7	by18 . 8 . 4.	— 745 . 4. 10.2.4.

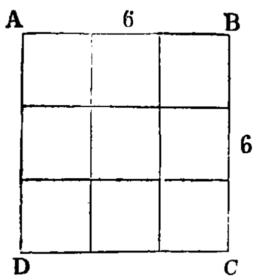
NOTE.—All these can be solved by the method of aliquot parts, thus :—

15. Multiply 368 F. 7′ 5″ by 137 F. 8′ 4″

	F.	′	″	‴	⁗
	368	7	5		
	137	8	4		
	2576				
	1104				
	368				
6′ = $\frac{1}{2}$ . .	184	3	8	6	
2′ = $\frac{1}{3}$ . .	61	5	2	10	
4″ = $\frac{1}{6}$ . .	10	2	10	5	8
6′ = $\frac{1}{2}$ . .	68	6			
1′ = $\frac{1}{6}$ . .	11	5			
4″ = $\frac{1}{3}$ . .	3	9	8		
1″ = $\frac{1}{4}$ . .	0	11	5		
Ans.	50756	7	10	9	8

PROBLEM I.

To find the area of a square.



Rule. Multiply the length of the side by itself, and the product will be the area.*

1. Let the side of the square A B C D be 6 : what is its area?

Ans. $6 \times 6 = 36$, the area.

2. What is the area of a square whose side is 15 chains? *Ans.* 225.

3. What is the area of a square whose side is 7 feet 9 inches? *Ans.* $60\frac{1}{16}$.

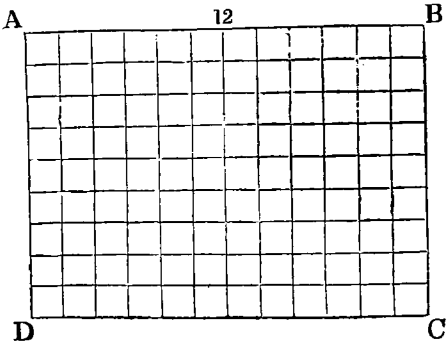
4. What is the area of a square whose side is 4769 links? *Ans.* 22743361.

* See Appendix, Demonstration 8.

PROBLEM II.

To find the area of a rectangle.

RULE. Multiply the length of the rectangle by its breadth, and the product will be the area.*



1. Let the sides of the rectangle A B C D be 12 and 9, what is its area? *Ans.* $12 \times 9 = 108$, the area.

2. What is the superficial content of a plank, whose length is 5 feet 6 inches, and breath 7 feet 8 inches?
Ans. 42 feet 2 inches.

3. What is the area of a field whose boundaries form a rectangle, its length being 176 links and breadth 154 links?
Ans. ·27104 of an acre.

4. What is the superficial content of a floor, whose length is 40 feet 6 inches, and breadth 28 feet 9 inches?
Ans. 1164 feet, 4 inches, 6 parts.

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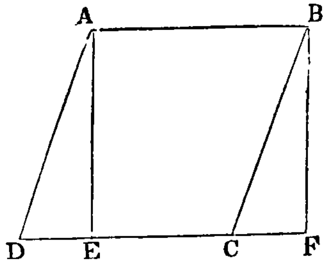
* See Appendix, Demonstration 8.

PROBLEM III.

To find the area of a rhombus.



RULE. Multiply the length by the perpendicular breadth, and the product will be the area.*

1. What is the area of a rhombus, whose side is 16 feet, and perpendicular breadth 10 feet. *Ans.* 16 × 10 = 160 feet the area.

2. What is the content of a field in the form of a rhombus, whose length is 7·6 chains, and perpendicular height 5·7 chains? *Ans.* 43·32 chains.

3. What is the area of a rhombus, whose side is 7 feet 6 inches, and perpendicular height 3 feet 4 inches? *Ans.* 25 feet.

4. What is the area of a rhombus whose length is 3 yards, and perpendicular height 2 feet 3 inches? *Ans.* 20 feet 3 inches.

PROBLEM IV.

To find the area of a triangle.

RULE. Multiply the base by the perpendicular height, and divide the product by two for the area.†

1. The base of a triangle is 76·5 feet, and perpendicular 92·2 feet; what is its area?
Ans. 76·5 × 92·2 ÷ 2 = 3526·65 square feet, the area.

---

* See Appendix, Demonstration 9.
† See Appendix, Demonstration 10.

2. The base of a triangle is 72·7 yards, and the perpendicular height of 36·5 yards?

Ans. 1326·775 yards.

3. The base of a triangular field is 1276 links; and perpendicular 976 links; how many acres in it?

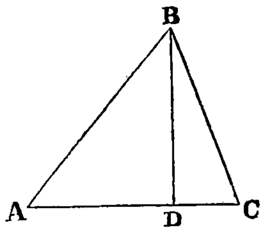
Ans. 6 acres 36·3008 perches.

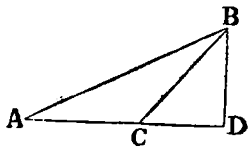
4. The base of a triangle measures 15 feet 6 inches, and the perpendicular 12 feet 7 inches; what is its area?

Ans. 97 feet 6¼ inches.

PROBLEM V.

Having the three sides of any triangle given, to find its area.





Rule I. From half the sum of the three sides subtract each side separately, then multiply the half sum and the three remainders together, and the square root of the last product will be the area of the triangle.*

Rule II. Divide the difference between the squares of two sides of the triangle by the third side; to half this third side add half the quotient, and deduct the square of this sum from the square of the greater side, the remainder will be the square of the perpendicular, the square root of which, multiplied by half the base, will give the area of the triangle.†

* See Appendix, Demonstration 11.
† See Appendix, Demonstration 12.

1. Given the side A B = 9·2, B C 7·5, and A C = 5·5; required the area of the triangle ?

9·2
7·5
5·5

Sum 22·2

$\frac{1}{2}$ Sum $11·1 - 9·2 = 1·9$: then $\surd(11·1 \times 1·9 \times 3·6 \times 5·6)$
$11·1 - 7·5 = 3·6$ $= \surd\ 425·1744 = 20·619$ the area
$11·1 - 5·5 = 5·6$ by Rule I.

Again, $9·2^2 - 7·5^2 = 84·61 - 56·25 = 28·39$; then $28·39 \div 5·5 = 5·161818$, quotient.

Now $(5·161818 \div 2) + (5·5 \div 2) = 2·580909 + 2·75 = 53·309$ = half quot. plus half third side : then $84·64 - 28·41869481 = 56·22150519$, and $\surd\ 56·22150519 = 7·498$ = perpendicular; then $7·498 \times 2·75 = 20·619$ the area as before.

2. What is the area of a triangle whose sides are 50, 40, and 30 ? *Ans.* 600.

3. The sides of a triangular field are 4900, 5025, and 2569 links; how many acres does it contain ?
Ans. 61 acres, 1 rood, 30·68 perches.

4. What is the area of an isosceles triangle, whose base is 20, and each of its equal sides 15 ? *Ans.* 117·803.

5. How many acres are there in a triangle, whose three sides are 380, 420, and 765 yards ?
Ans. 9 acres 38 poles.

6. How many square yards are in a triangle, whose three sides are 13, 14, and 15 feet ? *Ans.* $9\frac{1}{3}$ square yards.

7. How many acres, &c., in a triangle, whose three sides are 49, 50·25, and 25·69 chains ?
Ans. 61 acres, 1 rood, 39·68 perches.

PROBLEM VI.

To find the area of an equilateral triangle.

Rule. Square the side, and from this square deduct its fourth part; then multiply the remainder by the fourth part of the square of the side, and the square root of the product will give the area.* Or multiply $\frac{AB^2}{4}$ by $\sqrt{3}$ for the area.†

1. Each side of a triangular field, A B C, measures 4 perches, what is its area?

$4^2 = 16$, then $16 \div 4 = 4$ and $16 - 4 = 12$: then $12 \times \frac{16}{4} = 12 \times 4 = 48$, and $\sqrt{48} = 6{\cdot}928$, the area.

2. How many acres in a field of a triangular form, each of whose sides measures 70 perches?

Ans. 13 acres, 1 rood, 1 perch.

3. The perimeter of an equilateral triangle is 27 yards, what is its area? *Ans.* 35·074.

Note. When the triangle is isosceles, the perpendicular is equal to the square root of the difference between the squares of either of the equal sides, and half the base.

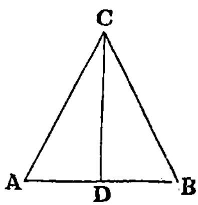
PROBLEM VII.

Given the area and altitude of a triangle to find the base.

Rule. Divide the area by the altitude or perpendicular, and double the quotient will give the base.

1. Given the area of a triangle = 12 yards, and altitude = 4; what is its base?

Ans. $12 \div 4 = 3$; then $3 \times 2 = 6$ yards, the base A B.



2. A surveyor having lost his field book, and requiring

* See Appendix. Demonstration 13.
† See Appendix, Demonstration 14.
‡ See Appendix, Demonstration 15.

the base of a triangular field, whose content he knew from recollection was 14 acres, and altitude 7 yards, how much is the base? *Ans.* 19360 yards.

PROBLEM VIII.

Given the area of a triangle and its base, to find its altitude.

Rule. Divide the area by the given base, and double the quotient will give the perpendicular.

The reason of this rule is manifest, from the last.

1. Given the area of a triangle = 12, and its base = 6; what is its perpendicular height?

Ans. $12 \div 6 = 2$; then $2 \times 2 = 4$ the altitude.

PROBLEM IX.

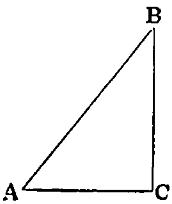
Given any two sides of a right angled triangle, to find the third side, and thence its area.

Rule.

I. To the square of the perpendicular add the square of the base, and the square root of the sum will give the hypothenuse.

II. The square root of the difference of the squares of the hypothenuse, and either side will give the other.

III. Or multiply the sum of the hypothenuse, and either side, by their difference; and the square root of the product will give the other.*



1. Given the base A C 3, the perpendicular C B 4; required the hypothenuse A B?

$3^2 + 4^2 = 25$; then $\sqrt{25} = 5$, the hypothenuse A B.

2. Given A B 5, A C 3; required C B?

$5^2 - 3^2 = 16$; then $\sqrt{16} = 4$, the side B C; or, $(5 + 3) \times (5 - 3 = 8 \times 2 = 16$; then $\sqrt{16} = 4$, as before.

3. Given A B 5, B C 4; required A C?

$5^2 - 4^3 = 9$, then $\sqrt{9} = 3$, the side A C; or $(5 + 4) \times (5 - 4) = 9 \times 1 = 9$; then $\sqrt{9} = 3$, as before. And $3 \times 4 \div 2 = 6$ the area of the triangle.

---

* See Appendix, Demonstration 16.

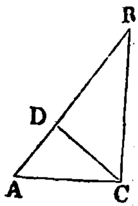
4. The wall of a building on the brink of a river is 120 feet, and the breadth of the river is 70 yards; what is the length of the chord in feet that will reach from the top of the building across the river? *Ans.* 241·86 feet.

5. A ladder 60 feet long, will reach to a window 40 feet from the flags on one side of a street, and by turning the ladder over to the other side of the street, it will reach a window 50 feet from the flags; required the breadth of the street? *Ans.* 77·8875 feet.

6. The roof of a house, the side walls of which are the same height, forms a right angle at the top, the length of one rafter being 10 feet, and its opposite one 14 feet; what is the breadth of the house? *Ans.* 17·204.

PROBLEM X.

Given the base and perpendicular of a right angled triangle, to find the perpendicular let fall on the hypothenuse from the right angle; and also the segments into which the hypothenuse is divided by this perpendicular.



Rule. Find the hypothenuse by Prob. IX. Then divide the square of the greater side by the hypothenuse, and the quotient will give the greater segment, which deducted from the entire will give the less. Having found the segments, multiply them together, and the square root of the product will give the perpendicular.*

1. Given A C 3 yards, and C B 4 yards; required the segments B D, D A, and the perpendicular D C.

$3^2 + 4^2 = 25$: then $\sqrt{25} = 5 =$ A B.

$4^2 \div 5 = 16 \div 5 = 3.2, =$ B D; then $5 - 3{\cdot}2 = 1{\cdot}8 =$ AD.

Again, $3{\cdot}2 \times 1{\cdot}8 = 5{\cdot}76$; then $\sqrt{5{\cdot}76} = 2{\cdot}4 =$ D C.

2. The roof of a house whose side walls are each 30 feet high, forms a right angle at the top; now if one of the rafters be 10 feet long, and its opposite yoke-fellow 12, required the breadth of the building, the length of the prop set upright to support the ridge of the roof, and the part of the floor at which it must be placed?

Ans. Breadth of the building 15·6204 feet, greater segment

* See Appendix, Demonstration 17.

9·2186 feet, lesser segment 6·4018 feet, and length of the prop 57·68 feet.

PROBLEM XI.

To find the area of a trapezium.

Rule. Divide the trapezium into two triangles, by joining two of its opposite angles; find the area of each triangle, and the sum of both areas will give the area of the trapezium.

Or,

Draw two perpendiculars from the opposite angles to the diagonal; then multiply the sum of these perpendiculars by the diagonal, and half the product will give the area.*

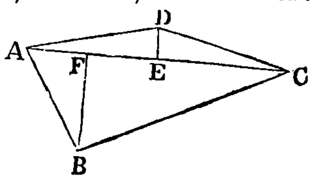
1. In the trapezium A B C D, the diagonal A C is 100 yards, the perpendicular DE 35, and BF 30; what is its area?

D E = 35
B F = 30

65
100

2)6500

3250 the area.



2. What is the area of a field, whose south side is 2740 links, east side 3575 links, north side 3755 links, west side 4105 links, and the diagonal from south-west to north-east 4835 links? *Ans.* 123 acres 11·8633 perches.

3. In the trapezium A B C D, the side A D is 15, D C 13, C B 14, and A B 12; also the diagonal A C 16; what is its area? *Ans.* 172·5247.

4. In the trapezium A B C D, there are given A B 220 yards, D C 265 yards, and A C 378 yards; also A F 100 yards, and A C 70 yards; what is its area?

Ans. 85342·2885 yards = 17 acres, 2 roods, 21 perches.

5. In the trapezium A B C D, there are given A B 220 yards, D C 265 yards, B F 195·959 yards, D E 255·5875 yards; also F E 208 yards; required the area of the trapezium? *Ans.* 85342·2885 yards.

* See Appendix, Demonstration 18.

6. Suppose in the trapezium A B C D, on account of obstacles, I can only measure A B, D C, B F, D E, and F D, which are respectively 22 yards, 26 yards, 19 yards, 25 yards, and 32 yards, required the area ?

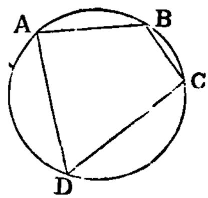
Ans. 840·55 square yards.

PROBLEM XII.

To find the area of a trapezium inscribed in a circle, or of any one whose opposite angles are together equal to two right angles.

RULE. Add the four sides together, and take half the sum; from this half sum deduct each side separately; and the square root of the product of the four remainders will give the area of the trapezium.*

1. What is the area of a four-sided field, whose opposite angles are together equal to two right angles, the length of the four sides being as follows, viz., A B 12·5, A D 17, D C 17·5, and B C 8 yards ?



$$\begin{array}{r} 12{\cdot}5 \\ 17 \\ 17{\cdot}5 \\ 8 \\ \hline 2)55 \\ \hline 27{\cdot}5 \end{array}$$

$$\begin{array}{cccc} 27{\cdot}5 & 27{\cdot}5 & 27{\cdot}5 & 27{\cdot}5 \\ 12{\cdot}5 & 17 & 17{\cdot}5 & 8 \\ \hline 15 & 10{\cdot}5 & 10 & 19{\cdot}5 \end{array}$$

$15 \times 10{\cdot}5 \times 10 \times 19{\cdot}5 = 30712{\cdot}50$; then $\sqrt{30712{\cdot}50} = 175{\cdot}25$, the area in yards.

2. There is a trapezium whose opposite angles are together equal to two right angles; the sides are as follows, viz., A B 25, A D 34, D C 35 and B C 16; required its area ?

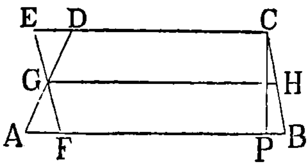
Ans. 700·99.

* See Appendix, Demonstration 19

PROBLEM XIII.

To find the area of a trapezoid.

RULE. Multiply half the sum of the two parallel sides by the perpendicular distance between them, and the product will give the area.*



1. Let A B C D be a trapezoid, the side A B = 40, D C = 25, C P = 18; required the area?

40
25
—
65 ÷ 2 = 32·5 × 18 = 585 area.

2. What is the area of a trapezoid, whose parallel sides are 750 and 1225 links, and the perpendicular height 1540 links? *Ans.* 15 acres 33·2 perches.

3. What is the area of a trapezoid, whose parallel sides are 4 feet 6 inches, and 8 feet 3 inches; and the perpendicular height 5 feet 8 inches? *Ans.* 36 feet $1\frac{1}{2}$ inches.

4. What is the area of a trapezoid whose parallel sides are 1476 and 2073 yards, and perpendicular height 976 yards? *Ans.* 220 acres, 3 roods, 25 perches, 7 yards Irish.

PROBLEM XIV.

To find the area of an irregular polygon.

RULE. Divide the figure into triangles and trapeziums, and find the area of each separately, by Problem IV. or XI. Add these areas together, and the sum will be the area of the polygon.†

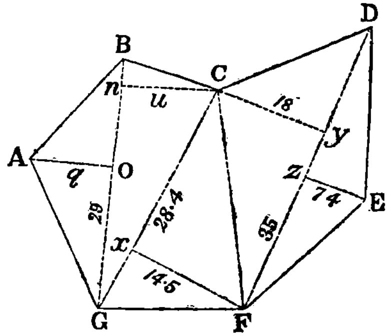
1. What is the area of the irregular polygon ABCDEFG A the following lines being given?

---

* See Appendix, Demonstration 20.

† In finding the area of an irregular figure, draw a line through the extreme angles of the figure, on which let fall perpendiculars from all the other angles of the polygon, which will divide it into triangles and trapezoids; then find the area of these by Problems IV. and XIII.

A O = 9
G B = 29
C n = 11
G C = 28·4
F x = 14·5
C y = 13
F D = 35
E z = 7·4



A O = 9
O n = 11

2)20 sum

10 half
20 diag. G B

290 area of A B C G A.
C y = 13
E z = 7·4

2)20·4 sum

1·02
35

357·0 area of F C D E F.
F x = 14·5
½ G C = 14·2

205·9 area of G F C.

290 = area of A B C G A
357 = area of F C D E F
205·9 = area of G F C

Ans. 852·9 = area of A B C D E F G A.

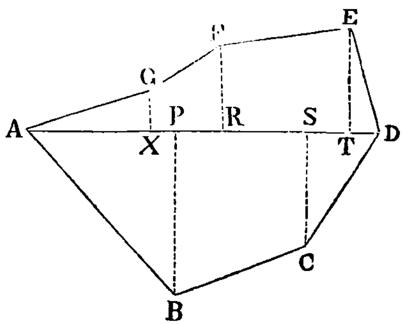
2. In a five-sided field G C D E F G there is G C = 28 perches, Fx = 14 perches, Cy = 13 perches, zE = 7 perches, and FD = 35 perches; required its area?

Ans. 3 acres, 1 rood, 26 perches.

3. In the annexed figure, there are given in perches.

A X = 15	A P = 17	F R = 10
X R = 8	P S = 14	E T = 12
R T = 14	S D = 12	B P = 20
T D = 6	G X = 5	C S = 14

Required the area? *Ans.* 4 acres, 3 roods, 19½ perches.



PROBLEM XV.

To find the area of a regular polygon.

RULE I. Add all the sides together and multiply half the sum by the perpendicular drawn from the centre of the polygon to the middle of one of the sides, and the product will give the area. This perpendicular is the radius of the inscribed circle.

RULE II. Multiply the square of the side of the polygon by the number standing opposite to its name in the following table, under the word area, and the product will give the area of the polygon.

RULE III. Multiply the side of the polygon by the number standing opposite to its name in the column of the following table, headed "Radius of inscribed Circle," and the product will be the perpendicular from the centre of the polygon to the middle of one of its sides; then multiply half the sum of the sides by this perpendicular, and the product will give the area.*

TABLE II.

When the side of the polygon is 1.

No. of sides.	Radius of inscribed Circle.	Area of Polygon.	
3	0·2886751	0·4330127 =	$\frac{3}{4}$ tan. $30\frac{1}{4}° = \sqrt{3}$
4	0·5000000	1·1000000 =	$\frac{4}{4}$ tan. $45° = 1 \times 1$
5	0·6881910	1·7204774 =	$\frac{5}{4}$ tan. $54° = \frac{5}{4}\sqrt{(1+\frac{2}{5}\sqrt{5})}$
6	0·8660254	2·5980762 =	$\frac{6}{4}$ tan. $60° = \frac{6}{4}\sqrt{3}$
7	1·0382617	3·6339124 =	$\frac{7}{4}$ tan. $64°\frac{2}{7}$
8	1·2071068	4·8281271 =	$\frac{8}{4}$ tan. $67°\frac{1}{2} = 2 \times (1+\sqrt{2})$
9	1·3737387	6·1818242 =	$\frac{9}{4}$ tan. $70°$
10	1·5388418	7·6042088 =	$\frac{10}{4}$ tan. $72° = \frac{5}{2}\sqrt{(5+2\sqrt{5})}$
11	1·7028437	9·3656404 =	$\frac{11}{4}$ tan. $73°\frac{7}{11}$
12	1·8660254	11·1961524 =	$\frac{12}{4}$ tan. $75° = 3 \times (2+\sqrt{3})$

NOTE. The radius of the circumscribed circle, when the side of the polygon is 1, may be seen in Table I.

The expressions in the fourth column may be seen in *Trigonometry*, to which the pupil is referred for a full investigation of them. The tangents of the angle O *a* C in the heptagon, nonagon, and undecagon, are extremely difficult to be found without a table of tangents.

1. The side of a pentagon is 20 yards, and the perpendicular from the centre to the middle of one of the sides is 13·76382; required the area?

By RULE I. 20 × 5 × 13·76382 ÷ 2 = 1376·382 ÷ 2 = 688·191. *Ans.*

By RULE II. 20 × 20 × 1·720477 = 688·19, the area as before.

2. The side of a hexagon is 14, and the perpendicular from the centre 12·1243556; required the area? *Ans.* 509·2229352.

3. The side of an octagon is 5·7, required its area?

Ans. 156·875596479.

* See Appendix, Demonstration 21

4. The side of a heptagon is 19·38 yards, what is its area? *Ans.* 1364·84.

5. The side of an octagon is 10 feet, what is its area? *Ans.* 482·84271.

6. The side of a nonagon is 50 inches, what is its area? *Ans.* 15454·5605.

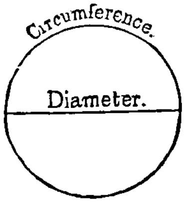
7. The side of an undecagon is 20, what is its area? *Ans.* 3746·25616.

8. The side of a dodecagon is 40 yards, what is its area? *Ans.* 17913·84384.

PROBLEM XVI.

Given the diameter of a circle, to find the circumference; or the circumference to find the diameter, and thence the area.

RULE.*



I. Say as 7 : 22 :: the given diameter : circumference.

Or, as 113 : 355 :: the diameter : the circumference.

Or, as 1 : 3·416 :: the diameter : the circumference.

II. Say as 22 : 7 :: the given circumference : the diameter.

Or, as 355 : 113 :: the circumference : the diameter.

Or, as 3·1416 : 1 :: the circumference : the diameter.

1. The diameter of a circle is 15, what is its circumference?

7 : 22 :: 15 : 22 × 15 ÷ 7 = 330 ÷ 7 = 47·142857.

Or, 113 : 355 :: 15 : 355 × 15 ÷ 113 = 5325 ÷ 113 = 47·124.

Or, 1 : 3·1416 :: 15 : 3·1416 × 15 = 47·124.

2. The circumference of a circle is 80, what is its diameter?

22 : 7 :: 80 : 7 × 80 ÷ 22 = 25·45.

355 : 113 :: 80 : 113 × 80 ÷ 355 = 25·4647.

3·146 : 1 :: 80 : 80 ÷ 3·1416 = 25·4647.

* See Appendix, Demonstration 22.

3. What is the circumference of a circle whose diameter is 10 ? *Ans.* 31·4285.

4. What is the diameter of a circle whose circumference is 50 ? *Ans.* 15·909.

5. The diameter of the earth is 7958 miles, what is its circumference ? *Ans.* 25000·8528 miles.

6. The circumference of the earth being 25000·8528 miles, what is its diameter ? *Ans.* 7958 miles.

PROBLEM XVII.

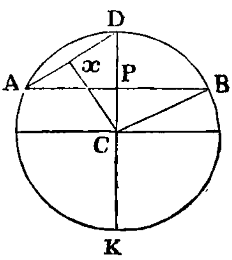
To find the length of an arc of a circle.

RULE. I. Multiply the radius of the circle by the number of degrees in the given arc, and that product by ·01745329, and the last product will be the length of the arc.*

RULE II. From eight times the chord of half the arc, subtract the chord of the whole arc, one-third of the remainder will give the length of the arc, nearly.†

1. If the arc A B contain 30 degrees, the radius being 2 feet, what is the length of the arc?

$30 \times 9 = 270$, and $270 \times .01745329 = 4·7124$. *Ans.*



2. If the chord AD of half the arc ADB be 20 feet, and the chord A B of the whole arc 38; what is the length of the arc?

$20 \times 8 - 38 = 122$; then $122 \div 3 = 46\frac{2}{3}$ feet. *Ans.*

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* See Appendix, Demonstration 23.
† See Appendix, Demonstration 24

3. The chord of an arc is 6 feet, and the chord of half the arc is $3\frac{1}{2}$; required the length of the whole arc?

Ans. $7\frac{1}{3}$.

4. The chord of the whole arc is 40, and the versed sine* or height of the segment 15; what is the length of the arc?

Ans. $53\frac{1}{3}$.

5. The chord A B of the whole arc is 48·74, and the chord A D of half the arc 30·25; required the length of the arc?

6. A B = 30, D P = 8; required the length of the arc?

Ans. $35\frac{1}{3}$.

PROBLEM XVIII.

To find the area of a circle.

RULE I. Multiply half the circumference by half the diameter, for the area.†

RULE II. Multiply the square of the diameter by ·7854, for the area.‡

RULE III. Multiply the square of the circumference by ·07958.§

RULE IV. As 14 to 11, so is the square of the diameter to the area.

RULE V. As 88 to 7, so is the square of the circumference to the area.

1. To find the area of a circle whose diameter is 100 and circumference 314·16.

By RULE I.	By RULE II.	By RULE III.
31416	·7854	98696·5 sq. cir.
100	$100^2 = 10000$	·07958
4)31416	Area 7854	7854· Area.
Area 7854		

* By "versed sine," in works on mensuration, is not meant the trigonometrical versed sine of the whole arc, but of half the arc.

† See Appendix, Demonstration 22.

‡ See Appendix, Demonstration 22.

§ See Appendix, Demonstration 26.

By Rule. IV.	By Rule V.
$1000^2 = 10000$	98696·5 sq. cir.
11	7
2)110000	8)690875·5
7)55000	11)86359·4
Area 7857	7850·85

2. What is the area of a circle whose diameter is 7?
Ans. 38½ nearly.

3. How many square yards are in a circle whose diameter is 1⅙ yard? *Ans.* 1·069.

4. The surveying wheel turns twice in the length of 16½ feet; in going round a circular bowling green it turns exactly 200 times; how many acres, roods, and perches in it?
Ans. 4 acres, 3 roods, 35·8 perches.

5. The circumference of a fish pond is 56 chains, what is its area? *Ans.* 239·56288.

6. What is the area of a quadrant, the radius being 100?
Ans. 7854.

7. Required the length of a chord fastened to a stake at one end, and to a cow's horns at the other, so as to allow her to feed on an acre of grass and no more?
Ans. 39¼ yards.

8. The circumference of a circle is 91, what is its area?
Ans. 659·00198.

9. The diameter of a circle is 15 perches, what is its area? *Ans.* 176·715.

10. What is the area of the semicircle of which 20 is the radius? *Ans.* 628·32.

PROBLEM XIX.

Given the diameter of a circle to find the side of a square equal in area to the circle.

Rule. Multiply the diameter by ·8862269, and the product will be the side of a square equal in area to the circle.*

* See Appendix, Demonstration 26.

1. If the diameter of a circle be 100, what is the side of a square equal in area to the circle? *Ans.* 88·62269.

2. The diameter of a circular fish-pond is 200 feet, what is the side of a square fish-pond equal in area to the circular one? *Ans.* 177·24538.

PROBLEM XX.

Given the circumference of a circle to find the side of a square equal in area to the circle.

RULE. Multiply the circumference by ·282 948, and the product will be the side of the square.*

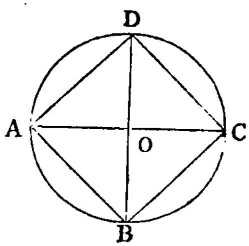
1. The circumference of a circle is 100, what is the side of a square equal in area to the circle? *Ans.* 28·2 948.

2. The circumference of a round fish-pond is 200 yards, what is the side of a square fish-pond equal in area to the round one? *Ans.* 56·41896.

PROBLEM XXI.

Given the diameter, to find the side of the inscribed square.

RULE. Multiply the diameter by ·7071068, and the product will give the side of the inscribed square.†



1. The diameter of a circle is 100, what is the side of the inscribed square? *Ans.* 70.71068.

2. The diameter of a circle is 200, what is the side of the inscribed square? *Ans.* 141·42136.

PROBLEM XXII.

Given the area of a circle, to find the side of the inscribed square.

RULE. Multiply the area by ·6366197, and extract the square root of the product, which will give the side of the inscribed square.‡

* See Appendix, Demonstration 27.
† See Appendix, Demonstration 28
‡ See Appendix, Demonstration 29.

1. The area of a circle is 100, what is the side of the inscribed square ? *Ans.* 7·97884.

2. The area of a circle is 200, what is the side of the inscribed square ?

200 × ·6366197 = 127·3239400; then $\sqrt{127\cdot3239400}$ = 11·2837. *Ans.*

PROBLEM XXIII.

Given the side of a square, to find the diameter of the circumscribed circle.

Rule. Multiply the side of the square by 1·4142136, and the product will give the diameter of the circumscribed circle.*

1. If the side of a square be 10, what is the diameter of the circumscribed circle ? *Ans.* 14·142136.

2. If the side of a square be 20, find the diameter of the circumscribed circle ? *Ans.* 28·284272.

PROBLEM XXIV.

Given the side of a square to find the circumference of the circumscribed circle.

Rule. Multiply the side of the square by 4·4428934, and the product will be the circumscribed circle.†

1. If the side of a sqnare be 100, what is the circumference of the circumscribed circle ? *Ans.* 444·28934.

2. If the side of the square be 30, what is the circumference of the circumscribed circle ? *Ans.* 133·286802.

PROBLEM XXV.

Given the side of a square, to find the diameter of a circle equal in area to the square.

Rule. Multiply the side of the square by 1·1283791, and the product will be the diameter of a circle equal in area to the square whose side is given.‡

* See Appendix, Demonstration 30.
† See Appendix. Demonstration 31.
‡ See Appendix, Demonstration 32.

1. If the side of a square be 100, what is the diameter of the circle whose area is equal to the square whose side is 100? *Ans.* 112·83791.

2. What is the diameter of a circle equal in area to a square whose side is 200? *Ans.* 225·67582.

PROBLEM XXVI.

Given the side of a square, to find the circumference of a circle whose area is equal to the square whose side is given.

RULE. Multiply the side of the square by 3·5449076, and the product will give the circumference of a circle equal in area to the given square.*

1. What is the circumference of a circle, whose area may be equal to a square whose side is 100? *Ans.* 354·49076.

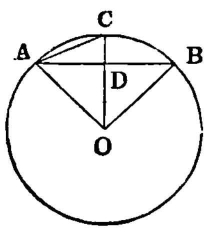
2. Find the circumference of a circle equal in area to a square whose side is 300? *Ans.* 1063·47228.

PROBLEM XXVII.

To find the area of a sector of a circle.

RULE. Multiply half the length of the arc by the radius of the circle, and the product is the area of the sector.†

RULE II. As 360 is to the degrees in the arc of the sector, so is the area of the whole circle to the area of the sector.‡



1. Let ACBO be a sector less than a semicircle whose radius AO is 20 feet, and chord AB 30 feet; what is the area?

First, $\sqrt{(AO^2 - AD^2)} = \sqrt{(400 - 225)} = 13·228 = OD$; then $OC - OD = 20 - 13·228 = 6·772 = CD$.

Again, $\sqrt{(AD^2 + CD^2)} = \sqrt{(225 + 45·859984)} = 16·4578 = AC$, the chord of half the arc.

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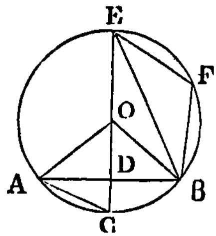
* See Appendix, Demonstration 33.
† See Appendix, Demonstration 34.
‡ See Appendix, Demonstration 35.

Hence, by problem XVII. the arc A B is 33·8874; then $\frac{33 \cdot 8874}{2} \times 20 = 338 \cdot 874$, the area required.

2. Let A E F B O A be a sector greater than a semicircle, whose radius A O is 20, the chord E B 38, and chord B F of half E F B 23; required the area?



$$\begin{array}{r} 23 = \text{chord B F} \\ 8 \\ \hline 184 \\ 38 = \text{chord B E} \\ \hline 3)146 \\ \hline 48 \cdot 666 \text{ \&c.} = \text{arc B F E} \\ 20 \\ \hline 973\tfrac{1}{2} \text{ area.} \end{array}$$

3. What is the area of a sector whose arc contains 18 degrees, the diameter being 3 feet?

$$\begin{array}{r} \cdot 7854 \\ 9 \\ \hline \end{array}$$

Then 360 : 18 :: 7·0686 : the area of the sector;
Or, 20 : 1 :: 7·0686 : ·35343. *Ans.*

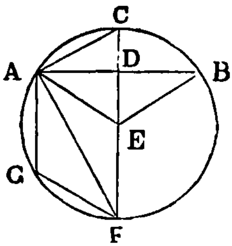
4. What is the area of a sector whose arc contains 147 degrees 29 minutes, and radius 25? *Ans.* 804·3986.

5. What is the area of a sector whose arc contains 18 degrees, the radius being 3 feet? *Ans.* 1·41372.

PROBLEM XXVIII.

To find the area of the segment of a circle.

RULE I. Find the area of the sector having the same arc with the segment, by the last problem; find also the area of the triangle, formed by the chord of the segment and the two radii of the sector. Then add these two areas together, when the segment is greater than the semicircle, but find their difference when it is less than a semicircle, the result will evidently be the answer.



1. What is the area of the segment A C B D A, its chord A B being 24, and radius A E or E C 20?

$(\surd AE^2 - AD^2) = \surd(400 - 144) = 16 = DE$; $EC - ED = 20 - 16 = 4 = CD$; $\surd(AD^2 + DC^2) = \surd(144 + 16) = 12{\cdot}64911 = AC$; then $\frac{(AC \times 8, - 24}{3} =$ $25{\cdot}7309$ = arc A C B,

And 12·8654 = half arc	12 = A D
20 = radius	16 = D A
257·308 = area of sector EBCA.	192 = area of $\triangle$ABE
192· = area of $\triangle$ A B E	
65·308 = area of segment A B C A.	

2 Let A G F B A be a segment greater than a semicircle, there are given the chord A B 20·5, F D 17·17. A F 20, F G 11·5 and AE, 11·64, required the area of the segment?

$\frac{(FG \times 8) - AF}{3} = \frac{(11{\cdot}5 \times 8) - 20}{3} = 24$ the length of the arc A G F (Problem XVII.); then $24 \times 11{\cdot}64 = 279{\cdot}36$, area of sector A E B F G A (Problem XXVII). Again, $FD - EF = 17{\cdot}17 - 11.64 = 5.53 = ED$; then $\frac{AB \times ED}{2} = \frac{20{\cdot}5 \times 5{\cdot}553}{2} = 56{\cdot}6825$ the area of the triangle A B E, which being added to the area of the sector before found will give the area of the segment, viz. $279{\cdot}36 + 56{\cdot}6825 = 336{\cdot}0425$ **the area of the segment A G F B A,** RULE II. To two-thirds of the product of the chord and versed sine of the segment, add the cube of the versed sine divided by twice the chord, and the sum will give the area of the segment, nearly.

When the segment is greater than the semicircle, find the area of the remaining segment, and deduct it from the area of the whole circle, the remainder will give the area of the segment.*

3. What is the area of the segment A C B, less than a semicircle, its chord being 18·9, and height or versed sine D C 2·4?

$AB \times DC = 18{\cdot}9 \times 2{\cdot}4 = 45{\cdot}36$, and $\frac{2}{3}AB \times DC = \frac{2}{3} \times 45{\cdot}36 = 30{\cdot}24$; then $\frac{2{\cdot}4^3}{2 \times 18{\cdot}9} = {\cdot}36571$; hence $30{\cdot}24 + {\cdot}36571 = 30{\cdot}60571$ the area.

NOTE. If two chords of a circle cut one another, the rectangle contained by the segments of one of them is equal to the rectangle contained by the segments of the other. This is the 35th Proposition of Book III. of Euclid.

4. Required the area of the segment A G F B whose height F D is 20, and chord A B 20?

$\frac{AB}{2} = \frac{20}{2} = 10 = AD$, and $AD^2 = 100$; but $AD^2 = FD \times DC \therefore CD = \frac{AD^2}{FD} = \frac{100}{20} = 5$.

The area of the segment A C B is, by the last case, 69·7916; and the area of the whole circle, by Problem XVIII. is 490·875; then 490·875 — 69·7916 = 421·0834 = area of the segment AGFB.

5. What is the area of the segment A G F B, greater than a semicircle, whose chord A B is 12, and versed sine 18?

Ans. 297·81034.

* See Appendix, Demonstration 36

RULE III. Divide the height of the segment by the diameter of the circle, to three places of decimals. Find the quotient in the column Height of the Table at the end of the practical part of this treatise, and take out the corresponding Area Seg., which multiply the square of the diameter, and the product will be the area of the segment required.*

NOTE I. If the quotient of the height by the diameter be greater than ·5 subtract it from 1, and find the Area Seg. corresponding to the remainder, which subtract from ·7854 for the correct Area Seg.

NOTE II. If the quotient of the height by the diameter does not terminate in three figures, find the Area Seg. corresponding to the first three decimal figures of the quotient, subtract it from the next greater Area Seg., multiply the remainder by the fractional part of the quotient, and add the product to the area segment first taken out of the table. When great accuracy is not required, the fractional part may be omitted.

6. Let the diameter be 20, and the versed sine 2, required the area of the segment ?

$\frac{2}{20} = \cdot 1$, to which answers ·040875
Square of diameter, 400

16·35 area.

7. What is the area of a segment, whose diameter is 52, and versed sine 2 ?

$\therefore \frac{2}{52} = \cdot 038\frac{6}{13}$ which is the tabular versed sine. Then to ·038 answers ·009763, and the difference between this area and the next is ·000385, which multiplied by $\frac{6}{13}$ gives ·000177 which added to ·009763 gives ·009940, which is the area corresponding to the versed sine ·038 $\frac{6}{13}$. Then $52^2 \times \cdot 009940 = 26 \cdot 87776$ is the area required.

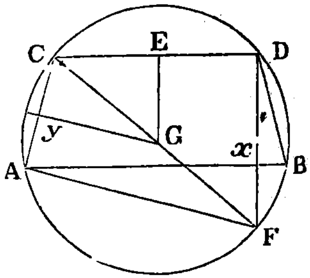
PROBLEM XXIX.

To find the area of a zone, or the space included by two parallel chords and the arcs contained between them.

RULE. Join the extremities of the parallel chords towards the same parts, and these connecting lines will cut off two

* See Appendix, Demonstration 37.

equal segments, the areas of which added to the area of the trapezoid then formed will give the area of the zone.



1. Suppose the greater chord A B = 30, the less C D 20, and the perpendicular distance $Dx = 25$, required the area of the zone A B D C.

$\frac{1}{2}(AB - CD) = xB = \frac{1}{2}(30 - 20) = 5$: then $\sqrt{(xD^2 + xB^2)} = DB = \sqrt{(25^2 + 5^2)} = 25{\cdot}49$. $AB - Bx = Ax = 30 - 5 = 25$, and $(Ax \times Bx) \div Dx = Fx = (25 \times 5) \div 25 = 5$. $Dx + Fx = DF = 25 + 5 = 30$; $\frac{1}{2}\sqrt{(CD^2 + DF^2)} = \frac{1}{2}CF = Gz = \frac{1}{2}\sqrt{(20^2 + 30^2)} = 18{\cdot}027$, the radius of the circle ; $(DB \times Ax) \div 2Dx = Gy$* $= (25{\cdot}49 \times 25) \div (2 \times 25) = 12{\cdot}745$; $Gz - Gy = zy = 18{\cdot}027 - 12{\cdot}745 = 5{\cdot}282$, the height of the segment A z C. 36·05) 5·28 (·146, the tabular area segment answering to which is ·071033, then $\cdot071033 \times (36{\cdot}05)^2 = 92{\cdot}315$ = the area of the segment A z C.

$\frac{1}{2}(AB + CD) \times Dx = \frac{1}{2}(30 + 20) \times 25 = 625$ the area of the trapezoid A B D C : then $625 + 92{\cdot}315 \times 2 = 809{\cdot}63$ = the area of the zone.

2. Let the chord A B be 48, the chord C D 30, the chord A C 15·8114; what is the area of the zone A B D C ?

Ans. The diameter C F = 50, height of the segment A z C = 1·2829, area by the table of segments = 13·595. Area of the zone A B D C = 534·19.

3. Let A B = 20, C D = 15, and their distance = 17½; required the area ? *Ans.* 395·4369.

4. Let A B = 96, C D = 60, and their distance = 26; required the area ? *Ans.* 2136·7527.

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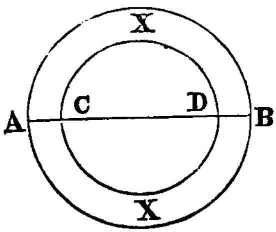
* See Appendix, Demonstration 38.

PROBLEM XXX.

To find the area of a circular ring, or of the space included between two concentric circles.

RULE. Multiply the sum of the two diameters by their difference, and the product arising by ·7854 for the area of the ring.*

1. The diameter A B is 30, and C D 20; what is the area of the ring XX?




```
   30
   20
   —
   50 sum
   10 difference
  ———
  500
 ·7854
————————
392·7000 area of ring XX.
```

2. What is the area of the circular ring, when the diameters are 40 and 30? *Ans.* 549·78.

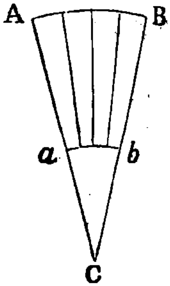
3. What is the area of a circular ring, when the diameters are 50 and 45? *Ans.* 373·065.

PROBLEM XXXI.

To find the area of a part of a ring, or of the segment of a sector.

RULE. Multiply half the sum of the bounding arcs by their distance asunder, and the product will give the area.†

* See Appendix, Demonstration 39.
† See Appendix, Demonstration 40.



1. Let A B be 50, and $a\ b$ 30, and the distance a A 10; what is the area of the space $a\ b$ B A?

$$Ans.\ \frac{50+30}{2}\times 10=400.$$

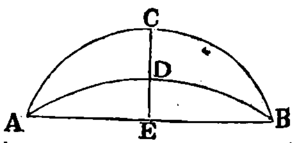
2. Let A B=60, $a\ b$=40, and a A =2; required the area of the space $a\ b$ B A? *Ans.* 100.

3. Let A B=25, $a\ b$ =15, and a A =6; required the area of the segment of the sector? *Ans.* 120.

PROBLEM XXXII.

To find the area of a lune, or the space included between the intersecting arcs of two eccentric circles.

Rule. Find the areas of both segments which form the lune, and deduct the less from the greater; the remainder will evidently be the area required.



1. Let the chord A B= 40, E C=12, and E D=4; what is the area of the lune A D B C A?

By note page 50, (A E^2 ÷E C) + E C = diameter of the circle of which A C B is an arc; and (A E^2÷E D) +E D=the diameter of the circle of which A D B is an arc; hence $(20^2 \div 12) + = 45{\cdot}3$; and $(20^2 \div 4) + 4 =104$ are the two diameters.

$12\div 45{\cdot}3={\cdot}264.$ $\qquad 4\div 104={\cdot}038.$

The Area Seg. answering to ·264 is ·165780, and $(45{\cdot}3)^2 \times {\cdot}165780 = 340{\cdot}1954802$ = area of the segment A E B C A?

The Area Seg. answering to ·038 is ·009763, and

$(104)^2 \times \cdot 009763 = 105 \cdot 596608$ = area of the segment A E B D A; then $340 \cdot 1954802 - 105 \cdot 596608 = 234 \cdot 5988722$ = the area of the lune.

2. Let the chord A B be 40, and the heights of the segments E C and E D 15 and 2; required the area of the lune? *Ans.* 388·5.

PROBLEM XXXIII.

TO MEASURE LONG IRREGULAR FIGURES.

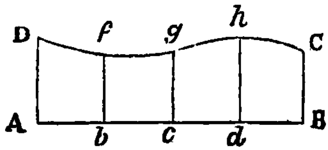
When irregular figures, not reducible to any known figure, present themselves, their contents are best found by the method of equi-distant ordinates.

RULE. Take the breadths in several places, at equal distances and divide the sum of the first and last of them by 2 for the arithmetical mean between those two. Add together this mean and all the other breadths, omitting the first and last, and divide their sum by the number of parts so added, the quotient will give the mean breadth of the whole, which being multiplied by the given length will give the area of the figure, very nearly.

It is not necessary sometimes to take the breadths at equal distances, but to compute each trapezoid separately, and the sum of all the separate areas thus found will give the area of the entire, nearly.

Or, add all the breadths together and divide by the number of them for a mean breadth, which being multiplied by the length, as before, will give the area, nearly.

1. Let the ordinate A D be 9·2, *b f* 7, *c g* 9, *d h* 10, B C 8·8 and the length A B 30; required the area?



9·2 A D
8·8 B C

2)18

9 mean breadth of first and last.
7 *b f*
9 *c g*
10 *d h*

4)35 sum

8·75 mean breadth of all.
30

262·50 area of the whole figure.

2. The length of an irregular figure is 39 yards, and its breadths, in five equi-distant places, are 4·8, 5·2, 4·1, 7·3, and 7·2; what is its area? *Ans.* 215·475 square yards.

3. The length of an irregular figure is 50 yards, and its breadths, at seven equi-distant places, are 5·5, 6·2, 7·3, 6, 7·5, 7, and 8·8; what is its area? *Ans.* 342·05 square yards.

4. The length of an irregular figure being 37·6, and the breadths, at nine equi-distant places, 0, 4·4, 6·5, 7·6, 5·4, 8, 5·2, 6·5, 6·1; what is the area? *Ans.* 218·315.

EXERCISES.

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1. Find the area of a square whose side is 35·25 chains.
Ans. 124 acres, 1 rood, 1 perch.

2. Find the area of a rectangular board, whose length is $12\frac{1}{2}$ feet, and breadth, 9 inches. *Ans.* $9\frac{3}{8}$ feet.

3. The sides of three squares being 4, 5, and 6 feet, what is the length of the side of a square which is equal to all three? *Ans.* 8·7749 feet.

4. Required the area of a rhomboid whose length is 10·51 chains, and breadth, 4·28 chains?
Ans. 4 acres, 1 rood, 39 perches.

5. There is a triangle whose base is 12·6 chains, and altitude 6·4 chains, what is its area? *Ans.* 40·32.

6. Find the area of a triangle whose sides are 30, 40 and 50 yards. *Ans.* 300 square yards.

7. There is a triangular corn field whose sides are 150, 200, and 250 yards, determine the number of acres contained in the field, and the expense of reaping the corn at 9*s.* 6*d.* per acre?
Ans. Content of the field, 3 acres, 15 perches; expense of reaping, £1 9*s.* 5*d.*

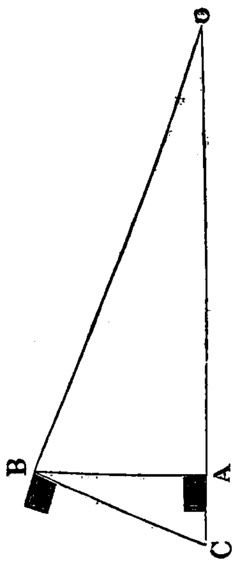
8. What must the base of a triangle be to contain 36 square feet, whose vertex is to be 9 feet from the base?
Ans. 8 feet.

9. What must be the altitude of a triangle equal in area to the last, whose base is 12 feet? *Ans.* 6 feet.

10. The height of a precipice standing close by the side of a river is 103 feet, and a line of 320 feet will reach from the top of it to the opposite bank; required the breadth of the river? *Ans.* 302·97 feet.

11. A ladder $12\frac{1}{2}$ feet in length stands upright against a wall, how far must the bottom of it be pulled out from the wall so as to lower the top 6 inches? *Ans.* $3\frac{1}{2}$ feet,

12. A person wishing to measure the distance from a point A, at one side of a canal, to an object O, at the other, and having no instrument but a book, placed a corner of it on the point A, and directed an edge of it, as in the figure, in a straight line with the object O, and drew the straight lines A B, A C; he then placed the book so that a corner of it rested on the point B, at the distance of eight times its length from the point A, and directed an edge of it, as before, to the object O, and drew the straight line B C which met A C at the distance of three times the length of the book from A; how many times the length of the book is the object O from the points A and B?



Ans. $21\frac{1}{3}$ and 22·78 times.

13. What is the area of a trapezium whose diagonal is 70·5 feet, and the two perpendiculars 26·5 and 30·2 feet? *Ans.* 1998·675 square feet.

14. What is the area of a trapezium whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches? *Ans.* 6347 feet 36 inches.

15. What is the area of a trapezoid whose two parallel sides are 75 and 122 links, and the perpendicular distance 154 links? *Ans.* 13629 square links.

16. A field in the form of a trapezoid, whose parallel sides are 6340 and 4380 yards, and the perpendicular distance between them 121 yards, lets for £207 14*s.* per annum; what is that per acre? *Ans.* £1 11*s.*

17. Two opposite angles of a four sided field are together equal to two right angles, and the sides are 24, 26, 28, and 30 yards; what is its area?

Ans. 723·99 square yards, nearly.

18. Required the area of a figure similar to that annexed to the first question under Problem XIV., whose dimensions are double of those there given?

Ans. 3411·6.

19. What is the side of an equilateral triangle equal in area to a square, whose side is 10 feet?

Ans. 15·196 feet, nearly.

20. Required the area of a regular nonagon, one of whose sides is 8 feet, and the perpendicular from the centre = 10·99 feet? *Ans.* 395·64 square feet.

21. Required the area of a regular decagon, one of whose sides is 20·5 yards? *Ans.* 3233·491125 square yards.

22. A wheel of a car turns round 4400 times in a distance of 10 miles; what is its diameter?

Ans. 3.819708 feet.

23. If the diameter of a circle be 9 feet, what is the length of the circumference? *Ans.* $28\frac{2}{7}$ feet, nearly.

24. Required the length of an arc of 60°; the radius of the circle being 14 feet? *Ans.* 14·660772 feet.

25. The chord of an arc is 30 feet and the height is 8 feet, what is the length of the arc? *Ans.* 35½ feet, nearly.

26. The diameter of a circle is 200, what is the area of the quadrant? *Ans.* 7854.

27. The diameters of two concentric circles are 15 and 10, what is the area of the ring formed by those circles?

Ans. 98·175.

28. The circumference of a circle is 628·32 yards, what is the radius of a concentric circle of half the area?

Ans. 141·42.

29. What is the side of a square equal in area to the circle whose diameter is 3? *Ans.* 2·6586807.

30. The two parallel chords of a zone are 16 and 12 and their perpendicular distance is 2, what is the area of the zone? *Ans.* 28·376.

31. The length of a chord is 15, and the heights of two segments of circles on the same side of it are 7 and 4; what is the area of the lune formed by those segments?

Ans. 38, nearly.

32. The base and perpendicular of a right-angled triangle are each 1, what is the area of a circle having the hypothenuse for its diameter. *Ans.* 1·5708.

33. If the area of a circle be 100, what is the area of the inscribed square? *Ans.* 63·66.

CONIC SECTIONS.

SECTION III.

OF THE ELLIPSIS.*

PROBLEM I.

The transverse and conjugate diameters of an ellipsis being given, to find the area.

RULE. Multiply the transverse and conjugate diameters thgether, and the product arising by ·7854, and the result will be the area.†

1. Let the transverse axis be 35, and the conjugate axis 25; required the area?

$35 \times 25 \times {\cdot}7854 = 687{\cdot}225$ *Ans.*

2. The longer diameter of an ellipse is 70, and the shorter 50; what is the area? *Ans.* 2748·9.

3. What is the area of an ellipse whose longer axis is 80, and shorter axis is 60? *Ans.* 3769·92.

4. What is the area of an ellipse, whose diameters are 50 and 45? *Ans.* 1767·15.

PROBLEM II.

To find the area of an elliptical ring.

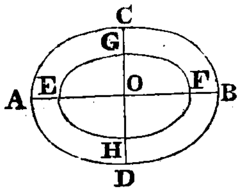
RULE. Find the area of each ellipse separately, and their difference will be the area of the ring.

* For definitions of the ellipsis (or, as it is frequently written, ellipse) and the other Conic Sections. See Appendix, Properties of the Conic Sections.

† See Appendix, Demonstration 41.

Or, From the product of the two diameters of the greater ellipse deduct the product of the two diameters of the less and multiply the remainder by ·7854 for the area of the ring.*

1. The transverse diameter A B is 70, and the conjugate C D 50; and the transverse diameter E F of another ellipse



having the same centre O, is 35, and the conjugate G H is 25; required the area of the elliptical space between their circumferences?

70 × 50 × ·7854 = 2748·9; and 35 × 25 × ·7854 = 687·225; then 2748·9 — 687·225 = 2061·675 = area of the elliptical ring.

70 × 50 = 3500
35 × 25 = 875

2625 × ·7854 = 2061·675 = area.

2. The transverse and conjugate diameters of an ellipse are 60 and 40, and of another 30 and 10; required the area of the space between their circumferences? *Ans*. 1649·34.

3. A gentleman has an elliptical flower garden, whose greater diameter is 30, and less 24 feet; and has ordered a gravel walk to be made round it of 5 feet 6 inches in width; required the area of the walk? *Ans*. 371·4942 feet.

PROBLEM III.

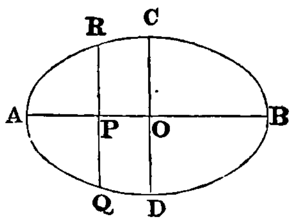
Given the height of an elliptical segment, whose base is parallel to either of the axes of the ellipse, and the two axes of the ellipse, to find the area.

Rule. Divide the height of the segment by that diameter of which it is a part, to three places of decimals, find the

* See Appendix, Demonstration 42.

quotient in the column Height of the Table referred to in page 51, and take out the correspondent Area Seg. Multiply the Area Seg. thus found and both the axes of the ellipsis together, and the result will give the area required.*

1. Required the area of an elliptical segment R A Q



whose height A P is 20; the tranverse axis A B being 70, and the conjugate axis C D 50?

$20 \div 70 = \cdot285\frac{5}{7}$ = the tabular versed sine, the corresponding segment answering to which is ·185166; then $\cdot185166 \times 70 \times 50 = 648\cdot081$, the area.

2. What is the area of an elliptical segment cut off by a chord parallel to the shorter axis, the height of the segment being 10, and the two diameters 35 and 25?

Ans. 162·0202.

3. What is the area of an elliptical segment cut off by a chord parallel to the longer axis, the height of the segment being 10, and the two diameters 40 and 30?

Ans. 275·0064.

4. What is the area of an elliptical segment cut off by a chord parallel to the shorter diameter, the height being 10, and the two diameters 70 and 50? *Ans.* 240·884.

* See Appendix, Demonstration 43.

PROBLEM IV.

To find the circumference of an ellipse, by having the two diameters given.

RULE. Multiply the sum of the two diameters by 1·5708, and the product will give the circumference nearly; that is, putting t for the transverse, c for the conjugate, and p for 3·1416; the circumference will be $(t+c,) \times \frac{1}{2} p$.*

1. Let the transverse axis be 24, and the conjugate 18; required the area?

$(24 + 18) \times 1·5708 = 42 \times 1·5708 = 65·9736$ is the circumference, nearly.

2. Required the circumference of an ellipse whose transverse axis is 30, and conjugate 20? *Ans.* 78·54.

3. Required the circumference of an ellipse whose diameters are 60 and 40? *Ans.* 157·08.

4. What is the circumference of an ellipse whose diameters are 6 and 4? *Ans.* 15·708.

5. What is the circumference of an ellipse whose diameters are 3 and 2? *Ans.* 7·854.

PROBLEM V.

To find the length of any arc of an ellipse.

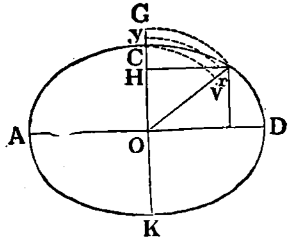
RULE. Find the length of the circular arc $x\ y$, intercepted by O C, O B, and whose radius is half the sum of O C, O B: and it will be equal to the elliptical arc B C, nearly.†

NOTE. The nearer the axes of the ellipse approach towards equality, the more exact the result of the operation by this Rule; and the less the elliptical arc, the nearer its exact length will approach the arc $x\ y$.

* See Appendix, Demonstration 44.
† See Appendix, Demonstration 45.

1. Let the axis A D be 24, C K 18, and O T 3; required the length of the arc B C?

Here we have T D = 9, and A T = 15; then from the property of the ellipsis, we have A O² : O C² :: A T ×



$$T\,D : T\,B^2 = \frac{9^2 \times 9 \times 15}{12 \times 12} = \frac{9 \times 9 \times 15}{16},$$ and O B =

$$\surd\,(O\,T^2 \times T\,B^2) = \surd(9 + \frac{9 \times 9 \times 15}{16}) = 9{\cdot}21616,$$ the

radius of the circle of which G B is an arc; but O C is the radius of the circle of which C V is an arc; therefore the radius of the circle of which $x\,y$ is an arc, is $\frac{1}{2}$ O C + $\frac{1}{2}$ O B = 9·10808. But by *Trigonometry,** H B ÷ O B = 3 ÷ 9·21616 = ·325515, is the sine of the angle C O B, or arc $x\,y$, to the radius 1, answering to 18·9968 degrees. Therefore, by Problem XVII. Rule I, the length of the arc $x\,y$ is ·01745 × 18·9968 × 9·10808 = 3·0192, which is also equal to the length of the elliptical arc C B, nearly.

2. Given A D 30, C K 20, and O T 5; required the length of the arc B C? *Ans.* 5·03917786255.

3. Given A D 40, C D 30, and O T 5; required the length of the arc B C? *Ans.* 5·033880786.

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* It may be done without Trigonometry, by first finding the length of the arc G B by Rule II. Prob. XVII. Sec. 2, then O G : O Y : : G B : Y Z.

PROBLEM VI.

Given the diameter and abscissas, to find the ordinate.

RULE. Say, as the transverse is to the conjugate, so the square root of the rectangle of the two abscissas, the ordinate.*

1. In the ellipse A C D K, the transverse diameter A is 100, the conjugate diameter C K 80, and the abscis D T 10; required the length of the ordinate T B?

$100 : 80 :: \surd(90 \times 10) : TB = 24$. (See the last figure

2. Let the transverse axis be 35, the conjugate 25, a the abscissa 7; required the ordinate? *Ans.* 10.

3. Given the two diameters 70 and 60, and the abscis 10; required the ordinate? *Ans.* 20·9956.

PROBLEM VII.

Given the transverse axis, conjugate and ordinate, to fi the abscissas.

RULE. As the conjugate is to the transverse diameter, is the square root of the difference of the squares of t ordinate and semi-conjugate, to the distance between t ordinate and centre. Then this distance being added and subtracted from, the semi-diameter, will give the t abscissas.†

1. Let the diameters be 35 and 25, and the ordinate 1 required the abscissas?

By the Rule $\frac{35}{2} \pm \frac{35}{25} \surd\left(\left[\frac{25}{2}\right]^2 - 10^2\right) = \frac{35 \pm 21}{2} =$

and 7 the two abscissas.

2. Let the diameters be 120 and 40, and the ordina 16; required the abscissas? *Ans.* 96 and 24.

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* See Appendix, Demonstration 46.
† See Appendix, Demonstration 47.

PROBLEM VIII.

Given the conjugate axis, ordinate, and abscissas, to find the transverse axis.

RULE. Find the square root of the difference of the squares of the semi-conjugate axis and the ordinate, which add to, or subtract from, the semi-conjugate, according as the less abscissa or greater is given.

Then say, as the square of the ordinate is to the rectangle of the conjugate, and the abscissa, so is the sum or difference found above to the transverse required.*

1. Let the ordinate be 10, and the less abscissa 7; what is the diameter, allowing the conjugate to be 25?

$\sqrt{\left(\left[\frac{25}{2}\right]^2 - 10^2\right)} = 7{\cdot}5$; then $7{\cdot}5 + 12{\cdot}5 = 20$; then $10^2 : 25 \times 7 :: 20 : 35$ the transverse required.

2. Let the ordinate be 10, the greater abscissa 28, and the conjugate 25; required the transverse diameter?

Ans. 35.

PROBLEM IX.

Given the transverse axis, ordinate, and abscissa, to find the conjugate.

RULE. The square root of the product of the two abscissas is to the ordinate, as the transverse axis is to the conjugate.†

1. Let the transverse axis be 35, the ordinate 10, and the abscissas 28 and 7; required the conjugate?

$\sqrt{(28 \times 7)} : 10 :: 35 : \frac{35 \times 10}{\sqrt{(28 \times 7)}} = \frac{35 \times 10}{14} =$ 25, the conjugate.

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* See Appendix, Demonstration 48.

† See Appendix, Demonstration 49.

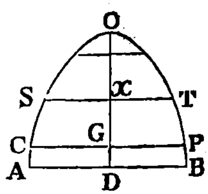
2. Let the transverse diameter be 120, the ordinate 16, and the abscissas 24 and 96; required the conjugate?
Ans. 40.

OF THE PARABOLA.

PROBLEM X.

Given the base and height of a parabola to find its area.

NOTE. Any double ordinate, A B, to the axis of a parabola may be called its base, and the abscissa O D, to that ordinate its height.



RULE. Multiply the base by the height, and $\frac{2}{3}$ of the product will be the area.*

1. Required the area of a parabola, whose height is 6 and base 12?

$$6 \times 12 \times \tfrac{2}{3} = 48 \text{ the area.}$$

2. What is the area of a parabola, whose base is 24, and height 4? *Ans.* 64.

3. What is the area of a parabola, whose base is 12, and height 2? *Ans.* 16.

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* See Appendix, Demonstration 50.

PROBLEM XI.

To find the area of the zone of a parabola, or the space between two parallel double ordinates.

RULE I. When the two double ordinates, their distance, and the altitude of the whole parabola are given; find the area of the whole parabola, and find also the area of the upper segment, their difference will be the area of the zone.

II. When the two double ordinates and their distance are given; to the sum of the squares of the two double ordinates, add their product, divide the sum by the sum of the two double ordinates, multiply the quotient by $\frac{2}{3}$ of the altitude of the zone, and the product will be the area of the zone.*

1. Given A B 20, S T 12, and D x 8; what is the area of the zone A S T B, the altitude D O being 12·5 ?

$(20 \times 12 \cdot 5) \times \frac{2}{3} = 166\frac{2}{3}$ = area of the parabola A B O, and $(12 \cdot 5 - 8) \times 12 = 54$, and $54 \times \frac{2}{3} = 36$; hence $166\frac{2}{3} - 36 = 130\frac{2}{3}$ the area.

III. When the altitude of the whole parabola is not given.

2. Suppose the double ordinate A B = 10, the double ordinate S T = 6, and their distance D x = 4 ; what is the area of the zone A S T B ?

$$\frac{10^2 + 6^2 + 10 \times 6}{10 + 6} = 12\frac{1}{4}; \text{ then } 12\frac{1}{4} \times 4 \times \frac{2}{3} = 33\frac{2}{3},$$

the area as before.

3. Let the double ordinate A B = 30, C P = 25, and their distance D G = 6 ; required the area of the zone A B P C? *Ans.* $165\frac{5}{11}$.

PROBLEM XII.

To find the length of the curve, or arc of a parabola, cut off by a double ordinate to the axis.

RULE.

I. **Divide the double ordinate by the parameter, and call** the quotient q.

* See Appendix, Demonstration 51.

II. Add 1 to the square of the quotient q, and call the square root of the sum s.

III. To the product of q and s, add the hyperbolic logarithm of their sum, then the last sum multiplied by half the parameter, will give the length of the whole curve on both sides of the axis.

Putting c for the curve, q for the quotient of the double ordinate divided by the parameter, s for $\sqrt{(1 + q^2)}$ and a for half the parameter; then

$$c = a \times \{q\,s + \text{hyp. log. of } (q + s.)\}^*$$

NOTE. The common logarithm of any number multiplied by 2·302585093 gives the hyperbolic logarithm of the same number.

1. What is the length of the curve of a parabola, cut off by a double ordinate to the axis, whose length is 12, the abscissa being 2?

$x = 2$ and $y = 6$; then $a = y\,\frac{y^2}{2x} = \frac{36}{4} = 9$,

and $q = \frac{y}{a} = \frac{6}{9} = \frac{2}{3}$, also $s = \sqrt{(1 + q^2)} = \sqrt{(1 + \frac{4}{9})}$ $= \sqrt{(\frac{13}{9}} = \frac{1}{3}\sqrt{(13} = 1{\cdot}2018504 = s$. Then $\frac{2}{3} + 1{\cdot}2018504 = 1{\cdot}868517$, whose common logarithm is ·271497, which being multiplied by 2·302585093, produces ·6251449 for its hyperbolic logarithm; and also $\frac{2}{3} + 1{\cdot}2018504 = {\cdot}8012336$; the sum of these two is 1·4263785, therefore $9 \times 1{\cdot}4263785 = 12{\cdot}8374065$, is the length of the curve required.

RULE II. Put y equal to the ordinate, and q equal the quotient arising from the division of the double ordinate by the parameter, or from the division of double the abscissa by the ordinate; then the length of the double curve will be expressed by the infinite series.

$$2\,y \times \left(1 + \frac{q^2}{2.3} - \frac{q^4}{2.4.5} + \frac{3\,q^6}{2.4.6.7}, \text{ \&c.}\right)$$

* See Appendix, Demonstration 52.

NOTE. This series will converge no longer than till $q = 1$. For when q is greater than 1, the series will diverge.

Let the last example be resumed, in which the abscissa is 2, and the ordinate 6.

Hence, $2 \times 2 \div 6 = \frac{2}{3} = q$; then employing $\frac{2}{3}$ instead of q in the last series, we get

$$12 \times \left(1 + \frac{(\frac{2}{3})^2}{2.3} - \frac{(\frac{2}{3})^4}{2.4.5} + 3 \times \frac{(\frac{2}{3})^6}{2.4.6.7}\right) = 12{\cdot}837$$ the length of the curve as before.

RULE III. To the square of the ordinate, add $\frac{4}{3}$ of the square of the abscissa, and the square root of the sum will be the length of the single curve, the double of which will be the length of the double curve, nearly.*

NOTE. The two first rules are not recommended in practice.—The practical application of this is much simpler, and is therefore to be employed in preference to either.

Retaining the same example, in which $x = 2$, and $y = 6$, we shall get $v = \sqrt{(y^2 + \frac{4}{3} x^2)} = \sqrt{(36 + \frac{16}{3})} = 6{\cdot}1291$, and $C = 12{\cdot}8582$, nearly.

2. Required the length of the parabolic curve, whose abscissa is 3, and the ordinate 8? *Ans.* 17·435.

PROBLEM XIII.

Given any two abscissas and the ordinate to one of them, to find the corresponding ordinate to the second abscissa.

RULE. Say, as the abscissa, whose ordinate is given, is to the square of the given ordinate, so is the other given abscissa to the square of its corresponding ordinate.†

1. If the abscissa x O = 10, and the ordinate x S = 8, what is the ordinate A D, whose abscissa D O is 20?

$x\,O : x\,S^2 :: D\,O : A\,D^2$, viz. $10 : 64 :: 20 : 128$, the square root of which is 11·313, &c. = A D.

* See Appendix, Demonstration 53.
† See Appendix, Demonstration 54.

2. If 6 be the ordinate corresponding to the abscissa 9, required the ordinate corresponding to the abscissa 16?

Ans. 8.

PROBLEM XIV.

Given two ordinates and the abscissa corresponding to one of them, to find the abscissa corresponding to the other.

RULE. Say, as the square of the ordinate whose abscissa is given, is to the given abscissa, so is the square of the other ordinate to its corresponding abscissa.*

1. Given S x = 6, x O = 9, and A D = 8; required the abscissa O D?

36 : 9 :: 64 : 16 = O D.

2. Given S x = 8, x O = 10, and A D = 9; required O D?

Ans. 12·656.

PROBLEM XV.

Given two ordinates perpendicular to the axis and their distance, to find the corresponding abscissas.

RULE. Say, as the difference of the squares of the ordinates is to their distance, so is the square of either of them to the corresponding abscissa.†

1. Given S x = 6, A D = 8, and x D = 7; required the abscissas?

(64 — 36) : 7 :: 64
28 : 7 :: 64 : 16 = O D, and
28 : 7 :: 36 : 9 = O x.

2. Given S x = 3, A D = 4, and x D = 2; required the abscissas?

Ans. $4\frac{4}{7}$ and $2\frac{4}{7}$.

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* See Appendix, Demonstration 54.
† See Appendix, Demonstration 55.

OF THE HYPERBOLA.

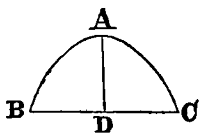
PROBLEM XVI.

Given the transverse and conjugate diameters, and any abscissa, to find the corresponding ordinate.

RULE. As the transverse is to the conjugate, so is the mean proportional between the abscissas to the ordinate.*

1. If the transverse be 24, the conjugate 21, and the less abscissa A D 8; required the ordinate?

NOTE. The less abscissa added to the transverse gives the greater.



$$24 : 21 :: \surd (32 \times 8) : \frac{21 \surd (32 \times 8)}{24} = 14$$ the ordinate.

2. If the transverse axis of an hyperbola be 120, the less abscissa 40, the conjugate 72; required the ordinate? *Ans.* 48.

3. The transverse axis being 60, the conjugate 36, and the less abscissa 20; what is the ordinate? *Ans.* 24.

* See Appendix, Demonstration 56.

PROBLEM XVII.

Given the transverse, conjugate, and ordinate, to find the abscissa.

RULE. To the square of half the conjugate, add the square of the ordinate, and extract the square root of the sum. Then say,

As the conjugate is to the transverse, so is that square root to half the sum of the abscissas.

Then to this half sum, add half the transverse, for the greater abscissa; and from the half sum take half the transverse for the less abscissa.*

1. If the transverse be 24, and the conjugate 21; required the abscissas to the ordinate 14?

$$\begin{array}{ll} 10{\cdot}5 = \tfrac{1}{2}\text{ conjugate} & 14 = \text{ordinate.} \\ 10{\cdot}5 & 14 \\ \hline 110{\cdot}25 & 196 \\ 196 & \\ \hline \end{array}$$

306·25 the square root of which is 17·5; then 21 : 24 :: 17·5 : 20 = half sum, 20 + 12 = 32 the greater abscissa, and 20 — 12 = 8 the less abscissa.

2. The transverse is 120, the ordinate 48, and the conjugate 72; required the abscissas? *Ans.* 40 and 160.

PROBLEM VIII.

Given the conjugate, ordinate, and abscissas, to find the transverse.

RULE. To or from the square root of the sum of the squares of the ordinate and semi-conjugate, add or subtract the semi-conjugate, according as the less or greater abscissa is used;

* See Appendix, Demonstration 57.

then, as the square of the ordinate is to the product of the abscissa and conjugate, so is the sum or difference, above found, to the transverse.*

1. Let the conjugate be 21, the less abscissa 8, and its ordinate 24; required the transverse?

$$\frac{21 \times 8 \times \sqrt{(14^2 + \frac{21^2}{4})} + 10\frac{1}{2}}{14^2} =$$

$= 3 \times \sqrt{(3^2 + 4^2)} + 3) = 3 \times (5 + 3) = 24$ the transverse.

2. The conjugate axis is 72, the less abscissa 40, the ordinate 48; required the transverse? *Ans.* 120.

3. The conjugate is 36, the less abscissa 20, and its ordinate 24; required the transverse? *Ans.* 60.

PROBLEM XIX.

Given the abscissa, ordinate, and transverse diameter, to find the conjugate.

Rule. As the mean proportional between the abscissas is to the ordinate, so is the transverse to its conjugate.†

1. What is the conjugate to the transverse 24, the less abscissa being 8, and its ordinate 14?

$$\frac{24 \times 14}{\sqrt{(32 \times 8)}} = 21 \text{ the conjugate.}$$

2. The transverse diameter is 60, the ordinate 24, and the less abscissa 20; what is the conjugate? *Ans.* 36.

PROBLEM XX.

Given any two abscissas, X, x, and their ordinates, Y, y, to find the transverse to which they belong.

Rule. Multiply each abscissa by the square of the ordinate belonging to the other; multiply also the square of each abscissa by the square of the other's ordinate; then

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* See Appendix, Demonstration 68.

† See Appendix, Demonstration 69.

divide the difference of the latter products by the difference of the former; and the quotient will be the transverse diameter to which the ordinates belong.*

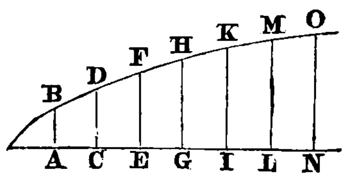
1 If two absciassas be 1 and 8, and their corresponding ordinates $4\frac{3}{8}$ and 14, required the transverse to which they belong?

Here $\frac{8^2 \times 4\frac{3}{8} \times 4\frac{3}{8} - 1^2 \times 14^2}{1 \times 14^2 - 8 \times 4\frac{3}{8} \times 4\frac{3}{8}} = \frac{35 \times 35 - 14 \times 14}{14 \times 14 - 35 \times 4\frac{3}{8}}$
$= \frac{5 \times 5 - 2 \times 2}{2 \times 2 - 5 \times \frac{5}{8}} = \frac{21 \times 8}{7} = 24$, the transverse.

PROBLEM XXI.

To find the area of a space A N O B, *bounded on one side by the curve of a hyperbola, by means of equi-distant ordinates.*

Let A N be divided into any given number of equal parts, A C, C E, E G, &c., and let perpendicular ordinates A B, C D, E F, &c., be erected, and let these ordinates be terminated by any hyperbolic curve B D F, &c.; and let A = A B + N O, B = C D + G H + L M, &c., and C = E F + I K, &c.; then the common distance A C, of the ordinates, being multiplied by the sum arising from the addition of A, 4 B, and 2 C, and one-third of the product taken will



be the area, very nearly. That is, $\frac{A + 4\,B + 2\,C}{3} \times D =$ the area putting D = A C.†

* See Appendix, Demonstration 60.
† See Appendix, Demonstration 61.

1. Given the lengths of 9 equi-distant ordinates, viz., 14, 15, 16, 17, 18, 20, 22, 23, 25 feet, and the common distance 2 feet; required the area? *Ans.* $300\frac{2}{3}$ feet.

2. Given the lengths of 3 equi-distant ordinates, viz., A B = 5, C D = 7, and E F = 8, also the length of the base A E 10; what is the area of the figure A B F E?

Ans. $68\frac{1}{3}$ feet.

3. If the length of the asymptote of a hyperbola be 1, and there be 11 equi-distant ordinates between it and the curve, the common distance of the ordinates will then be $\frac{1}{10}$, and from the nature of the curve their lengths will be $\frac{10}{10}$, $\frac{10}{11}$, $\frac{10}{12}$, $\frac{10}{13}$, $\frac{10}{14}$, $\frac{10}{15}$, $\frac{10}{16}$, $\frac{10}{17}$, $\frac{10}{18}$, $\frac{10}{19}$, $\frac{10}{20}$; what is the area of the curved figure? *Ans.* ·69315021.

This formula will answer for finding the area of all curves by using the sections perpendicular to the axis. The greater the number of ordinates employed, the more accurate the result; but in real practice three or five are in most cases sufficient.

PROBLEM XXII.

To find the length of any arc of an hyperbola beginning at the vertex.

RULE.

I. To 19 times the square of the transverse, add 21 times the square of the conjugate; also to 9 times the square of the transverse add, as before, 21 times the square of the conjugate, and multiply each of these sums by the abscissa.

II. To each of these two products, thus found, add 15 times the product of the transverse and the square of the conjugate.

III. Then, as the less of these results is to the greater, so is the ordinate to the length of the curve, nearly.*

* See Appendix, Demonstration 62.

1. In the hyperbola B A C, the transverse diameter is 80, the conjugate 60, the ordinate B D 10, and the abscissa A D 2; required the length of the arc B A C? (Fig. p. 73.)

Here $2(19 \times 80^2 + 21 \times 60^2) = 2(121600 + 75600) = 394400$.

And $2(9 \times 80^2 + 21 \times 60^2) = 2(57600 + 75600) = 266400$.

Whence $15 \times 80 \times 60^2 + 394400 = 4320000 + 314400 = 4714400$.

And $15 \times 80 \times 60^2 + 266400 = 4320000 + 266400 = 4586400$.

Then $4586400 : 4714400 :: 10 : \frac{47144000}{4586400} = 10{\cdot}279 = A\,B$.

Hence $A\,B\,C = 10{\cdot}279 \times 2 = 20{\cdot}558$.

2. In the hyperbola B A C, the transverse diameter is 80, the conjugate 60, the ordinate B D 10, and the abscissa A D 2·1637; required the length of the arc A B?

Ans. 10·3005.

PROBLEM XXIII.

Given the transverse axis of a hyperbola, the conjugate, and the abscissa, to find the area.

RULE.

I. To the product of the transverse and abscissa, add $\frac{5}{7}$ of the square of the abscissa, and multiply the square root of the sum by 21.

II. Add 4 times the square root of the product of the transverse and abscissa, to the preceding product, and divide the sum by 75.

III. Divide 4 times the product of the conjugate and abscissa by the transverse ; this quotient, multiplied by the former quotient, will give the area of the hyperbola, nearly.*

1. In the hyperbola B A C, (see figure, page 73,) the transverse axis is 30, the conjugate 18, and the abscissa A D is 10 ; what is the area ?

Here $21\sqrt{(30 \times 10 + \frac{5}{7} \times 10^2)} = 21\sqrt{(300 + 71{\cdot}42857)} = 21\sqrt{(371{\cdot}42857)} = 21 \times 19272 = 404{\cdot}712$;

And $\frac{4\sqrt{(30 \times 10)} + 404{\cdot}712}{75} = \frac{4 \times 17{\cdot}3205 + 404{\cdot}712}{75}$

$= \frac{69{\cdot}282 + 404{\cdot}712}{75} = \frac{473994}{75} = 6{\cdot}3199.$

Whence $\frac{18 \times 10 + 4}{30} \times 5{\cdot}3199 = 24 + 6{\cdot}3199 = 151{\cdot}6776$, the area required.

2. What is the area of an hyperbola whose abscissa is 25, the transverse and conjugate being 50 and 30 ?

Ans. 805·0909.

3. The transverse axis is 100, the conjugate 60, and abscissa 50 ; required the area ? *Ans.* 322·3633584.

* See Appendix, Demonstration 63.

MENSURATION OF SOLIDS.

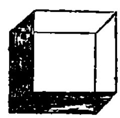
SECTION IV.

DEFINATIONS.

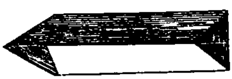
1. A solid is that which has length, breath, and thickness.

2. The solid content of any body is the number of cubic inches, feet, yards, &c., it contains.

3. A *cube* is a solid, having six equal sides at right angles to one another.

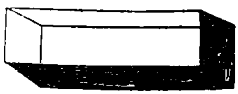


4. A *prism* is a solid whose ends are plane figures which are parallel, equal, and similar. Its sides are parallelograms.



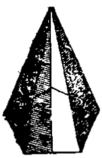
It is called a triangular prism, when its ends are triangles; a square prism, when its ends are squares; a pentagonal prism, when its ends are pentagons; and so on.

5. A *parallelopipedon* is a solid having six rectangular sides, every opposite pair of which are equal and parallel.



6. A *cylinder* is a round solid, having circular ends, and may be conceived to be described by the revolution of a rectangle about one of its sides, which remains fixed.

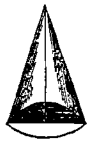
7. A *pyramid* is a solid, having a plane figure for its base; and whose sides are triangles meeting in a point, called the vertex.



Pyramids have their names from their bases, like prisms.

When the base is a triangle, the solid is called a triangular pyramid; when the base is a square, it is called a square pyramid; and so on.

8. A *cone* is a round pyramid, having a circle for its base.



9. A *sphere* is a round solid, which may be conceived to be formed by the revolution of a semicircle about its diameter which remains fixed

10. The axis of a solid is a line joining the middle of both ends.

11. When the axis is perpendicular to the base, the solid is called a right prism or pyramid, otherwise it is oblique.

12. The height or altitude of a solid, is a line drawn from its vertex, perpendicular to its base, and is equal to the axis of a right prism or pyramid; but in an oblique one the altitude is the perpendicular of a right-angled triangle, whose hypothenuse is the axis.

13. When the base is a regular figure it is called a regular prism or pyramid; but when the base is an irregular figure, the solid on it is called irregular.

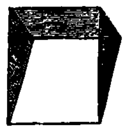
14. The segment of any solid, is a part cut off from the top by a plane parallel to its base.

15. A frustum is the part remaining at the bottom, after the segment is cut off.

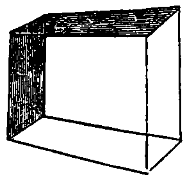
16. A zone of a sphere is a part intercepted between two planes, which are parallel to each other.

17. A *circular spindle* is a solid generated by the revolution of a segment of a circle about its chord, which remains fixed.

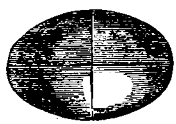
18. A *wedge* is a solid, having a rectangular base, and two of its opposite sides meeting in an edge.



19. A *prismoid* is a solid, having for its two ends two right-angled parallelograms, parallel to each other, and its upright sides are four trapezoids.



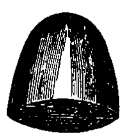
20. A *sepheroid* is a solid, generated by the rotation of a semi-ellipsis about one of its axis, which remains fixed.



When the ellipsis revolves round the transverse axis, the figure is called a prolate, or oblong spheroid; but when the ellipsis revolves round the short axis, the fignre is called an oblate spheroid.

21. An elliptical spindle is a solid, generated by the rotation of a segment of an ellipsis about its chord.

22. A *parabolic conoid*, or paraboloid, is a solid generated by the rotation of a semi-parabola about its axis.



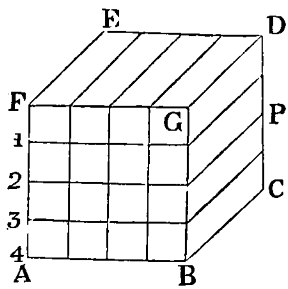
23. An *ungula* or *hoof*, is a part cut off a solid by a plane oblique to the base.

PROBLEM I.

To find the solidity of a cube.

RULE. Multiply the side of a cube by itself, and that product again by the side, for the solidity required.*

1. If the side of a cube be 4 inches, required its solidity?



Here, $4 \times 4 = 16$, the number of cubes of 1 inch deep is the square E F G D, and as the entire solid consists of four such dimensions, its content is $16 \times 4 = 64$ cubic inches.

2. What is the solidity of a cubical piece of marble each side being 5 feet 7 inches? *Ans.* 174 feet, nearly.

3. A cellar is to be dug, whose length, breadth, and depth, are each 12 feet 3 inches; required the number of solid feet in it? *Ans.* 1838 feet 3 inches, nearly.

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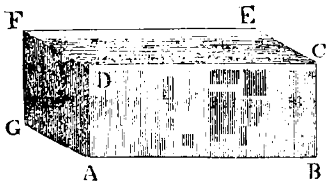
* See Appendix, Demonstration 64.

PROBLEM II.

To find the solidity of a parallelopipedon.

Rule. Multiply continually the length, breadth, and depth together for the solidity.*



1. What is the solidity of a parallelopipedon A B C D E F G, the length of A B being 10 feet, the breadth A G 4 feet, and thickness A D 5 feet?

$$A B \times A G \times A D = 10 \times 4 \times 5 = 200 \text{ feet.}$$

2. A piece of timber is 26 feet long, 10 inches broad, and 8 inches deep; required its solid content? *Ans.* $14\frac{4}{9}$ feet.

3. A piece of timber is 10 inches square at the ends, and 40 feet long; required its content? *Ans.* $27\frac{7}{9}$ feet.

4. A piece of timber 15 inches square at each end, and 18 feet long, is to be measured; required its content, and how far from the end must it be cut across, so that the piece cut off may contain 1 solid foot?

Ans. The solidity is 28·125 feet; and 7·68 in length will make one foot.

5. What length of a piece of square timber will make one solid foot, being 2 feet 9 inches deep, and 1 foot 7 inches broad?

Ans. 2·756 inches in length will make one solid foot.

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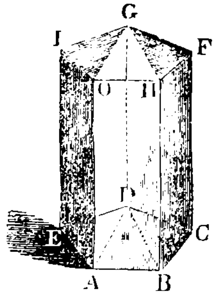
* See Appendix, Demonstration 64.

PROBLEM III.

To find the solidity of a prism.

RULE. Multiply the area of the base by the perpendicular height, and the product will be the solidity.*

1. What is the solidity of a prism A B C F I E, whose



base C A is a pentagon, each side of which being 3·75, and height 15 feet ?

When the side of a pentagon is 1, its area is 1·720477 (Table II.); therefore $1·720477 \times 3·75^2 = 24.1942 =$ the area of the base in square feet ; hence $24·1942 \times 15 =$ 362·913 solid feet, the content.

2. What is the solidity of a square prism, whose length is $5\frac{1}{2}$ feet, and each side of its base $1\frac{1}{3}$ foot.

Ans. $9\frac{7}{9}$ solid feet.

3. What is the solidity of a prism, whose base is an equilateral triangle, each side being 4 feet, and height 10 feet ?

Ans. 69·282 feet.

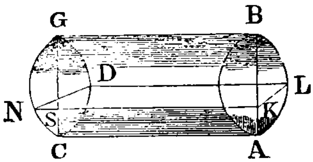
4. What quantity of water will a prismatic vessel contain, its base being a square, each side of which is 3 feet, and height 7 feet ? *Ans.* 63 feet.

* See Appendix, Demonstration 64.

PROBLEM IV.

To find the solidity of a cylinder.

RULE. Multiply the area of the base by its height, and the product will be the solid content.*



1. What is the capacity of a right cylinder A B G C, whose height, and the circumference of its base, are each 20 feet?

First $\frac{20}{3 \cdot 1416}$ = the diameter, half of which multiplied by half the circumference will give the area of the base (Prob. XVIII. Sec. II.), that is, $10 \times \frac{10}{3 \cdot 1416} = \frac{25}{\cdot 7854}$ = the area of the end; then $\frac{25}{\cdot 7854} \times 20 = 636 \cdot 61828$, the content.

2. What is the content of the oblique cylinder A B F E, the circumference of whose base is 20 feet, and altitude A C 20 feet?

As before, the area of the base is $\frac{25}{\cdot 7854}$; then $\frac{25}{\cdot 7854} \times 20 = 636 \cdot 61828$, the solid content, as before.

3. The length of a cylindrical piece of timber is 18 feet, and its circumference 96 inches; how many solid feet in it?

Ans. 91·676 feet.

4. Three cubic feet are to be cut off a rolling-stone 44 inches in circumference; what distance from the end must the section be made? *Ans.* 33·64 inches.

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* See Appendix, Demonstration 64

PROBLEM V.

To find the content of a solid formed by a plane passing parallel to the axis of a cylinder.

RULE. Find by Prob. XXVIII., Sec. II., the area of the base, which, multiplied by the height, will give the solidity.*

1. In the cylinder A B G C, whose diameter is 3, and height 20 feet; let a plane L N pass parallel to the axis, and 1 foot from it; what is the solidity of each of the two prisms into which the cylinder is divided?—(See the last figure.)

$\frac{S\ C}{G\ C} = (\frac{3}{2} - 1) \div 3 = \frac{\frac{1}{2}}{3} = \frac{1}{6} = \cdot 166\frac{2}{3}$ the tabular versed sine, to which, in the Table of Circular Segments, corresponds the area . . . . ·08604117
which taken from . . . . . . ·78539816
leaves the other segment . . . . . ·69935699

Then $3^2 = 9$ which $\times$ ·08604117 $=$ 7·7437053 $=$ seg D C N.

Also 9 $\times$ ·69935699 $=$ 6·29421291 $=$ seg. D G N.

Hence 20 $\times$ 7·7437053 $=$ 15·4874 $=$ the slice L K A C N D; and 20 $\times$ 6·29421699 $=$ 125·88434 $=$ the slice L K B G N D.

2. Suppose the right cylinder, whose length is 20 feet, and diameter 50 feet, is cut by a plane parallel to, and at the distance of, 21·75 feet from its axis; required the solidity of the smaller slice? *Ans.* 1082·95 feet.

PROBLEM VI.

To find the solidity of a pyramid.

RULE. Multiply the area of the base by the one-third of the height, and the product will be the solidity.†

* See Appendix, Demonstration 64.
† See Appendix, Demonstration 65.

1. What is the solidity of a square pyramid, each side of its base being 4 feet, and height 12 feet?

$4 \times 4 = 16$ the area of the base:

Then $16 \times \frac{12}{3} = 64$ feet, the solidity.

2. Each side of the base of a triangular pyramid is 3, and height 30; required its solidity? *Ans.* 38·97117.

3. The spire of a church is an octagonal pyramid, each side at the base being 5 feet 10 inches, and its perpendicular height 45 feet; also each side of the cavity, or hollow part, at the base is 4 feet 11 inches, and its perpendicular height 41 feet; it is required to know how many solid yards of stone the spire contains.

Ans. 32·19738 yards.

4. The height of a hexagonal pyramid is 45 feet, each side of the hexagon of the base being 10; required its solidity? *Ans.* 3897·1143.

PROBLEM VII.

To find the solidity of a cone.

Rule. Multiply the area of the base by one-third of the height, and the product will be the solidity?*

1. The diameter of the base of a cone is 10 feet, and its perpendicular height 42 feet; what is its solidity?

$10^2 = 100 \times {\cdot}7854 = 78{\cdot}54$; then $78{\cdot}54 \times \frac{42}{3} = 1099{\cdot}56$ feet.

2. The diameter of the base of a cone is 12 feet, and its perpendicular height 100; required its solidity?

Ans. 3769·92 feet.

3. The spire of a church, of a conical form, measures 37·6992 feet round its base; what is its solidity, its perpendicular height being 100 feet? *Ans.* 3769·92 feet.

4. How many cubic yards in an upright cone, the circumference of the base being 70 feet, and the slant height 30? *Ans.* 134·09.

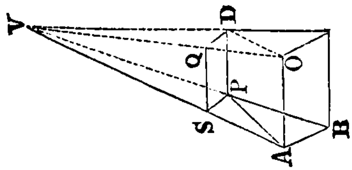
* See Appendix. Demonstration 66.

5. How many cubic feet in an oblique cone, the greatest slant height being 20 feet, the least 16, and the diameter of the base 8 feet? *Ans.* 254·656588 feet.

PROBLEM VIII.

To find the solidity of the frustum of a pyramid.

RULE. Add the areas of two ends and the mean proportional between them together; then multiply the sum by one-third of the perpendicular height, and the product will give the solidity.*



1. In a square pyramid, let A O = 7, P D = 5, and the height O Q = 6; the solidity of the frustum is required.

7 × 7 = 49 = the area of the base.

5 × 5 = 25 = the area of the section S D.

7 × 5 = 35 = the mean proportional between 49 and 25.

Therefore, $\frac{49 + 35 + 25}{3} \times 6 = 218$ = the content of the frustum.

2. What is the content of a pentagonal frustum, whose height is 5 feet, each side of the base 1 foot 6 inches, and each side of the less end 6 inches.

Ans. 9·31925 cubic feet.

3. What is the content of a hexagonal frustum, whose height is 6 feet, and the side of the greater end 18 inches, and of the less 12 inches? *Ans.* 24·681724.

4. How many cubic feet in a squared piece of timber, the areas of the two ends being 504 and 372 inches, and its length 31½ feet? *Ans.* 95·447 feet.

---

* See Appendix, Demonstration 67.

5. What is the solidity of a squared piece of timber, its length being 18 feet, each side of the greater base 18 inches, and each side of the small end 12 inches?

Ans. 28·5.

PROBLEM IX.

To find the solidity of the frustum of a cone.

RULE. Add the two ends, and the mean proportional between them together, then multiply one-third of the sum by the perpendicular height, and the product will be the content.*

1. How many solid feet in a tapering round piece of timber, whose length is 26 feet, and the diameter of the ends 22 and 18 inches respectively?

Here $22^2 \times \cdot 7854 = 380 \cdot 134$ inches, the area of the greater end, and

$18^2 \times \cdot 7854 = 254 \cdot 47$ inches = the area of the less end, $(380 \cdot 134 \times 254 \cdot 47)^{\frac{1}{2}} = 311 \cdot 018$ = the mean proportional between the areas of the ends; then by the rule

$$\frac{254 \cdot 47 + 380 \cdot 134 + 311 \cdot 018}{3} \times 26 \times 12 = 98345 \text{ cubic}$$

inches = 56·9 cubic feet, the answer.

2. How many cubic feet in a round piece of timber, the diameter of the greater end being 18 inches, and that of the less 9 inches, and length 14·25 feet?

Ans. 14·68943 feet.

3. What is the solid content of the frustum of a cone, whose height is 1 foot 8 inches, and the diameters of the ends 2 feet 4 inches, and 1 foot 8 inches?

Ans. 5·284.

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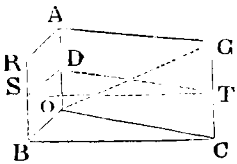
* See Appendix, Demonstration 68.

PROBLEM X.

To find the solidity of a wedge.

RULE I. Add the three parallel edges together, and multiply one-third of the sum by the area of that section of the wedge which is perpendicular to these three edges, and the product will give the content.*



NOTE. When the quadrangular sides are parallelograms, the wedge is a triangular prism, having for its base the triangle B O C; when the quadrangles are rectangular, A O is the height of the prism, and the area of the triangle B O C multiplied by A O will give its content; when the triangle B O C is isosceles and perpendicular to the plane A C, the wedge is of the common kind; C G is its edge, and A R B O its back.

RULE II. To twice the length of the base, add the length of the edge, multiply the sum by the breadth of the base, and the product by the height of the wedge, and one-sixth of the last product will be the solidity, that is, $(2\,L + l) \times \frac{1}{6}\,b\,h$, by putting $L =$ R B, the length off the base $l =$ G C, the length of the edge, $b =$ A R, the breadth of the base, $h =$ the perpendicular height of the wedge.†

1. Let A O = 4, G C = 3, R B $2\frac{1}{2}$, the perpendicular D T = 12, and p the perpendicular distance of B R from the plane of the face A C = $3\frac{1}{2}$ feet; required the solid content?

$$\frac{4 + 3 + 2\frac{1}{2}}{3} \times 12 \times \frac{3\frac{1}{2}}{2} = 66\frac{1}{2} \text{ cubic feet.}$$

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* See Appendix, Demonstration 69.
† See Appendix, Demonstration 70.

2. The perpendicular height from the point T to the middle of the back A B is 24·8, the length of the edge C G 110 inches, the base R B 70 inches, and its breadth A R 30 inches; required the solidity?

Ans. 31000 cubic inches.

3. How many cubic inches in a wedge whose altitude is 14 inches, its edge 21 inches, the length of its base 32 inches, and its breadth $4\frac{1}{2}$ inches?

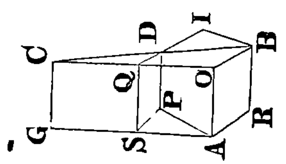
Ans. 892·5 cubic inches.

PROBLEM XI.

To find the solidity of a prismoid, which is the frustum of a wedge.

RULE. By either of the foregoing rules, find the solidity of two wedges whose bases are the two ends of the frustum, and height the distance between them, and the sum of both will be the solidity of the prismoid or frustum.*

1. In the prismoid A B P Q, there is given R B = 18, A O = 27, P D = 21, S Q = 24, B O = 12, D Q = 4, and B I = 30; what is its solidity?



$\frac{18 + 27 + 21}{3} \times \frac{30 \times 12}{2} = 3960$ = the content of the greater wedge, and $\frac{24 + 27 + 21}{3} \times \frac{30 \times 4}{2} = 1440$, the content of the other; then $3960 + 1440 = 5400$, the content of the frustum.

* See Appendix, Demonstration 71.

2. What is the solidity of a piece of wood in the form of a prismoid, whose ends are rectangles, the length and breadth of one being 1 foot 2 inches and 1 foot respectively, and the corresponding sides of the other 6 and 4 inches respectively; the perpendicular height being $30\frac{1}{2}$ feet?

Ans. 18·074 cubic feet.

NOTE. The following rule will answer for any prismoid, of whatever figure each end may be.

RULE. If the bases be dissimilar rectangles, take two corresponding dimensions, and multiply each by the sum of double the other dimension of the same end, and the dimension of the other end corresponding to this last dimension; then multiply the sum of the products by the height, and one-sixth of the last product will be the solidity.*

PROBLEM XII.

To find the solidity of a cylindroid; or the frustum of an elliptical cone.

RULE.

I. To the longer diameter of the greater end, add half the longer diameter of the less end, and multiply the sum by the shorter diameter of the greater end.

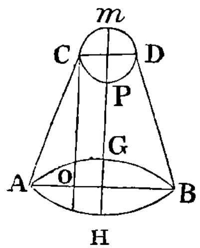
II. To the longer diameter of the less end, add half the longer diameter of the greater end, and multiply the sum by the shorter diameter of the less end.

III. Add the two preceding products together, and multiply the sum by ·2618 (one-third of ·7854) and then by the height; the last product will be the solidity.†

1. Let A B C D be a cylindroid, the base of which is an ellipsis, whose two diameters are 40 and 20 inches, the

* See Appendix, Demonstration 72.
† See Appendix, Demonstration 73.

top a circle, whose diameter is 30 inches; what is its solidity, allowing the height to be 10 feet?



$$(A\,B + \tfrac{1}{2}\,C\,D) \times G\,H = (40 + 15) \times 20 = 1100$$
$$(C\,D + \tfrac{1}{2}\,A\,B) \times m\,P = (30 + 20) \times 30 = 1500$$

sum = 2600

Then (2600 × ·2618 × 10) 6806·8, which, divided by 144, gives 47·27 feet, the answer.

2. The transverse diameter of the greater base of a cylindroid is 13, and conjugate 8; the transverse diameter of the less base 10, and conjugate 5·2; what is the solidity of the cylindroid, its height being 12? *Ans.* 721·93968.

3. The transverse diameter at the top of the cylindroid is 12 inches, and conjugate 7; the longer diameter at the bottom is 14 inches, and shorter 12, and its height 10 feet; required its solidity? *Ans.* 6·78 feet.

PROBLEM XIII.

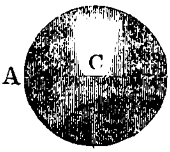
To find the solidity of a sphere.

Rule I. Multiply the cube of the diameter by ·5236, and the product will be the content.

Rule II. Multiply the diameter by the circumference of the sphere, and the product multiplied by one-sixth part of the diameter will be the solidity.*

* See Appendix, Demonstration 74.

1. Suppose the earth to be a perfect sphere, and its diameter $7957\frac{3}{4}$ miles, how many solid miles does it contain?



$7957\frac{3}{4} \times 3{\cdot}1416$ = the circumference of the earth (Prob. XVI., Sec. II.); then

$7957\frac{3}{4} \times 3{\cdot}1416 \times 7957\frac{3}{4} = 198943750$ = the surface of the sphere; then

$198943750 \times 7957\frac{3}{4} \times \frac{1}{6} = 263857437760$ miles the solidity by Rule II.

Again, $\cdot5236 \times d^3 = \cdot5236 \times (7957\frac{3}{4})^3 = 263858149120$ miles, the solidity by Rule I., which gives the result too great on account of taking ·5236 a little too great.

2. What is the solidity of a sphere, whose diameter is 24 inches? *Ans.* 7238·2464 cubic inches.

3. What is the solid content of the earth, allowing its circumference to be 25000 miles?

Ans. 263858149120 miles.

4. Required the solidity of a globe whose diameter is 30 feet? *Ans.* 14137·2.

PROBLEM XIV.

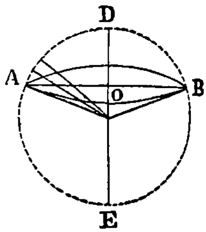
To find the solidity of the segment of a sphere.

Rule I. From three times the diameter of the sphere deduct twice the height of the segment; multiply the remainder by the square of the height, and that product by ·5236; the last product will be solidity.*

Rule II. To three times the square of the radius of the segment's base add the square of its height; multiply this sum by the height, and the product by ·5236; the last result will be the solidity.

* See Appendix, Demonstration 75.

1. What is the solidity of each of the frigid zones, the diameter of the earth being $7957\frac{3}{4}$ miles and half the breadth, or arc of the maridian intercepted between the polar circle and the pole $23\frac{1}{2}$ degrees; that is, A D = $23\frac{1}{2}$ degrees, supposing A B to represent the polar circle.



By Rule I.

As 1 (= tabular radius): $3978\frac{7}{8}$ (= radius of the earth) :: ·0829399 (= tabular versed sine of $23\frac{1}{2}$ degrees) : 330·0074946, the versed sine, or height of the segment.

Then $·5236\ h^2 = (3\ d - 2\ h) = ·5236 \times 330·0074946^2 \times 23213·2350108 = 1323679710$, the solid content.

By Rule II.

As 1 : $3978\frac{7}{8}$ :: ·3987491 (= the tabular sine of $23\frac{1}{2}$ degrees) : 1586·57282526, the radius of the base.

Then $·5236\ h \times (3\ r^2 + h^2) = ·5236 \times 330·0074946 \times 7660544·936 = 1323680299·69$, the solidity.

2. Let A B D O be the segment of the sphere, whose solidity is required. The diameter A B of the base is 16 inches, and the height O D 4 inches.

Ans. 435·6352 cubic inches.

3. Required the solidity of the segment of a sphere, whose diameter is 20 feet, and the height of the segment 5 feet?

Ans. 654·5 feet.

PROBLEM XV.

To find the solidity of the frustum or zone of a sphere.

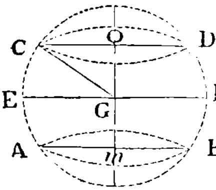
RULE.

I. To the sum of the squares of the radii of the two ends, add $\frac{1}{3}$ of the square of their distance, or of the height of the zone; this sum multiplied by the height of the zone, and the product again by 1·5708, will be the solidity,

II. For the middle zone of a sphere. To the square of the diameter of the end add two-thirds of the square of the height; multiply this sum by the height, and then by ·7854, the last result will be the solidity.

Or, From the square of the diameter of the sphere, deduct one-third of the square of the height of the middle zone; multiply the remainder by the height, and then by ·7854, the last result will be the solidity.*

1. Required the solidity of the frustum of a sphere, the diameter of whose greater end is 4 feet, the diameter of the less end 3 feet, and the height $2\frac{1}{2}$ feet?



$(2^2 + 1·5^2 + \frac{1}{3} \times 2·5^2) \times 1·5708 \times 2·5 = 8\frac{1}{3} \times 3·927 = 32·725$, the solidity of the frustum.

2. What is the solidity of the temperate zone, its breadth being 43 degress, the radius of the top being 1586·57282526, and the radius of the base 3648·86750538, and height 2062 2655?

$(3648·86750538^2 + 1586·57282526^2 + \frac{1}{3} \times 2062·2655^2) \times 2062·2655 \times 1·5708 = 17249136 \times 2062·2955 \times 1·5708 = 55877778668$, the solidity of each temperate zone.

3. Required the solidity of the torrid zone, which extends $23\frac{1}{2}$ degrees on each side of the equator, the diameter being $7957\frac{3}{4}$ miles, and the height 3173·14565052?

$(7957·75^2 - \frac{1}{3} \times 3173·14565052^2) \times 3173·14565052 \times ·7854 = 149455081137$, the answer.

4. What is the solidity of the middle zone of a sphere, whose top and bottom diameters are each 3 inches, and height 4 inches? *Ans.* 61·7848.

5. What is the solid content of a zone, whose greater diameter is 20 feet, less diameter 15 feet, and the height 10 feet? *Ans.* 189·58.

* See Appendix, Demonstration 76.

6. How many solid feet in a zone, whose greater diameter is 12 feet, and less diameter 10; the height being 2?

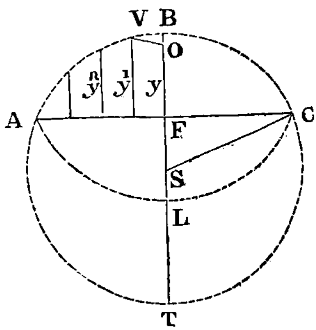
Ans. 195·8264.

PROBLEM XVI.

To find the solidity of a circular spindle.

RULE. Find the distance of the chord of the generating circular segment from the centre of the circle, and also the area of the segment.

Then, from one-third of the cube of half the length of the spindle or half chord of the segment, subtract the product of the central distance, and half the area of the segment; the remainder multiplied by 12·5664, will give the solidity.*



1. Let the axis A C of a circular spindle be 40 inches, and its greater diameter B L 30 inches; what is its solidity?

$20^2 \div 15 = 26\frac{2}{3}$, then $26\frac{2}{3} + 15 = 41\frac{2}{3}$, the diameter of the circle. Again, $\frac{41\frac{2}{3} - 30}{2 \quad 2} = 5\frac{5}{6}$, the central distance.

Now $15 \div 41\frac{2}{3} = \cdot 36$, the area segment corresponding to which is ·254550, which multiplied by the square of $41\frac{2}{3}$, produces 441·92708 the area of the generating segment A B C, the half of which is 220·96354.

Lastly, $(20^3 \div 3) - (5\frac{5}{6} \times 220 \cdot 96354) = 1377 \cdot 71268$, and this multiplied by 12·5664 produces 17312·88862 cubic inches, the solidity required.

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* See Appendix, Demonstration 77.

2. The axis of a circular spindle is 48, and the middle diameter 36; required the solidity of the spindle?

Ans. 29916·6714.

PROBLEM XVII.

To find the solidity of the middle frustum of a circular spindle.

RULE.

I. Find the distance of the centre of the middle frustum from the centre of the circle.

II. Find the area of a segment of a circle, the chord of which is equal to the length of the frustum, and height half the difference between its greatest and least diameters; to which add the rectangle of the length of the frustum and half its least diameter; the result will be the generating surface.

III. From the square of the radius subtract the square of the central distance, the square root of the remainder will give half the length of the spindle.

IV. From the square of half the length of the spindle take one-third of the square of half the length of the middle frustum, and multiply the remainder by the said half length.

V. Multiply the central distance by the generating surface, and subtract this product from the preceding; the remainder, multiply by 6·2832, will give the solidity.*

1 Required the solidity of the middle frustum of a circular spindle, the length D E being 40, the greater diameter Q F 32, and the least diameter P S 24?

First $20^2 \div 4 = 100$, and $100 + 4 = 104$, the diameter of the circle.

Again, $52 - 16 = 36$, the central distance. Also, $\frac{1}{2}(32 - 24) = 4$, and $4 \div 104 = ·038\frac{6}{13}$, the area segment cor-

* See Appendix, Demonstration 78.

responding to which is ·009940, which, multiplied by the square of 104, produces 107·51104, the area of P L Q; and $40 \times 12 = 480$ the area of the rectangle P D E L.

Hence $107·51104 + 480 = 587·51104$, the area of the generating surface P D L E.

Next $\sqrt{(52^2 - 36^2)} = \sqrt{(1408)} = 8\sqrt{(22)} =$ B O half the length of the spindle;

$$\text{And } (1408 - \frac{400}{3} \times 20 = 25493\tfrac{1}{3}.$$

$$\text{Then } 36 + 587·51104 = 21150·39744, \text{ and}$$

$(25493\tfrac{1}{3} - 21150·39744) \times 6·2832 = 27287·5347$, the required solidity.

2. What is the solidity of the middle frustum P S R L of a circular spindle, whose middle diameter F Q is 36, the diameter P S of the end 16, and its length D E 40?

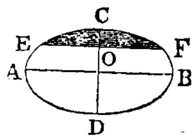
Ans. 29257·2904.

PROBLEM XVIII.

To find the solidity of a spheroid.

RULE. Multiply the square of the revolving axis by the fixed axis, and this product again by ·5236 for the solidity.*

1. What is the solidity of a prolate spheroid whose longer axis A B is 55 inches, and shorter axis C D 33?



Here $33^2 \times 55 \times ·5236 = 31361·022$ cubic inches, the answer.

2. What is the solidity of an oblate spheroid, whose longer axis is 100 feet, and shorter axis 6?

Ans. 31416 cubic feet.

3. What is the solidity of a prolate spheroid, whose axes are 40 and 50? *Ans.* 41888.

4. What is the solidity of an oblate spheroid, whose axes are 20 and 10? *Ans.* 2094·4.

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* See Appendix, Demonstration 79.

PROBLEM XIX.

To find the solidity of the segment of a spheroid, the base of the segment being parallel to the revolving axis of the spheroid.

CASE I.

RULE. From three times the fixed axis, deduct twice the height of the segment, multiply the remainder by the square of the height, and that product by ·5236.

Then say, as the square of the fixed axis is to the square of the revolving axis, so is the product found above to the solidity of the spheroidal segment.*

1. What is the content of the segment of a prolate spheroid, the height O C being 5, the fixed axis 50, and the revolving axis 30?—See last figure.

$50 \times 3 - 5 \times 2 = 150 - 10 = 140$; then
$140 \times 5^2 = 3500$, and $3500 \times \cdot 5236 = 1832{\cdot}6$; then
$25 : 9 :: 1832{\cdot}6 : 659{\cdot}736$, the answer.

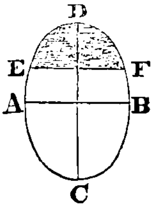
CASE II.

When the base is elliptical, or perpendicular to the revolving axis.

RULE. From three times the revolving axis, take double the height; multiply that difference by the square of the height, and the product again by ·5236.

Then as the revolving axis to the fixed axis, so is the last product to the content.†

2. What is the content of the segment of a spheroid, whose fixed axis is 50, revolving axis 30, and height 6?



$30 \times 3 - 2 \times 6 = 90 - 12 = 78$;
Then $78 \times 6^2 = 2808$; and $2808 \times \cdot 5236 = 1470{\cdot}2688$:
Then $30 : 50 :: 1470{\cdot}2688 : 2450{\cdot}448$, the answer.

* See Appendix, Demonstration 80.
† See Appendix, Demonstration 81.

3. In a prolate spheroid, the transverse or fixed axis is 100, the conjugate or revolving axis is 60, and the height of the segment, whose base is parallel to the revolving axis is 10; required the solidity?

Ans. 5277·888.

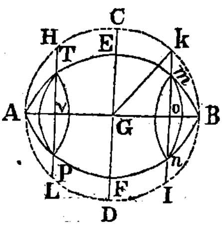
4. If the axis of a prolate spheroid be 10 and 6, required the content of the segment, whose height is 1, its base being parallel to the revolving axis? *Ans.* 5·277888.

PROBLEM XX.

To find the solidity of the middle zone of a spheroid, the diameter of the ends being perpendicular to the fixed axis, the middle diameter, and that of either end being given, together with the length of the zone.

Rule. To twice the square of the middle diameter, add the square of the diameter of the end; multiply the sum by the length of the zone, and the product again by ·2618 for the solidity.*

1. What is the solidity of the middle zone of an oblate spheroid, the middle diameter being 100, the diameter of the end 80, and the length 36?



$100^2 \times 2 + 80^2 = 26400$; then $26400 \times 36 = 950400$, and $950400 \times \cdot 2618 = 248814 \cdot 72$ the answer.

2. What is the solidity of the middle frustum of a spheroid, the greater diameter being 30, the diameter of the end 18, and the length 40?

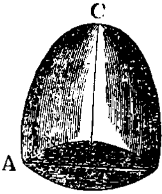
Ans. 22242·528.

* See Appendix, Demonstration 82.

PROBLEM XXI.

To find the solidity of a parabolic conoid.

RULE. Multiply the square of the diameter of its base by ·3927, and that product by the height; the last product will be the solidity.*



1. What is the solidity of the parabolic conoid, whose height is 10 feet, and the diameter of its base 10 feet?

$10^2 \times ·3927 = 39·27$; then $39·27 \times 10 = 392·7$, the solidity required.

2. What is the solidity of a parabolic conoid, whose height is 30, and the diameter of its base 40?

Ans. 18849·6.

3. What is the content of the parabolic conoid, whose altitude is 40, and the diameter of its base 12?

Ans. 2261·952.

4. Required the solidity of a parabolic conoid, whose height is 30, and the diameter of its base 8?

Ans. 753·984.

PROBLEM XXII.

To find the solidity of the frustum of a parabolic conoid.

RULE. Multiply the sum of the squares of the diameters of the two ends by the height, and that product by ·3927; the last product will be the solidity.†

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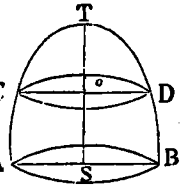
* See Appendix, Demonstration 83.
† See Appendix, Demonstration 84.

1. The greater diameter of the frustum is 10, and the less diameter 5; what is the solidity, the length being 12?



$10^2 = 100$
$5^2 = 25$

125. Then $125 \times 12 = 1500$, and $1500 \times \cdot 3927 = 589\cdot 05$, the solidity.

2. The greater diameter of the frustum of a parabolic conoid is 20, the less 10, and the height 12; what is the solidity? *Ans.* 2357·4.

3. The greater diameter of the frustum of a parabolic conoid is 30, the less 10, and the height 50; required the solidity? *Ans.* 19635.

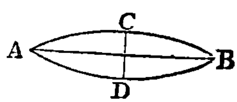
4. The greater diameter of the frustum of a parabolic conoid is 15, the less 12, and the height 8; required the solidity? *Ans.* 1159·8408.

PROBLEM XXIII.

To find the solidity of a parabolic spindle.

RULE. Multiply the square of the middle diameter by ·7854, and that product by the length; then $\frac{8}{15}$ of this product will be the solidity.*

1. The middle diameter C D, of a parabolic spindle is 10 feet, and the length A B is 40; required its solidity?



$10^2 \times \cdot 7854 \times 40 = 3141\cdot 6$ feet.
Then $\frac{8}{15} \times 3141\cdot 6 = 1675\cdot 52$ feet, the answer.

* See Appendix, Demonstration 85.

2. The middle diameter C D, of a parabolic spindle is 12 feet, and the length A B is 30; required the solidity?

Ans. 1809·5616.

3. The middle diameter of a parabolic spindle is 3 feet, and the length 9 feet; required its solidity?

Ans. 33·92928.

4. The middle diameter of a parabolic spindle is 6 feet, and the length 10; required its solidity?

Ans. 150·7968.

5. The middle diameter of a parabolic spindle is 30 feet, and the length 50; required its solidity?

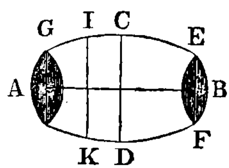
Ans. 18849·6.

PROBLEM XXIV.

To find the solidity of the middle frustum of a parabolic spindle.

RULE. To double the square of the middle diameter, add the square of the diameter of the end; and from the sum subtract $\frac{4}{10}$ of the square of the difference between these diameters; the remainder multiplied by the length, and that product by ·2618, will be the solidity.*

1. In a parabolic spindle, the middle diameter of the middle frustum is 16, the least diameter 12, and the length 20; required the solidity of the frustum?



Here $2 \times 16^2 + 12^2 - \frac{4}{10} \times 4^2 = 512 + 144 - 6·4 = 649·6$; hence $649·6 - 20 \times ·2618 = 3401·3056$, the solidity

* See Appendix, Demonstration 86.

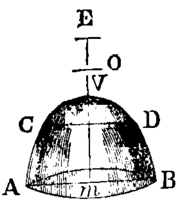
2. The bung diameter of a cask is 30 inches, the head diameter 20 inches, and the length 40 inches; required its content in ale gallons, allowing 282 cubic inches to be equal to one gallon? *Ans.* 80·211 gallons.

3. The bung diameter of a cask is 40 inches, the head diameter 30 inches, and the length 60; how many wine gallons does it contain, 231 cubic inches being equal to one gallon? *Ans.* 276·08 gallons.

PROBLEM XXV.

To find the solidity of a hyperbolic conoid.

RULE. To double the height of the solid add three times the transverse axis, multiply the sum by the square of the radius of the base, and that product by the height, and this last product by ·5236; the result divided by the sum of the height and transverse axis, will give the solidity.*



1. Required the solidity of a hyperbolic conoid, whose height V m is 50, the diameter A B 103·923048, and the transverse axis V E 100?

Here $(2 \times 50 + 3 \times 100) \times \frac{(103{\cdot}923048)^2}{2} = 400 \times 2700 =$ 1080000; and $\frac{1080000 \times 50 \times {\cdot}5236}{150} = 188496$, the solidity.

2. What is the content of an hyperboloid, whose altitude is 10, the radius of its base 12, and the transverse 30? *Ans.* 2073·451151369.

* See Appendix, Demonstration 87.

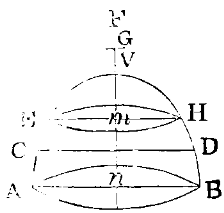
PROBLEM XXVI.

To find the solidity of the frustum of a hyperboloid, or hyperbolic conoid.

RULE. To four times the square of the middle diameter, add the sum of the squares of the greatest and least diameters; multiply the result by the altitude, and that product by ·1309 for the solidity.*

1. Required the solidity of the frustum A C E H D B of a hyperbolic conoid, whose greatest diameter A B is 96, least diameter E H 54, middle diameter C D 76·4264352, and the altitude m n 25 ?

Here 4 C D² + A B² + E H² = (5841 × 4) + 9216 + 2916 = 35496, and 35496 × 25 × ·1309 = 116160·66 the answer.



2. What is the solidity of a hyperboloidal cask, its bung diameter being 32 inches, its head diameter 24, and the diameter in the middle between the bung and head $\frac{8}{5}\sqrt{310}$, and its length 40 inches?

Ans. 24998·69994216 inches.

PROBLEM XXVII.

To find the solidity of a frustum of an elliptical spindle, or any other solid formed by the revolution of a conic section about an axis.

RULE. Add together the squares of the greatest and least diameters, and the square of double the diameter in the

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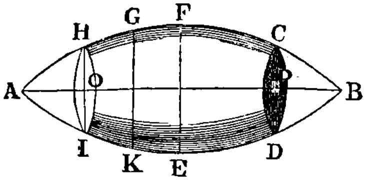
* See Appendix, Demonstration 89.

middle between the two; multiply the sum by the length, and the last product by ·1309 for the solidity.*

1. What is the content of the middle frustum C D I H of any spindle, the length O P being 40, the greatest, or middle



diameter E F 32, the least, or diameter at either end C D 24, and the diameter G K 30·157568?

Here $32^2 + (2 \times 30{\cdot}157568)^2 + 24^2 = 5237{\cdot}89$ sum;

Then $5237{\cdot}89 \times 40 = 209515{\cdot}6$, and

$209515{\cdot}6 \times {\cdot}1309 = 27425{\cdot}7$ the answer.

2. What is the content of the segment of any spindle, the length being 20, the greatest diameter 10, the least diameter at either end 5, and the diameter in the middle between these 8?

Ans. 997·458.

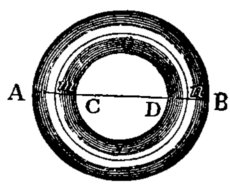
PROBLEM XXVIII.

To find the solidity of a circular ring.

RULE. To the thickness of the ring add the inner diameter; multiply the sum by the square of the thickness, and the product by 2.4674, for the solidity.†

1. The thickness of a cylindrical ring is 2 inches, and the diameter C D 5 inches; required its solidity?

$(2 + 5) \times 4 = 28$; then $28 \times 2{\cdot}4674 = 69{\cdot}0872$ cubic inches, the answer.



2. Required the solidity of an iron ring whose axis forms the cir-

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* See Appendix, Demonstration 89.
† See Appendix, Demonstration 90.

cumference of a circle; the diameter of a section of the ring 2 inches, and the inner diameter, from side to side, 18 inches?

Ans. 197·3925 cubic inches.

3. The thickness of a cylindrical ring is 7 inches, and the inner diameter 20 inches; required its solidity?

Ans. 3264·3702.

4. What is the solidity of a circular ring, whose thickness is 2 inches, and its diameter 12 inches?

Ans. 138·1744 cubic inches.

THE FIVE REGULAR BODIES.

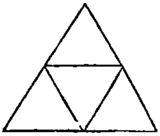
SECTION V.

DEFINITIONS.

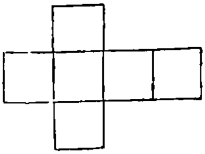
A regular body is a solid contained under a certain number of similar and equal plane figures.

Only five regular bodies can possibly be formed. Because it is proved in Solid Geometry that only three kinds of equilateral and equi-angular plane figures joined together can make a solid angle.

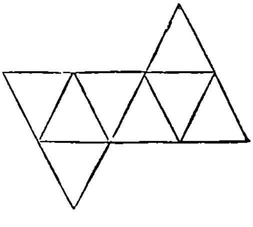
1. The *tetraedron*, or equi-lateral pyramid, is a solid having four triangular faces.*



2. The *hexaedron*, or cube, is a solid having six square faces.

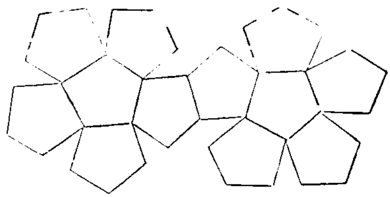


3. The *octaedron* is a regular solid having eight triangular faces.

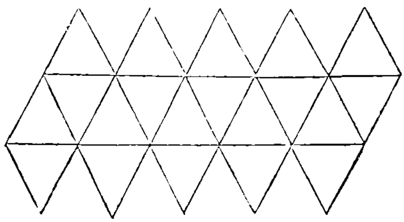


* If figures similar to those annexed to the definitions, be drawn on pasteboard, and cut out, by cutting through the bounding lines, and if the other lines be cut half through, and then the parts be turned up and glued together, the bodies defined will be formed.

4. The *dodecaedron* has twelve pentagonal faces.



5. The *icosaedron* has twenty triangular faces.



PROBLEM I.

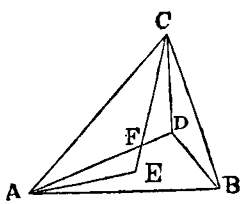
To find the solidity of a tetraedron.

RULE I. Multiply $\frac{1}{12}$ of the cube of the lineal side by the quare root of 2, and the product will be the solidity.

RULE II. Multiply the cube of the length of a side of the ody by the tabular solidity, and the product will be the solidity f the body.* This rule is general for all the regular bodies.

1. If the side of each face of a etraedron be 1; required its solidity?

Here $\frac{1}{12} \times 1^3 \times \sqrt{2} = \frac{1}{12} \times \sqrt{2} = \cdot 11785113$, the solidity.



2. The side of a tetraedron is 2; what is its solidity?

Ans. 203·6467.

* See Appendix, Demonstration 91.

PROBLEM II.

To find the solidity of a hexaedron, or a cube.

RULE. Cube the side for its solidity.*

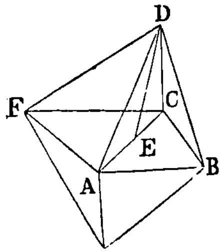
1. If the linear side of a hexaedron be 3, what is its content? *Ans.* $3 \times 3 \times 3 = 27$.

PROBLEM III.

To find the solidity of an octaedron.

RULE. Multiply the cube of the side by the square root of 2, and $\frac{1}{3}$ of the product will be the content.†

1. What is the solidity of an octaedron, when the linear side is 1?



$1^3 \times \sqrt{2} \times \frac{1}{3} = \frac{1}{3}\sqrt{2} =$ ·4714045.

2. What is the solidity of the octaedron, whose linear side is 2? *Ans.* 3·7712.

PROBLEM IV.

To find the solidity of a dodecaedron.

RULE. To 21 times the square root of 5 add 47, and divide the sum by 40; multiply the root of the quotient by 5 times the cube of the lineal side, and the product will be the solidity.‡

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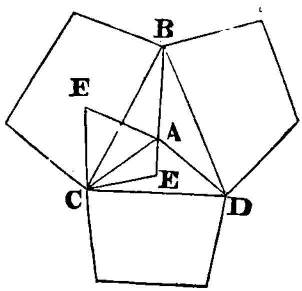
* See Appendix, Demonstration 64.
† See Appendix, Demonstration 92.
‡ See Appendix, Demonstration 93.



1. If the lineal side of the dodecaedron be 1, what is its lidity ?

Here A = 1, consequently $5\ A^3\sqrt{\frac{47 \times 21\sqrt{5}}{40}} =$ 66311896, the content.

2. The side of a regular dodecaedron is 12 inches; how iny cubic inches does it contain ?

Ans. 13241·8694592.

PROBLEM V.

To find the solidity of an icosaedron.

Rule. To 7 add three times the square root of 5, take lf the sum, multiply the square root of this half sum by of the cube of the lineal side, and the product will be solidity.*

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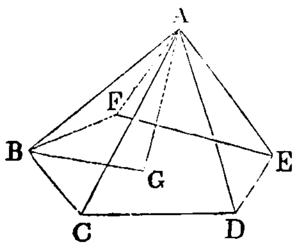
* See Appendix, Demonstration 94.



1. What is the solidity of an icosaedron, whose lineal side is 1?

Let the side be denoted by A. Then A = 1, and consequently

$$\tfrac{5}{6} A^3 \sqrt{\frac{7 + 3\sqrt{5}}{2}} = \tfrac{5}{6} \sqrt{\frac{7 + 3\sqrt{5}}{2}} = 2{\cdot}18169499,$$

the content.

2. What is the solidity of an icosaedron, whose lineal side is 12 feet? *Ans.* 3769·9689 feet.

Note. The following table may be collected from the examples given in the foregoing rules each of which has been demonstrated under its particular head. It has also been demonstrated that the cube of the lineal side of any regular solid multiplied by the tabular number corresponding to the figure, will give its content. It is particularly recommended to the pupil to employ the general rule given in Problem I. whenever the content of any of the five regular bodies is required.

TABLE III.

Showing the solidity of the five regular bodies, the length of a side in each being 1.

No. of sides.	Names.	Solidity.
4	Tetraedron ...	·1178511
6	Hexaedron ...	1·0000000
8	Octaedron ...	·4714045
20	Icosaedron ...	2·1816950
12	Dodecaedron ...	7·6631189

PROBLEM VI.

To find the surface of a tetraedron.*

Rule I. Multiply the square of the linear side by the quare root of 3, and the product will be the whole surface.†

Rule II. Multiply the square of the length of a side of the ody, by the tubular area corresponding to the figure, and the roduct will be the surface of the body. This is a general ule for finding the surfaces of the regular bodies.

1. If the side of a tetraedron be 1, what is its surface ?

Here, $1^2 \times \sqrt{3} = \sqrt{3} = 1{\cdot}7320508$ = the whole urface.

2. The side of the tetraedron is 12; what is its surface !

Ans. 249·4153152.

PROBLEM VII.

To find the surface of a hexaedron, or cube.

Rule. Square the side and multiply it by 6, and the product ill be the surface.‡

1. If the side be 1, what is the surface of a hexaedron ?

$1^2 \times 6 = 6$ the whole surface.

2. If the side be 4, what is the surface of a hexaedron ?

Ans. 96.

---

* Though the next section treats exclusively of the surfaces of solids, and would erefore seem to be the proper place for this problem and the following ones in is section, yet it has been thought more convenient to place together the rules oth for finding the solidities and surfaces of those curious bodies.

† See Appendix, Demonstration 95.

‡ See Appendix, Demonstration 96.

PROBLEM VIII.

To find the surface of an octaedron.

RULE. Multiply the square of the side by the square root of 3, and double the product will be the surface.*

1. If the side of an octaedron be 1, what is its surface?

$2 \times 1^2 \sqrt{3} = 2\sqrt{3} = 3{\cdot}4641016 =$ the whole surface.

2. If the side of an octaedron be 12, what is its superficies? *Ans.* 498·8306304.

3. If the side of an octaedron be 4, what is its surface? *Ans.* 55·4256256.

PROBLEM IX.

To find the superficies of a dodecaedron.

RULE. To 1 add $\frac{2}{5}$ of the root of 5; multiply the root of the sum by 15 times the square of the lineal side, and the product will be the surface.†

1. If the lineal side be 1, what is the surface of a regular dodecaedron?

Here $1^2 \times 15\sqrt{(1 + \frac{2}{5}\sqrt{5})} = 15\sqrt{(1 + \frac{2}{5}\sqrt{5})} =$ 20·645728807, the surface.

2. What is the surface of a dodecaedron, whose lineal side is 2? *Ans.* 82·58292.

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* See Appendix, Demonstration 97.
† See Appendix, Demonstration 98.

PROBLEM X.

To find the superficies of an icosaedron.

RULE Multiply five times the square of the lineal side by the square root of 3, and the product will be the surface.*

1. The side of an icosaedron is 1, what is its surface?

$$5 \times 1^2 \times \sqrt{3} = 5\sqrt{3} = 8 \cdot 66025403.$$

2. What is the surface of an icosaedron whose side is 2?
Ans. 34·641.

3. What is the surface of an icosaedron whose side is 3?
Ans. 77·9423.

Note. In finding the superficial content of the regular bodies, it is particularly recommended to employ the general rule given in Problem VI, in practice in preference to any other. The particular rules given for each solid are introduced merely to find the tabular numbers by which the pupil is to work.

From the examples given in the preceding rules, in which the lineal side of each regular solid is 1, the following tabular numbers may be collected.

TABLE IV.

Showing the surfaces of the five regular bodies, when the linear side is 1.

Number of sides.	Names.	Surface.
4	Tetraedron .	1·7320508
6	Hexaedron .	6·0000000
8	Octaedron .	3·4641016
12	Dodecaedron .	20·6457288
20	Icosaedron .	8·6602540

* See Appendix, Demonstration 90.

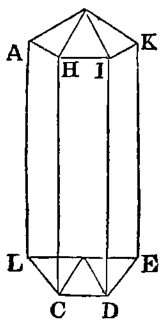
SURFACES OF SOLIDS.

SECTION VI.

PROBLEM I.

To find the surface of a prism.

RULE. Multiply the perimeter of the end of the solid by its length, to the product add the area of the two ends, and the sum will be the surface.*



* See Appendix, Demonstration 100.

1. If the side H I of the pentagon be 25 feet, and height I D 10, what is its surface ?

$25 \times 5 = 125$, the perimeter;

Then $125 \times 10 = 1250 =$ the upright surface;

$25^2 \times 1{\cdot}720477 = 1075{\cdot}298125 =$ the area of one end;

And $1075{\cdot}298125 \times 2 = 2150{\cdot}596250 =$ the area of both ends.

Then $2150{\cdot}5196250 + 1250 = 3400{\cdot}59625 =$ the entire surface;

2. If the side of a cubical piece of timber be 3 feet 6 inches, what is the upright surface and whole superficial content ?

Ans. { 49 feet upright surface.
73 feet 6 in. whole superficial content.

3. If a stone in the form of a parallelopipedon be 12 feet 9 inches long, 2 feet 3 inches deep, and 4 feet 8 inches broad, what is the upright surface and whole superficial content ?

Ans. { 176 feet 4 in. 6 sec. upright surface.
197 feet 4 in. 6 sec. whole sup. content.

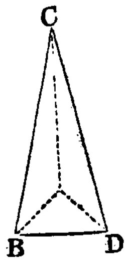
PROBLEM II.

To find the surface of a pyramid.

Rule. Multiply the slant height by half the circumference of the base, and the product will be the surface of the sides, to which add the area of the base for the whole surface.*

Note. The slant height of a pyramid is the perpendicular distance from the vertex to the middle of one of the sides, and the perpendicular height is a straight line drawn from the vertex to the middle of the base.

* See Appendix. Demonstration 101.



1. The slant height of a triangular pyramid is 10 feet, and each side of the base is 1; what is its surface?

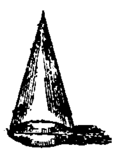
Half circumference	=	$\frac{3}{2}$
Slant height	=	10
Upright surface	=	15
Area of the base	=	·433013
The entire surface	=	15·433013

2. The perpendicular height of a heptagonal pyramid is 13·5 feet, and each side of the base 15 inches; required its surface. *Ans.* 65·0128 feet.

PROBLEM III.

To find the surface of a cone.

RULE. Multiply the slant height by half the circumference of the base, and the product, with the area of the base, will be the whole surface.*



* See Appendix, Demonstration 103.

1. What is the surface of a cone whose side is 20, and the circumference of its base 9?

Here $20 \times \frac{9}{2} = 90 =$ the convex surface.
$9^2 \times \cdot 07958 = 6\cdot 44598 =$ the area of the base.
Then $90 + 6\cdot 44598 = 96\cdot 44598 =$ the whole surface.

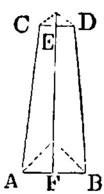
2. The perpendicular height of a cone is 10·5 feet, and the circumference of its base is 9 feet: what is its superficies? *Ans.* 54·1336 feet.

PROBLEM IV.

To find the superficies of the frustum of a right, regular pyramid.

RULE. Add the perimeters of the two ends together, and multiply half the sum by the slant height, the product will be the upright surface; to which add the areas of both ends, and the sum will be the whole surface.*

1. What is the superficies of the frustum of a square pyramid, each side of the greater base A B being 10 inches, and each side of the less base C D 4 inches, and slant height 20 inches?



Here $10 \times 4 = 40$ the perimeter of the greater base.
And $4 \times 4 = 16$ the perimeter of the less end.

Sum 56, the half of which is 28.
Then $28 \times 20 = 560 =$ the upright surface.
$10 \times 10 = 100 =$ the area of the greater base.
$4 \times 4 = 16 =$ the area of the less end.
Hence $560 + 100 + 16 = 676 =$ the whole surface.

2. What is the superficies of the frustum of an octagonal pyramid, each side of the grater base being 9 inches, each side of the less base is 5 inches, and the height 10·5 feet? *Ans.* 52·59 feet.

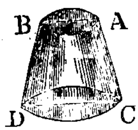
---

* See Appendix, Demonstration 103.

PROBLEM V.

To find the superficies of the frustum of a cone.

Rule. Add the perimeters of both ends together, and multiply half the sum by the slant height, to which add the areas of both ends, for the whole superficies.*



1. If the diameters of the two ends C D and A B are 7 and 3, and the slant height D B 9, what is the whole surface of the frustum A B C D?

$$\frac{7+3}{2} \times 3{\cdot}1416 \times 9 = 141{\cdot}372, \text{ the convex surface.}$$

$7 \times 7 \times {\cdot}7854 = 38{\cdot}4846$, the area of the base C D.

$3 \times 3 \times {\cdot}7854 = 7{\cdot}0686$, the area of the end A B.

Then $141{\cdot}372 + 45{\cdot}5532 = 186{\cdot}9252 =$ the whole surface of the frustum.

2. What is the superficies of the frustum of a cone, whose greater diameter is 18 inches, and less diameter 9 inches and the slant height 171·0592 inches?

Ans. 7572·981.

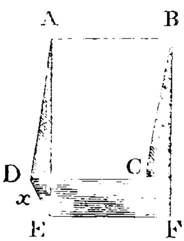
* See Appendix, Demonstration 104.

PROBLEM VI.

To find the superficies of a wedge.

RULE. Find the area of the back, which is a right-angled parallelogram; find the areas of both ends, which are triangles; and also of both sides, which are trapezoids; all these areas added together will evidently be the whole surface.*

1. The back of a wedge is 10 inches long, and 2 inches broad, each of its faces is 10 inches from the edge to the back; required its whole surface?



$10 \times 2 = 20 =$ the area of the back.

$10 \times 10 \times 2 = 200$ the areas of both faces.

$\sqrt{(AE^2 - Ex^2)} = \sqrt{(100 - 1)} = 9{\cdot}949 = Ax$; then

$9{\cdot}949 \times 2 = 19{\cdot}898 =$ areas of both ends.

Hence $200 + 20 + 19{\cdot}889 = 239{\cdot}898 =$ the whole surface of the wedge.

2. The back of a wedge is 20 inches long, and 2 inches broad; each of its faces is 10 inches from the back to the edge; what is its whole surface? *Ans.* 459·898.

PROBLEM VII.

To find the area of the frustum of a wedge

RULE. Find the areas of the back and top sections; of the two faces; and of the two ends; the sums of all the separate results will evidently be the whole surface.

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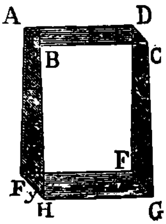
* See Appendix, Demonstration 105.



1. The length and breadth of the back are 10 and 2 inches, the length and breadth of the upper section are 10 and 1 inches, the length of the edge from the back to the upper section is 10 inches; required the whole surface?

$10 \times 2 = 20 =$ the area of the back.
$10 \times 1 = 10 =$ the area of the upper section.

$10 \times 10 \times 2 = 200 =$ the areas of both faces.

$\frac{2-1}{2} = \frac{1}{2} = \cdot 5$, and $\sqrt{(100 - \cdot 25)} = 9 \cdot 98 = \text{B}\,y$.

Then $(2 + 1) \times 9 \cdot 98 = 29 \cdot 94 =$ areas of both ends.
Hence $20 + 10 + 200 + 29 \cdot 94 = 259 \cdot 94$ inches, the answer.

2. The length and breadth of the back are 10 and 4, the length and breadth of the upper section are 5 and 2, and the length of each of the faces is 20; required the whole superfices? *Ans.* 470·78.

PROBLEM VIII.

To find the surface of a globe or sphere.

Rule. Multiply the diameter of the sphere by its circumference, and the product will be its convex surface.*

1. What is the surface of a globe, whose diameter is 24 inches?

$24 \times 3 \cdot 1416 = 75 \cdot 3984$, the circumference:
$75 \cdot 3984 \times 24 = 1809 \cdot 5616$ inches, the answer.

2. What is the surface of the earth, its diameter being 7957¾, and the circumference 25000 miles?

Ans. 198943750 square miles.

† See Appendix, Demonstration 107.

PROBLEM IX.

To find the convex surface of any segment, or zone of a sphere.

Rule. Multiply the circumference of the whole sphere by the height of the segment, or zone, and the product will be the convex surface.*

1. If the diameter of the earth be 7970 miles, the height of the frigid zone will be 252·361283 miles, what is its surface?

Here $7970 \times 3·1416$ = the circumference; then $7970 \times 3·1416 \times 252·361283 = 6318761·107182216$ miles.

2. If the diameter of the earth be 7970 miles, the height of the temperate zone will be 2143·6235535 miles; what is its surface? *Ans.* 53673229·812734532 miles.

3. If the diameter of the earth be 7970 miles, the height of the torrid zone will be 3178·030327 miles; what is its surface? *Ans.* 79573277·600166504 miles.

Note. By adding the surfaces of both frigid zones and both temperate zones to the surface of the torrid zone, the sum 199557259·44, is the surface of the earth in square miles.

4. The diameter of a sphere is 3, the height of the segment 1; what is its convex surface? *Ans.* 9·4248.

5. The circumference of a sphere is 33, the height of the segment is 4; what is its convex surface? *Ans.* 132.

PROBLEM X.

To find the surface of a cylinder.

Rule. Multiply the circumference by the length, and the product will be the convex surface; to which add the area of the two ends, and the sum will be the surface of the entire solid.†

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* See Appendix, Demonstration 108.
† See Appendix, Demonstration 109.

1. What is the entire surface of a cylinder, whose length is 10 feet, and its diameter 5 feet?

$$\begin{array}{r} 3{\cdot}1416 \\ 5 \\ \hline 15{\cdot}7080 \end{array}$$

15·7080, then 15·708 × 10 = 157·08 the convex surface.

5 × 5 × ·7854 = the area of the base; then

2 × 5 × 5 × ·7854 = 50 × ·7854 = 39·2700 the area of both bases; then

157·08 + 39·27 = 196·35, the answer.

2. Required the superficial content of a cylinder, whose diameter is 21·5 inches, and height 16 feet. *Ans.* 95·1 ft.

3. What is the surface of a cylinder whose diameter is 20·75 inches, and its length 55 inches? *Ans.* 29·595 ft.

PROBLEM XI.

To find the superficies of a circular cylinder.

Rule. Add the inner diameter to the thickness of the ring, multiply the sum by the thickness, and that product by 9·8696 for the superfices.*

1. The thickness A C of a cylindrical ring is 2 inches, the inner diameter C D 5 inches; required its superficial content.

Here (2 + 5) × 2 = 14; then 14 × 9·8696 = 138.1744 square inches.

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* See Appendix, Demonstration 110.

PROBLEM XII.

To find the surface of a parallelopipedon.

RULE. Find the area of the sides and ends, and their sum will be the surface.

1. What is the surface of a parallelopipedon, whose length is 10 feet, breadth 4, and depth 2? *Ans.* 136 feet.

$10 \times 4 = 40 =$ the area of one face.
$10 \times 4 = 40 =$ the area of its opposite face.
$10 \times 2 = 20 =$ the area of one face.
$10 \times 2 = 20 =$ the area of its opposite face.
$4 \times 2 = 8 =$ the area of one end.
$4 \times 2 = 8 =$ the area of its opposite end.

$136 =$ the surface of the whole solid.

2. The length of a parallelopipedon is 5, breadth 4, and depth 3; what is its surface? *Ans.* 94.

SECTION VII.

DESCRIPTION OF THE CARPENTER'S RULE.

This instrument is sometimes called the sliding rule, and is used in measuring timber and artificer's works. By it dimensions are taken and contents computed.

It consists of two equal pieces of boxwood, each one foot long, connected by a folding joint.

One face of the rule is divided into inches and half quarters, or eighths. On the same side or face are several plane scales, divided by diagonal lines into twelfths; these are chiefly used in planning dimensions which are taken in feet and inches. The edge of the rule is divided decimally; that is, each foot is divided into ten equal parts, and each of those again into 10 equal parts. By means of this last scale, dimensions are taken in feet, tenths, and hundredths; and then multiplied as common decimal numbers.

In one of these equal pieces there is a slider, on which are marked the two letters B, C; on the same face are marked the letters A, D. The same numbers serve for both these two middle lines, the one being above the numbers, and the other below.

Three of these lines, viz., A, B, C, are called double lines, as they proceed from 1 to 10 twice over. These three lines are exactly alike both in division and numbers, and are numbered from the left hand towards the right, 1, 2, 3, 4, 5, 6, 7, 8, 9 to 1, which stands in the middle; the numbers then go on, 2, 3, 4, 5, 6, 7, 8, 9 to 10, which stands at the right-hand end of the rule.

These four lines are logarithmic ones; the lower line D, is a single one, proceeding from 4 to 40, and is called the girt line, from its use in finding the content of timber.

Upon it are also marked W G at 17·15, A G at 18·95, and 1 G at 18·8. These are the wine, ale, and imperial guage points.

On this face is a table of the value of a load, or 50 cubic feet, of timber, at all prices from 6 pence to 2 shillings per foot.

To ascertain the values of the figures on the rule, which have no determinate value of their own, but depend upon the value set on the unit at the left hand of that part of the rule marked 1, 2, 3, &c.; if the first unit be called 1, the 1 in the middle will be 10, the other figures that follow will be 20, 30, 40, &c., and the 10 at the right-hand end will be 100. If the left-hand unit be called 10, the 1 in the middle will be 100, and the following figures will be 200, 300, 400, 500, &c.; and the 10 at the right hand end will be 1000. If the 1 at the left-hand end be called 100, the middle 1 will, be 1000, and the following figures will be 2000, 3000, 4000, &c., and the 10 at the right hand will be 10,000. From this it appears that the values of all the figures depend upon the value set on the first unit.

The use of the double line A, B, is to find a fourth proportional, and also to find the areas of plane figures.

The use of the several lines described here is best learned in practice.

If the rule be unfolded, and the slider moved out of the grove, the back part of it will be seen divided like the edge of the rule, all measuring 3 feet in length.

Some rules have other scales and tables delineated upon them; such as a table of board measure, one of timber measure, another for showing what length for any breath will make a square foot. There is also a line showing what length for any thickness will make a solid foot.

THE USE OF THE SLIDING RULE.

PROBLEM I.

To multiply numbers together.

Set 1 on B to the multiplier on A; then against the mu tiplicand on B, stands the product on A.

1. Multiply 12 and 18 together.

Set 1 on B to 12 on A; then against 18 on B stands th product 216 on A.

2. Multiply 36 by 22.

Set 1 on B, to 36 on A; then as 22 on B goes beyon the rule, look for 2·2 on B, and against it on A stands 79·2 but as the real multiplier was divided by 10, the produc 79·2 must be multiplied by 10, which is effected by takin away the decimal point, leaving the product 792.

PROBLEM II.

To divide one number by another.

Set the divisor on A, to 1 on B; then against the div dend on A, stands the quotient on B.

1. Divide 11 into 330.

Set the divisor 11 on A, to 1 on B; then against th dividend 330 on A, stands the quotient 30 on B.

2. Divide 7680 by 24.

Set 24 on A, to 1 on B; then because 7680 goes beyon the rule on A, look for 768 (the tenth of 7680) on A, an against it stands 32 on B; but as the tenth of the dividen was taken that the number should fall within the compass o the scale A, the quotient 32 must be multiplied by 10, whic gives 320 for the answer.

PROBLEM III.

To square any number.

Set 1 upon C, to 10 upon D; then if you call the 10 upon D 1, the 1 on the C will be 10; if you call the 10 on D, 10, then the 1 on C will be 100; if you call the 10 on D, 100, then the 1 on C will be 1000; this being understood, you will observe that against every number on D, stands its square on C.

1. What are the squares of 25, 30, 12, and 20?

Proceeding according to the above directions, 625 stands against 25, 900 against 30, 144 against 12, 400 against 20.

PROBLEM IV.

To extract the square root of a number.

Set 1 or 100, &c., on C, to 1 or 10, &c., on D; then against every number found on C, stands its root on D.

1. What are the square roots of 529 and 1600?

Proceeding according to the above directions, opposite 529 stands 23; opposite 1600 stands 40, and so on.

PROBLEM V.

To find a mean proportional between two numbers as 9 and 25.

Set the number 9 on C, to the same 9 on D; then against 25 on C, stands 15 on D, the required mean proportional.

The reason of this may be seen from the proportion, viz., 9 : 15 :: 15 : 25.

1. What is the mean proportional between 29 and 430?

Set one number 29 on C, to the same on D; then against the other number 430 on C, stands 112 on D, which is the mean proportional, nearly.

PROBLEM VI.

To find a third proportional to two numbers, as 21 *and* 32.

Set the first number 21, on B, to the the second, 82, on A; then against the second, 32, on B, stands 48·8 on A, which is the required third proportional.

PROBLEM. VII.

To find a fourth proportional to three given numbers.

Set the first term on B, to the second on A; then against the third term on B, stands the fourth on A.

If either of the middle numbers fall beyond the line, take one-tenth part of that number, and increase the fourth number found, ten times.

1. Find a fourth proportional to 12, 28, and 114.

Set the first term, 12, on B, to the second term, 28, on A; then against the third term 114 on B, stands 266 on A, which is the answer.

TIMBER MEASURE.

PROBLEM I.

To find the superficial content of a board or plank.

RULE. Multiply the length by the breadth, and the product will be the area.

NOTE. When the plank is broader at one end than at the other, add both ends together, and take half the sum for a mean breadth.

BY THE CARPENTER'S RULE.

Set 12 on B, to the breadth in inches on A; then against the length in feet, on B, will be found the superficies on A in feet.

1. If a board be 12 feet 6 inches long, and 2 feet 3 inches broad, how many feet are contained in it?

12 . 6	12·5
2 . 3	2·25
25 . 0	625
3 . 1 . 6	250
	250
28 . 1 . 6 *Ans.*	
	28.125 *Ans.*

BY THE CARPENTER'S RULE.

As 12 on B : 27 on A :: 12·5 on B : 28·125 on A.

2. What is the value of a board whose length is 8 feet 6 inches, and breadth 1 foot 3 incnes, at 5*d.* per foot?
Ans. 4*s.* 5*d.*

3. What is the value of a board whose length is 12 feet 9 inches, and breadth 1 foot 3 inches, at 5*d.* per foot?
Ans. 6*s.* 7½*d.*

4. What is the value of a plank whose breadth at one end is 2 feet, and at the other end 4 feet, at 6*d.* per foot, the length being 12 feet? *Ans.* 18*s.*

5. How many square feet in a board, whose breadth at one end is 15 inches, and at the other 17 inches the length being 6 feet? *Ans.* 8.

6. How many square feet in a plank, whose length is 20 feet, and mean breadth 3 feet 3 inches? *Ans.* 65.

PROBLEM II.

To find the solid content of squared or four-sided timber.

RULE. Take half the sum of the breadth and depth in the middle (that is, the quarter girt), square this half sum, and multiply it by the length for the solid content.*

BY THE CARPENTER'S RULE.

As 12 on D : length on C :: quarter girt on D : the solid content on C.

1. If a piece of squared timber be 3 feet 9 inches broad, 2 feet 7 inches deep, and 20 feet long ; how many solid feet are contained therein ?

```
      3 . 9
      2 . 7
    -------
   2) 6 . 4
    -------
      3 . 2 quarter girt.
      3 . 2
    -------
      9 . 6
          6 . 4
    -----------
   10 . 0 . 4 square of the quarter girt.
           20 length of the piece.
    -----------
  200 . 6 . 8 solid content.
```

BY THE CARPENTER'S RULE.

As 12 on D : 20 on C :: 38 on D : $200\frac{1}{2}$ on C.

2. A squared piece of timber is fifteen inches broad, 15 inches deep, and 18 feet long ; how many feet does it contain ?

Ans. $28\frac{1}{8}$ feet, which is the accurate content, as the breadth and depth are equal.

3. What is the solid content of a piece of timber whose breadth is 16 inches, depth 12 inches, and length 12 feet?

Ans. 16 feet.

* This rule, which is generally employed in practice, is far from being correct, when the breadth and depth differ materially from each other, and the timber does not taper.

RULE II. Multiply the breadth in the middle by the depth in the middle, and that product by the length, for the solidity.*

4. The length of a piece of timber is 18 feet 6 inches, the breadths at the greater and less end 1 foot 6 inches, and 1 foot 3 inches, and the thickness at the greater and less end 1 foot 3 inches, and 1 foot; what is the solid content?

1·5	1·25
1·25	1
2)2·75	2)2·25
1.375 mean breadth.	1·125 mean depth.

1·125 mean depth.
1·375 mean breadth.

1·546875
18·5 length.

28·6171875 solid content.

BY THE SLIDING RULE.

	B		A		B		A	
As	1	:	$13\frac{1}{2}$	: :	$16\frac{1}{2}$	:	223	the mean square.
	C		D		C		D	
As	1	:	1	: :	223	:	14·9	quarter girt.
	C		D		D		C	
As	$18\frac{1}{2}$	:	12	: :	14·9	:	28·6	the content.

Note. When the piece to be measured tapers regularly from one end to the other, either take the mean breadth and depth in the middle, or take from the dimensions at both ends, and half their sum for the mean dimension. This, however, though very easy in practice, is but a very imperfect approximation.

When the piece to be measured does not taper regularly, but is thick in some parts and small in others, in this case take several dimensions; add them

* This rule is correct when the timber does not taper; but when the timber tapers considerably, and the breadth and depth are nearly equal, the rule is very erroneous. The measurer, therefore, ought to consider the shape of the timber he is about to measure before he applies either of the above rules.

all together, and divide their sum by the number of dimensions so taken, and use the quotient as the mean dimension.

RULE III. Multiply the sum of the breadths of the two ends by the sum of the depths, to which add the product of the breadth and depth of each end; one-sixth of this sum multiplied by the length, will give the exact solidity of any piece of squared timber tapering regularly.*

5. How many feet in a tree, whose ends are rectangles, the length and breadth of one being 14 and 12 inches, and the corresponding dimensions of the other 6 and 4 inches; also the length $30\frac{1}{2}$ feet.

14	12	$12 \times 14 = 168$
6	4	$6 \times 4 = 24$
—	—	$20 \times 16 = 320$
20	16	—
		512 square inches.

$= \frac{32}{9}$ square feet.

Then $\frac{1}{6} \times \frac{32}{9} \times 30\frac{1}{2} = 18\frac{2}{27}$ feet, the solidity.

6. How many solid inches in a mahogany plank, the length and breadth of one end being $91\frac{1}{2}$ and 55 inches, the length and breadth of the other end 41 and $29\frac{1}{2}$ inches, and the length of the plank $47\frac{1}{4}$ inches?

Ans. 126340·59375 cubic inches.

PROBLEM III.

Given the breadth of a rectangular plank in inches, to find how much in length will make a foot, or any other required quantity.

RULE. Divide 144, or the area to be cut off, by the breadth in inches, and the quotient will be the length in inches.

---

* This rule is correct, being that given for finding the solidity of the prismoid—which see.

Let B and b be the breadths of the two ends, D and d the depths, and L the length: $\frac{1}{6}$ (B D + (B + b) × (D + d) + $b\,d$) × L = the true solidity, as in the rule for the prismoid.

The Carpenter's rule is furnished with a scale which answers the purpose of this rule. It is called a table of board measure, and is in the following form :

0	0	0	0	5	0	$8\frac{1}{2}$	6	Inches.
12	6	4	3	2	2	1	1	Feet.
1	2	3	4	5	6	7	8	Breadth.

If the breadth be 1 inch, the length standing against it is 12 feet; if the breadth be 2 inches, the length standing against it is 6 feet; if the breadth be 5 inches, the length is 2 feet 5 inches, &c.

When the breadth goes beyond the limits of the table on the rule, it must be shut, and then you are to look for the breadth in the line of board measure, which runs along the rule from the table of board measure, and over against it on the opposite side, in the scale of inches, will be found the length required. For example, if the breadth be 9 inches, you will find the length against it to be 16 inches; if the breadth be 11 inches, the length will be found to be a little above 13 inches.

1. If a board be 6 inches broad, what length of it will make a square foot? *Ans.* 2 feet.

2. If a board be 8 inches broad, what length of it will make 4 square feet? *Ans.* 6 feet.

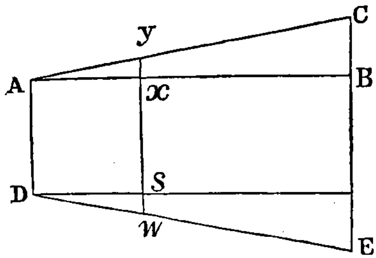
3. If a board be 16 inches broad, what length of it will make 7 square feet? *Ans.* $5\frac{1}{4}$ feet.

When the board is broader at one end than at the other, proceed according to the following :

RULE. To the square of the product of the length, and narrow end, add twice the continual product of these quantities, viz., the length, the difference between the breadths of the ends, and the area of the part required to be cut off; extract the square root of the sum; from the result deduct the product of the length and narrow end, and divide the remainder by the difference between the breadths of the ends.*

* See Appendix, Demonstration III.

If it were required to cut off 60 square inches from the smaller end of a board, A D being 3 inches, C E 6 inches, and A B 20 inches.



Here $A\,x = \frac{1}{2\,B\,C}(\sqrt{\{(B \times A\,D)^2 + 4\,B\,C \times A\,B \times 60\}} - A\,B \times A\,D) = \frac{1}{3}(\sqrt{\{(20 \times 3)^2 + 6 \times 20 \times 60\}} - 20 \times 3 = 14{\cdot}64$, the length required.

PROBLEM IV.

To find how much in length will make a solid foot, or any other required quantity, of squared timber, of equal dimensions from end to end.

Rule. Divide 1728, the solid inches in a foot or the solidity to be cut off, by the area of the end in inches, and the quotient will be the end in inches.

1. If a piece of timber be 10 inches square, how much in length will make a solid foot?

$10 \times 10 = 100$ the area to the end; then $1728 \div 100 = 17{\cdot}28$ *Ans.*

2. If a piece of timber be 20 inches broad, and 10 inches deep, how much of it will make a solid foot?

Ans. $8\frac{16}{25}$ inches.

3. If a piece of timber be 9 inches broad, and 6 inches deep, how much of it will make 3 solid feet. *Ans.* 8 ft.

On some carpenters' rules, there is a table to answer the purpose of the last rule; it is called a Table of Timber, and is in the following form:

0	0	0	0	0	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of square.

PROBLEM V.

To find the solidity of round or unsquared timber.

Rule I. Gird the piece of timber to be measured round the middle with a string, take one-fourth part of the girt and square it, and multiply this square by the length for the solidity.

BY THE SLIDING RULE.

As the length on C : 12 or 10 on D :: quarter girt, in 12ths or 10ths on D : content on C.

Note. When the tree is very irregular, divide it into several lengths and find the solidity of each part separately; or add all the girts together, and divide the sum by the number of them.

1. Let the length of a piece of round timber be 9 feet 6 inches, and its mean quarter girt 42 inches; what is its content?

3·5 quarter girt.	3 . 6 quarter girt.
3·5	3 · 6
12·25	10 . 6
9·5 length.	1 . 9
116375 content.	12 . 3
	9 . 6 length.
	110 . 3
	6 . 1 . 6
	116 . 4 . 6 content.

BY THE SLIDING RULE.

As 9·5 on C : 10 on D :: 35 on D : 116⅓ on C;
Or 9·5 : 12 :: 42 116⅓.

Rule II. Multiply the area corresponding to the quarter girt in inches, by the length of the piece in feet, and the product will be the solidity.

Note. It may sometimes happen that the quarter girt exceeds the limits of the table; in this case, take half of it, and four times the content thus found will give the required content.

A TABLE FOR MEASURING TIMBER.

Quarter Girt.	Area.	Quarter Girt.	Area.	Quarter Girt.	Area.
Inches.	*Feet.*	*Inches.*	*Feet.*	*Inches.*	*Feet.*
6	·250	12	1·000	18	2·250
6¼	·272	12¼	1·042	18½	2·376
6½	·294	12½	1·085	19	2·506
6¾	·317	12¾	1·129	19½	2·640
7	·340	13	1·174	20	2·777
7¼	·364	13¼	1·219	20½	2·917
7½	·390	13½	1·265	21	3·062
7¾	·417	13¾	1·313	21½	3·209
8	·444	14	1·361	22	3·362
8¼	·472	14¼	1·410	22½	3·516
8½	·501	14½	1·460	23	3·673
8¾	·531	14¾	1·511	23½	3·835
9	·562	15	1·562	24	4·000
9¼	·594	15¼	1·615	24½	4·168
9½	·626	15½	1·668	25	4·340
9¾	·659	15¾	1·722	25½	4·516
10	·694	16	1·777	26	4·694
10¼	·730	16¼	1·833	26½	4·876
10½	·766	16½	1·890	27	5·062
10¾	·803	16¾	1·948	27½	5·252
11	·840	17	2·006	28	5·444
11¼	·878	17¼	2·066	28½	5·640
11½	·918	17½	2·126	29	5·840
11¾	·959	17¾	2·187	29½	6·044

2. If a piece of round timber be 10 feet long, and the quarter girt $12\frac{1}{2}$ inches; required the solidity. *Ans.* 10·85.

To find the solid content by this table, look for the quarter girt $12\frac{1}{2}$, in the column marked, Quarter Girt, and in adjoining column marked, Area, will be found 1·085, which multiplied by the length, 10 feet, will give 10·85 feet for the solid content.

3. A piece of round timber is 20 feet long, and the quarter girt $14\frac{1}{4}$; how many feet are contained therein?

Ans. 28·2 feet.

4. How many solid feet are contained in a tree 40 feet long, its quarter girt being 9 inches? *Ans.* 22·48.

5. How many solid feet in a tree 32 feet long, its quarter girt being 8 inches? *Ans.* 14·208.

6. How many solid feet in a tree $8\frac{1}{2}$ feet long, its quarter girt being $7\frac{1}{2}$ inches? *Ans.* 3·315 feet.

7. Required the content of a tree, whose length is 40 feet, and quarter girt $27\frac{1}{2}$ inches? *Ans.* 210·08 feet.

8. What is the content of a tree, whose length is 30 feet 6 inches, and quarter girt $27\frac{1}{2}$ inches? *Ans.* 160·186 feet.

9. Required the content of a piece of timber, whose length is 25 feet 9 inches, and quarter girt $12\frac{3}{4}$ inches?

Ans. 29·071 feet.

10. What is the solid content of a piece of timber, whose length is 12 feet, and quarter girt $13\frac{1}{2}$ inches?

Ans. 15·18 feet.

11. What is the solid content of a piece of timber, whose quarter girt is $14\frac{3}{4}$ inches, and length 38 feet?

Ans. 57·418 feet.

When the square of the quarter is multiplied by the length, the product gives a result nearly one-fourth less than the true quantity in the tree. This rule, however, is invariably practised by timber merchants, and is not likely to be abolished. When the tree is in the form of a cylinder, its content ought to be found by Prob. IV. Sec. IV., which gives the content greater than that found by the last rule, nearly in the proportion of 14 to 11. Notwithstanding that the true content is not found by means of the square of the quarter girt, yet some allowance ought to be made to the purchaser on account of the waste in squaring the wood so as to be fit for use. If the cylindrical tree be reckoned no more than what the inscribed square will amount to, the last rule, which is said to give too little, gives too much. When the tree is not perfectly circular, the quarter girt is always too great, and therefore the content, on that account, will be too great.

Doctor Hutton recommends the following rule, which will give the content extremely near the truth :

Rule. Multiply the square of one-fifth of the girt, or circumference, by twice the length, and the product will be the content.

BY THE SLIDING RULE.

As double the length on C : 12 or 10 on D :: $\frac{1}{5}$ of the girt, in 12ths or 10ths on D : content on C.

12. Required the content of a tree, its length being 9 feet 6 inches, and its mean girt 14 feet.

```
                   ft. in. p.
14 ÷ 5 = 2·8 = 2 . 9 . 7 = 1/5 of the girt; then
             ft. in.
 2·8          9 . 6        2 . 9 . 7
 2.8              2        2 . 9 . 7
 ----        ------        ---------
 7·84        19 . 0        5 . 7 . 2
   19                      2 . 1 . 2 . 3
 ----                          1 . 7 . 7 . 1
148·96 content.            -------------
                           7 . 9 . 11 . 10 . 1
                                           19
                          ------------------
                         184 . 9 . 8 . 11 . 7 content.
```

```
   C    D    D      C
As 19 : 10 :: 28   : 149, content by the Sliding Rule.
Or 19 : 12 :: 33·6 : 149, content without it.
```

Dr. Gregory recommends the following rules given by Mr. Andrews :

Let L denote the length of the tree in feet and decimals, and G the mean girt in inches.

RULE I. Making no allowance for bark.

$\frac{LG^2}{2304}$ = cubic feet, customary; and $\frac{LG^2}{1807}$ = cubic feet true content.

RULE II. Allowing $\frac{1}{8}$ for bark.

$\frac{LG^2}{3009}$ = cubic feet, customary; $\frac{LG^2}{2360}$ = cubic feet, true content.

RULE III. Allowing $\frac{1}{10}$ for bark.

$\frac{LG^2}{2845}$ = cubic feet, customary; $\frac{LG^2}{2231}$ = cubic feet, true content.

RULE IV. Allowing $\frac{1}{12}$ for bark.

$\frac{LG^2}{2742}$ = cubic feet, customary; $\frac{LG^2}{2150}$ = cubic feet, true content.

What is the solid content of a tree, whose circumference or girt is 60 inches, and length 40 feet?

By Rule I.

$$\frac{40 \times 60^2}{2304} = 62\tfrac{1}{2} \text{ cubic feet, customary.}$$

$$\frac{40 \times 60^2}{1807} = 79\tfrac{3}{4} \text{ cubic feet, customary.}$$

By Rule II.

$$\frac{40 \times 60^2}{3009} = 47{\cdot}85 \text{ cubic feet, customary.}$$

$$\frac{40 \times 60^2}{2360} = 61 \text{ cubic feet, true content.}$$

By Rule III.

$$\frac{40 \times 60^2}{2845} = 50 \cdot 61 \text{ cubic feet, customary.}$$

$$\frac{40 \times 60^2}{2231} = 64 \cdot 54 \text{ cubic feet, true content.}$$

By Rule IV.

$$\frac{40 \times 60^2}{2742} = 52 \cdot 47 \text{ cubic feet, customary.}$$

$$\frac{40 \times 60^2}{2150} = 66 \cdot 97 \text{ cubic feet, true content.}$$

When the two ends are very unequal, calculate its content by the rule given for finding the solidity of the frustum of a cone, and deduct the usual allowance from the result.

When it is required to find the accurate content of an irregular body not reducible to any figure of which we have already treated, provide a cylindrical or prismatic vessel, capable of containing the solid to be measured ; put the solid into the vessel, and pour in water to cover it, marking the height to which the water reaches. Then take out the solid, and observe how much the water has descended in consequence of its removal ; calculate the capacity of the part of the vessel thus left dry, and it will evidently be equal to the solidity of the body whose content is required.

ARTIFICERS' WORK.

Artificers compute their works by several different measures :

Glazing and masonry by the foot.

Plastering, painting, paving, &c., by the yard of 9 square feet.

Partitioning, roofing, tiling, flooring, &c., by the square of 100 square feet.

Brick-work is computed either by the yard of 9 square feet, or by the perch or square rood, containing $272\frac{1}{4}$ square feet, or $30\frac{1}{4}$ square yards ; $272\frac{1}{4}$ and $30\frac{1}{4}$ being the squares of $16\frac{1}{2}$ feet and $5\frac{1}{2}$ yards respectively.

CARPENTERS AND JOINERS' WORK.

1. OF FLOORING.

To measure joists, multiply the breath, depth, and length together for the content.*

If a floor be 50 feet 4 inches long, and 22 feet 6 inches broad; how many squares of flooring are in that room?

50·333	50 . 4
22·5	22 . 6
251665	1107 . 4
100666	25 . 2
100666	100)11,32 . 6
100)1132·4925	11·325
11·3249 squares.	

Ans. 11 squares $32\frac{1}{2}$ feet.

2. If a floor be 51 feet 6 inches long, and 40 feet 9 inches broad, how many squares are contained in that floor?

Ans. 20·986 squares.

3. If a floor be 36 feet 3 inches long, and 16 feet 6 inches broad, how many squares are contained in that floor?

Ans. 5 squares $98\frac{1}{8}$ feet.

4. If a floor be 86 feet 11 inches long, and 21 feet 2 inches broad; how many squares are contained in it?

Ans. 18·3972.

5. In a naked floor the girder is 1 foot 2 inches deep, 1 foot broad, and 22 feet long; there are 9 bridgings, the scantling of each (viz. breadth and depth) being 3 inches, by 6 inches, and length 22 feet; 9 binding joists, the length of

* Joists receive various names, from their position; such as girders, binding-joists, trimming-joists, common joists ceiling-joists, &c. When girders and joists of flooring are designed to bear considerable weight, they should be let into the wall at each end about two-thirds of the thickness of the wall.

each being 10 feet, and scantlings 8 inches by 4 inches; the ceiling-joists are 25 in number, each 7 feet long, and their scantlings 4 inches by 3 inches; what is the solidity of the whole? *Ans.* 85 feet.

6. What would the flooring of a house three stories high come to, at £5 per square; the house measures 30 feet long, and 20 broad; there are seven fire-places,* two of which measure, each 6 feet by 4 feet, two others, each 6 feet by 5 feet 6 inches; two, each of 5 feet 6 inches by 4 feet; and the seventh 5 feet by 4; the well-hole for the stairs is 10 feet by 8? *Ans.* £69 2*s*.

OF PARTITIONING.

Partitions are measured by squares of 100 feet, as flooring; their dimensions are taken by measuring from wall to wall, and from floor to floor; then multiply the length and height for the content in feet, which bring to squares by dividing 100, as in flooring. When doors and windows are not included by agreement, deductions must be made for their amount.†

1. A partition measures 173 feet 10 inches in length, and 10 feet 7 inches in height; required the number of squares in it? *Ans.* 18·3972 squares.

2. A partition between two rooms measures 80 feet in length, and 50 feet 6 inches in height; how many squares in it? *Ans.* 40$\frac{2}{5}$ squares.

3. If a partition measure 10 feet 6 inches in length, and 10 feet 9 inches in height; how many squares in it? *Ans.* 1 square 12$\frac{1}{2}$ feet.

4. What is the number of squares in a partition, whose length is 50 feet 6 inches, and height 12 feet 9 inches? *Ans.* 6 squares, 43 feet, 10$\frac{1}{2}$ inches.

* Fire-places, &c., are of course to be deducted.

† The best and strongest partitions are those made with framed work. The king-posts are measured as roofing, the rest as flooring.

In roofing, the length of the rafters is equal to the length of a string stretched from the ridge down the rafter till it meets the top of the wall.

To find the content, multiply this length by the breadth and depth of the rafters, and the result will be the content of one rafter; and that multiplied by the number of them will give the content of all the rafters.*

1. If a house within the walls be 42 feet 6 inches long, and 20 feet 3 inches broad; how many squares of roofing in that house?

ft.	ft. in.
42·5	42 . 6
20·25	20 . 3
2125	840
850	$6\frac{1}{2}$ 10 . 1
8500	$3\frac{1}{4}$ 10 . 7
860·625 flat.	860 · 8 flat.
430·3125	430 . 4
100)1290·9375	100)1291
12·91 squares.	12 : 91

2. What cost the roofing of a house at 11*s.* per square; the length within the walls being 50 feet 9 inches, and the breadth 30 feet; the roof being of a true pitch?

Ans. £12 11*s.* $2\frac{1}{50}$*d.*

* Workmen generally take the flat and half the flat of any house, taken within the walls, to be the measure of the roof of the same house. This, however, is only when the roof is of a true pitch. The usual pitches are the common, or true pitches, in which the rafters are three-fourths of the breadth of the building: the Gothic pitch is when the length of the principal rafters is equal to the breadth of the building: the pediment pitch is when the perpendicular height is two-ninths of the breadth.

When the covering of the building is to be plain tiles or slates, the roof is generally of a true or common pitch; the Gothic pitch is used when the covering is of pantiles; the pediment pitch is used when the roof is covered with lead.

3. What number of squares are contained in a house, whose length within the walls is 40 feet, and breadth 18 feet; the roof being common pitch?

Ans. 10 squares and 80 feet.

4. How many squares in the roof of a building, the length of the house being 60 feet, and the length of the rafter 14 feet 6 inches? *Ans.* 17 squares and 40 feet.

5. How many squares in a building, whose length is 50 feet, and length of the rafter 15 feet?

Ans. 15 squares.

6. How many squares in the roof of a building, whose length is 37 feet, the length of the rafter being 13 feet?

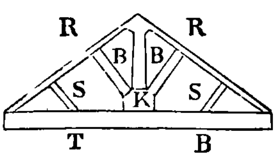
Ans. 9 squares and 62 feet.

7. How many squares in the roof of a building, whose length is 70 feet 6 inches, the length of the rafter being 14 feet 6 inches? *Ans.* 20 squares and 44½ feet.

8. How many squares in the roof of a building, whose length is 50 feet, and the length of a string reaching across the ridge from eave to eave being 30 feet?

Ans. 15 squares.

NOTE. All the timbers employed in roofing are measured like those used in flooring, except where there is a necessity for cutting out parallel pieces equal to, or exceeding 2½ broad and 2 feet long. In this case the amount of the pieces so cut out must be deducted from the content of the whole piece found from its greatest scantlings. When the pieces cut out do not amount to the above dimensions, they are considered as useless, and therefore no deduction is to be made for them.*



* In the above figure K is called the king-post, and in measuring the pieces cut out of it, the shortest length is to be taken. T B is called the tie-beam, which pre

10. Let the tie-beam T B be 36 feet long, 9 inches broad, d 1 foot 2 inches thick; the king-post K 11 feet 6 inches gh, 1 foot broad at the bottom, and 5 inches thick; out of is post are sawn two equal pieces from the sides, each 7 t long and 3 inches broad. The braces B B, are 7 feet nches long, and 5 inches by 5 inches square; the rafters R are 19 feet long, 5 inches broad, and 10 inches deep; e struts S S are 3 feet 6 inches long, 4 inches broad, and inches deep; what is the measurement for workmanship d also for materials?

ft.	in.	p.	
31 .	6 .	0	solidity of the tie-beam T B.
4 .	9 .	6	solidity of the king-post K.
2 .	7 .	3	solidity of the braces B B.
13 .	2 .	4	solidity of the rafters R R.
	11 .	8	solidity of the struts S S.
53 .	0 .	9	solidity for workmanship.
1 .	5 .	6	solidity cut from the king-post.
51 .	7 .	3	solidity for materials.

OF WAINSCOTTING.

Wainscotting is measured by the yard square, which is 9 are feet.

In taking the dimensions, the string is made to ply close r the cornice, swelling panels, moulding, &c. The height the room from the floor to the ceiling being thus taken, is dimension, and the compass of the room taken all round floor is the second dimension.

---

s the rafters R R from pressing out the wall. The braces B B serve to ngthen the rafters; the struts S S serve for a similar purpose. Besides ngthening the rafters, the braces and struts serve to bind the roof together. en head-room is required, the rafters are braced simply by R R.

Doors, windows, shutters, &c., where both their sides are planed, are considered as work and half; therefore in measuring the room, they need not be deducted; but the superficial content of the whole room found as if there were no door, window, &c., then the contents of the doors and windows must be found, and half thereof added to the content of the whole room.

When there are no shutters, the content of the windows must be deducted; chimneys, window-seats, check-boards, sopheta-boards, linings, &c., must be measured by themselves.

Windows are sometimes valued at so much per window, and sometimes by the superficial foot. The dimensions of a window are taken in feet and inches, from the under side of the sill to the upper side of the top-rail; and from the outside to outside of the jambs.

When the doors are panelled on both sides, take double the measure for the workmanship.

For the surrounding architrave, girt round it and inside the jambs, for one dimension, and add the length of the jambs to the length of the cap-piece, (taking the breadth of the opening for the length,) for the other dimension.

Weather-boarding is measured by the yard square, and sometimes by the square.

Frame-doors are measured by the foot, or sometimes by the yard square.

Staircases are measured by the foot superficial. The dimensions are taken with a string passing over the riser and tread for one dimension, and the length of the step for the other. By the length of the step is meant the length of the front and the returns at the two ends.

For the balustrade, take the whole length of the upper part of the hand-rail, and girt it over its end till it meet the top of the newel-post, for one dimension; and twice the length of the baluster upon the landing, with the girt of the hand-rail, for the other dimension.

Modillian cornices, coves, &c., are generally measured by the foot superficial.

Beads, stops, astragals, copings, fillets, boxings to windows, skirting-boards, and water-trunks, are paid for by lineal measure.

Frontispieces are measured by the foot superficial, and the architrave, frieze, and cornice, are measured separately.*

To find the contents of the foregoing work, multiply the two corresponding dimensions together for the superficial content.

1. A room, or wainscot, being girt downwards over the mouldings, measures 12 ft. 6 in, and 130 ft. 9 in. in compass; how many yards does that room contain?

ft. in.	ft.
130 . 9	130·75
12 . 6	12·5
1560	65375
65 . 4 . 6	26150
6 . 0 . 0	13075
3 . 0 . 0	9)1634·375 ft.
9)1634 . 4 . 6	181 yards, 5 ft.
181 . 5 *Ans*	

* Baluster is a small column or pillar, used for balustrades.

Balustrade is a row of balusters, joined by a rail; serving for a rest to the arms, or as an inclosure to balconies, staircases, altars, &c.

Cornice is the third and uppermost part of the entablature of a column, or the uppermost ornament of any wainscotting, &c.

Bead is a round moulding carved like beads in necklaces. There is also a kind of plain bead, often set on the edge of each fascia of an architrave, on the upper edge of skirting-boards, on the lining-board of a door-case, &c.

Architrave is that part of a column that bears immediately on the capital. It is supposed to represent the principal beam in timber buildings, in which it is sometimes called the master-piece or reason-piece. In chimneys it is called the mantel-piece. Architrave doors are those which have an architrave on the jambs and over the doors. Architrave windows of timber are usually raised out of the solid timber, and sometimes the mouldings are struck and laid on.

Astragal is a small round moulding, encompassing the top of the shaft of a column, like a ring or bracelet. The shaft terminates at the top with an astragal, and at bottom with a fillet, which in this place is called azia

2. If the wainscot of a room be 15 ft. 6 in. high, and the compass of the room 142 ft. 6 in.; how many yards are contained in it? *Ans.* $245\frac{5}{12}$ yards.

3. If the window-shutters about a room be 60 ft. 6 in. broad, and 6 ft. 4 in. high; how many yards are contained therein, at work and a half? *Ans.* $63\frac{31}{36}$ yards.

4. A rectangular room measures 129 feet 6 inches round, and is to be wainscotted at 3*s.* 6*d.* per square yard; after due allowance for girt of cornice, &c., it is 16 feet 3 inches high; the door is 7 feet by 3 feet 9 inches; the window-shutters, two pair, are 7 feet 3 inches by 4 feet 6 inches; the cheek-boards round them come 15 inches below the shutters, and are 14 inches in breadth; the lining-boards round the doorway are 16 inches broad: the door and window-shutters being worked on both sides, are reckoned as work and half, and paid for accordingly; the chimney 3 feet 9 inches by 3 feet, not being enclosed, is to be deducted from the superficial content of the room. The estimate of the charge is required. *Ans.* £43 4*s.* $6\frac{3}{4}$*d.*

5. The height of a room, taking in the cornice and mouldings, is 12 feet 6 inches, and the whole compass 83 feet 8 inches; the three window-shutters are each 7 feet 8 inches by 3 feet 6 inches, and the door 7 feet by 3 feet 6 inches; the door and shutter, being worked on both sides, are reckoned work and a half. Required the estimate, at 6*s.* per square yard. *Ans.* £36 12*s.* $2\frac{1}{2}$*d.*

OF BRICKLAYERS' WORK.

OF TILING OR SLATING.

Tiling and slating are measured by the square of 100 feet. There is no material difference between the method employed for finding the estimate of roofing and tiling; bricklayers sometimes require double measure for hips and valleys.

When gutters are allowed double measure, the usual mode is, to measure the length along the ridge tile, and add it to the contents of the roof: this makes an allowance of one foot in breadth along the hips or valleys. Double measure is usually allowed for the eaves, so much as the projector is over the plate, which is generally 18 to 20 inches.

When sky-lights and chimney-shafts are not large, no allowance is to be made for them; but when they are large, their amount is to be deducted.

1. There is a roof covered with tiles, whose depth on both sides (with the usual allowance at the eaves) is 30 feet 6 inches, and the length 42 feet; how many squares of tiling are contained therein?

ft. in.	ft.
30 · 6	30.5
42	42
1260	610
21	1220
100)1281	100)12,810
12 . 81	12 squares 81 feet.

2. There is a roof covered with tiles, whose depth on both sides (with the usual allowance at the eaves) is 40 feet 9 inches, and the length 47 feet 6 inches; required the number of squares contained therein?

Ans. 19 squares $35\frac{5}{8}$ feet.

3. What will the slating of a house cost at £1 5*s.* 6*d.* per square; the length being 43 feet 10 inches, and the breadth 27 feet 5 inches, on the flat; the eaves projecting 16 inches on each side—true pitch? *Ans.* £24 9*s.* $5\frac{1}{2}$*d.*

4. What is the content of a slated roof, the length being 45 feet 9 inches, and the whole girt 34 feet 3 inches?

Ans. 174·104 yards.

OF WALLING.

Brick-work is estimated at the rate of a brick and a half thick; so that if a wall be more or less than the standard thickness, it must be reduced to it: thus, multiply the superficial content of the wall by the number of half bricks in the thickness, and divide the product by 3.

The superficial content is found by multiplying the length by the height. Bricklayers estimate their work by the rod of $16\frac{1}{2}$ feet, or $272\frac{1}{4}$ square feet. Sometimes 18 feet are allowed to the rod, that is 324 square feet; sometimes the work is measured by the rod of 21 feet long, and three feet high, that is, 63 square feet; in this case, no regard is paid to the thickness of the wall in measuring, but the price is regulated according to the thickness.

When a piece of brick-work is to be measured, the first thing to be done is to ascertain which of the above measures is to be employed; then, having multiplied the length and breadth together (the dimensions being feet) the product is to be divided by the proper divisor, namely, 272·25, 324, or 63, according to the measure of the rod, and the quotient will be the measure in square rods of that measure.

To measure any arched way, arched window, or door, &c., the height of the window or door from the crown or middle of the arch to the bottom or sill, is to be taken, and likewise from the bottom or sill to the spring of the arch, that is, where the arch begins to turn. Then to the latter height add twice the former, and multiply the sum by the breadth of the window, door, &c., and one-third of the product will be the area, sufficiently near the truth for practice.

1. If a wall be 72 feet 6 inches long, and 19 feet 3 inches high, and 5 bricks and a half thick, how many rods of brick-work are contained therein, when reduced to the standard?

NOTE. The *standard* means a wall a brick and a half thick; therefore, to reduce any wall to the standard, multiply the superficial content of it by the number of half bricks in its thickness, and divide by 3.

```
        ft.  in.
        72 . 6
        19 . 3
      ----------
       648
       72
        18 . 1 . 6
         9 . 6 . 0
      ---------------
      1395 . 7 . 6
                 11
      ---------------
   3)15351 . 10 . 6
   -----------
272)5117(18 rods.
     ------
      2397
     ------
   68)221(3 quarters.
     -----
      17 feet.
```

NOTE. That 68·06 is the fourth part of 272·26, and 68 is one-fourth of 272.

In reducing feet into rods, it is usual to divide 272, rejecting the decimal ·25. By this method, the answer found above is about $4\frac{1}{2}$ feet too much.

2. How many rods of standard brick-work are in a wall whose length is 57 feet 3 inches, and height 24 feet 6 inches; the wall being $2\frac{1}{2}$ bricks thick?

Ans. 8·5866 rods.

3. The end wall of a house is 28 feet 10 inches long, and 55 feet 8 inches high to the eaves; 20 feet high is $2\frac{1}{2}$ bricks thick, another 20 feet high 2 bricks thick, and the remaining 15 feet 8 inches is $1\frac{1}{2}$ bricks thick, above which is a triangular gable one brick thick, which rises 42 courses of bricks, of which every 4 courses make a foot. What is the whole content in standard measure?

Ans. 253·62 yards.

OF CHIMNEYS.

When a chimney stands by itself, without any party-wall being adjoined, take the girt in the middle for the length, and the height of the story for the breadth; the thickness is to be the same as the depth of the jambs; if the chimney be built upright from the mantle-piece to the ceiling, no deduction is to be made for the vacancy between the floor (or hearth) and mantle-tree, on account of the gatherings of the breast and wings, to make room for the hearth in the next story.

When the chimney-back forms a party-wall, and is measured by itself, then the depth of the two jambs is to be measured, and the length of the breast for a length, and the height of the story for the breadth; the thickness is the same as the depth of the jambs. That part of the chimney which appears above the roof, called the chimney-shaft, is measured by girding it round the middle for the length, and the height is taken for the breadth.

In consideration of plastering and scaffolding, the thickness is generally reckoned half a brick more than it really is; and in some places double measure is allowed on account of extra trouble.

1. Let the dimensions of a chimney, having a double funnel towards the top, and a double shaft, be as follows, viz., in the parlour, the breast and two jambs measure 18 feet 9 inches, and the height of the room 12 feet 6 inches; in the first floor, the breast and two jambs girt 14 feet 6 inches, and the height 9 feet; in the second floor, the breast and the jambs girt 10 feet 3 inches, and the height is 7 feet; above the roof, the compass of the shaft is 13 feet 9 inches, and its height 6 feet 6 inches; lastly, the length of the middle partition, which parts the funnel, is 12 feet, and its thickness 1 foot 3 inches; how many rods of brick-work, standard measure, are contained in the chimney, double measure being allowed, the thickness 1½ bricks?

	ft. in.		ft. in. p.
1st.	18 . 9	5th.	1 . 3 . 0
	12 . 6		12
	225 . 0		15 . 0 partition.
	9 . 4 . 6		234 . 4 . 6 parlour
	234 . 4 . 6		130 . 6 . 0 first floor.
			71 . 9 . 0 second floor.
			89 . 4 . 6 shaft.
	ft. in.		
2nd.	14 . 6		541 . 0 . 0 sum.
	9		2
	130 . 6		272)1082 . 0 . 0 double.
	ft. in.		68)266(3 rods 3 quarters.
3rd.	10 . 3		
	7		62 feet.
	71 . 9		
	ft. in.		
4th.	13 . 9		
	6 . 6		
	82 . 6		
	6 . 10 . 6		
	89 . 4 . 6		

Ans. 3 rods, 3 quarters, and 62 feet.

MASONS' WORK.

To masonry belong all sorts of stone-work. The work is sometimes measured by the foot solid, sometimes by the foot in length, and sometimes by the foot superficial. Masons, in taking dimensions, girt all their mouldings in the same manner as joiners.

Walls, columns, blocks of stone or marble, &c., are measured by the solid foot, and pavements, slabs, chimney-pieces, &c., by the square foot.

In estimating for the workmanship, square measure is generally used, but for the materials, solid measure.

In the solid measure, the length, breadth, and thickness, are multiplied together.

In the superficial measure, there must be taken the length and breadth of every part of the projection, which is seen without the general upright face of the building.

1. If a wall be 82 feet 9 inches long, 20 feet 3 inches high, and 2 feet 3 inches thick; how many solid feet are contained in that wall?

	ft. in.	ft.
	82 . 9	82·75
	20 . 3	20·25
	1640	41375
3 = $\frac{1}{4}$	20 . 8 . 3	16550
6 = $\frac{1}{2}$	10 . 3 . 0	165500
3 = $\frac{1}{2}$	5 . 0 . 0	
		1675·6875
	1675 . 8 . 3	2·25
	2 . 3	
		83784375
	3351 . 4 . 6	33513750
3 = $\frac{1}{4}$	418 . 11 . 0$\frac{3}{4}$	33513750
	3770 . 3 . 6$\frac{3}{4}$	3770.296875 *Ans.*

2. If a wall be 120 feet 4 inches long, and 30 feet 8 inches high; how many superficial feet are contained therein?

Ans. 3690$\frac{2}{9}$.

3. If a wall be 112 feet 3 inches long, and 16 feet 6 inches high; how many superficial rods of 63 square feet are contained therein? *Ans.* 29 rods 25 feet.

4. What is the value of a marble slab, at 8*s.* per foot, the length being 5 feet 7 inches, and breadth 1 foot 10 inches?

Ans. £4 1*s.* 10$\frac{1}{2}$*d.*

PLASTERERS' WORK.

Plasterers' work is of two kinds, viz., ceiling which is plastering upon laths; and rendering, which is plastering upon walls. These are measured separately.

The content is sometimes estimated by the foot, sometimes by the yard, and sometimes by the square of 100 feet. Enriched mouldings are calculated by the running foot or yard.

Deductions are made for chimneys, doors, windows, &c.

In plastering timber partitions, where several of the large braces and other large timbers project from the plastering, a fifth is usually deducted.

Whitening and colouring are measured in the same manner as plastering. In timbered partitions, one-fourth, or one-fifth of the whole area is usually added, to compensate for the trouble of colouring the sides of the quarters and braces.

In arches, the girt round them is multiplied by the length for the superficial content.

1. If a ceiling be 40 feet 3 inches long, and 16 feet 9 inches broad, how many square yards contained therein?

	ft. in.	ft.
	40 . 3	40·25
	16 . 9	16·75
	640	20125
6 = $\frac{1}{2}$	20 . 1 . 6	28175
3 = $\frac{1}{2}$	10 . 0 . 9	24150
3 = $\frac{1}{4}$	4 . 0 . 0	4025
	9)674 . 2 . 3	9)674·1875
	Ans. 74 yards 8 feet.	*Ans.* 74·9097 yards.

2. The length of a room is 14 feet 5 inches, breadth 13 feet 2 inches, and height 9 feet 3 inches to the under side of the cornice,* which projects 5 inches from the wall, on the upper part next the ceiling; required the quantity of ren-

* Cornices, festoons, &c., are put on after the room is plastered and are not, of course, taken into account by the plasterer.

dering and plastering, there being no deduction but for one door, which is 7 feet by 4?

Ans. 53 yards 5 feet of rendering, 18 yards 5 feet of ceiling.

3. The circular vaulted roof of a church measures 105 feet 6 inches in the arch, and 275 feet 5 inches in length; what will the plastering come to, at 1*s.* per yard.

Ans. £161 8*s.* 5¾*d.*

4. The length of a room is 18 feet 6 inches, the breadth 12 feet 3 inches, and height 10 feet 6 inches; to how much amount the ceiling and rendering, the former at 8*d.*, and the latter at 3*d.* per yard; allowing for the door of 7 feet by 3 feet 8, and a fire-place of 5 feet square?

Ans. £1 13*s.* 3*d.*

PLUMBERS' WORK.

Plumbers' work is rated by the pound, or by the hundred weight of 112 lbs. Sheet lead, used in roofing, guttering, &c., weighs from 6 to 12 pounds per square foot, according to the thickness; and leaden pipes vary in weight per yard, according to the diameters of the bore.

The following table shows the weight of a square foot of sheet lead, according to its thickness; and the common weight of a yard of leaden pipe according to the diameter of its bore.

Thickness of lead.	Pounds to a square foot.	Bore of leaden pipes.	Pounds per yard.
Inch.			
$\frac{1}{10}$	5·899	0¾	10
$\frac{1}{9}$	6·554	1	12
$\frac{1}{8}$	7·373	1¼	16
$\frac{1}{7}$	8·427	1½	18
$\frac{1}{6}$	9·831	1¾	21
$\frac{1}{5}$	11·797	2	24

1. A piece of sheet lead measures 20 feet 6 inches in length, and 7 feet 9 inches in breadth; what is its weight at $8\frac{1}{4}$ lbs. to the square foot?

ft. in.	ft.
20 . 6	20·5
7 . 9	7·75
143 . 6	1025
15 . 4 . 6	1435
	1435
158 . 10 . 6	158·875
	$8\frac{1}{4}$
	1271·000
	39·719

cwt. qrs. lbs.
112)1310·719(11 . 2 . $22\frac{3}{4}$, nearly.
112
190
112
28)78(2
56
22

2. What weight of lead $\frac{1}{10}$ of an inch thick will cover a flat, 15 feet 6 inches long, and 10 feet 3 inches broad, the lead weighing 6lbs. to the square foot?

Ans. 8 cwt., 2 qrs. $1\frac{1}{4}$ lb.

3. What will be the expense of covering and guttering a roof with lead, at 18*s.* per cwt.; the length of the roof being 43 feet, and the girt over it 32 feet; the guttering being 57 feet in length, and 2 feet in breadth, allowing a square foot of lead to weigh $8\frac{3}{4}$ lbs.?

Ans. £104 15*s.* $3\frac{3}{4}$*d.*

4. What will be the expense of 130 yards of leaden pipe of an inch and half bore, at 4*d.* per lb., admitting each yard to weigh 18lbs.? *Ans.* £39.

PAINTERS' WORK.

Painters' work is computed in square yards Every part is measured where the colour lies, and the measuring line is forced into all the mouldings and corners. Double measure is allowed for curved mouldings, &c.

Windows are done at so much a-piece. Sash-frames at a certain price per dozen; sky-lights, window-bars, casements, &c., are charged at a certain price per piece.

To measure balustrades, take the length of the hand-rail for one dimension, and twice the height of the baluster upon the landing, added to the girt of the hand-rail, for the other dimension.

No general rule can be given for measuring trellis-work; but, however, double the area of one side is often taken for the measure of both sides.

1. If a room be painted whose height (being girt over the moulding) is 16 feet 4 inches, and the compass of the room 120 feet 9 inches; how many yards of painting in it?

ft. in.	ft.
120 . 9	120·75
16 . 4	16·3
1920	36225
$4=\frac{1}{3}$—40 . 3	72450
$6=\frac{1}{2}$ 8 . 0	12075
$3=\frac{1}{2}$ 4 . 0	9)1968·225
9)1972 . 3	*Ans.* 218·691 yards.
Ans. 219 yards 1 foot.	

2. A gentleman had a room to be painted, its length being 24 feet 6 inches, breadth 16 feet 3 inches, and height 12 feet 9 inches, also the size of the door 7 feet by 3 feet 6 inches, and the size of the window-shutters to each of the windows, there being two, is 7 feet 9 inches by 3 feet 6 inches; but the breaks of the windows themselves are 8 feet 6 inches high, and 1 foot 3 inches deep; what will be the expense of giving it three coats, at 2*d.* per yard each; the size of the fire-place to be deducted, being 5 feet by 5 feet 6 inches? *Ans.* £3 3*s.* 10½*d.*

3. The length of a room is 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches; how many yards of painting in it, deducting a fire-place of 4 feet by 4 feet 4 inches, and two window-shutters each 6 feet by 3 feet 2 inches? *Ans.* $73\frac{2}{27}$ yards.

GLAZIERS' WORK.

Glaziers take their dimensions either in feet, inches, and parts; or feet, tenths, and hundredths. They compute their work in square feet.

Windows are sometimes measured by taking the dimensions of one pane, and multiplying its superficies by the number of panes. But generally they take the length and breadth of the whole frame for the glazing. Circular windows are measured as if they were square, taking for their dimensions their greatest length and breadth.

1. If a pane of glass be 3 feet 6 inches and 9 parts long, and 1 foot 3 inches and 3 parts broad; how many feet of glass in that pane?

```
 3 . 6 . 9                      3·56
 1 . 3 . 3                      1·277
 ---------                      -----
 3 . 6 . 9                       2492
    10 . 8 . 3                  2492
         10 . 8 . 3             712
 ------------------            356
Ans. 4 . 6 . 3 . 11 . 3        ------
                          Ans  4·54612 feet.
```

2. If there be 10 panes of glass, each 4 feet 8 inches 9 parts long, and 1 foot 4 inches and 3 parts broad; how many feet of glass are contained in the 10 panes? *Ans.* 64·0407.

3. There are 20 panes of glass, each 3 feet 6 inches 9 parts long, and 1 foot 3 inches and 3 parts broad; how many feet of glass are in the 20 panes? *Ans.* 90·9224 ft.

4. If a window be 7 feet 6 inches high, and 3 feet 4 inches broad; how many square feet of glass contained therein? *Ans.* 25.

5. How many feet in an elliptical fan-light of 14 feet 6 inches in length, and 4 feet 9 inches in breadth?

Ans. 68 feet 10 inches.

6. What will the glazing of a triangular sky-light come to at 20*d.*; the base being 12 feet 6 inches, and the perpendicular height 6 feet 9 inches? *Ans.* £3 10*s.* 3½*d.*

PAVERS' WORK.

Paver's work is computed by the square yard; and the content is found by multiplying the length by the breadth.

1. What will be paid for paving a foot-path, at 4*s.* the yard, the length being 40 feet 6 inches, and the breadth 7 feet 3 inches?

ft. in.	ft.
40 . 6	40·5
7 . 3	7·25
283 . 6	2025
10 . 1 . 6	810
	2835
Ans. 293 . 7 . 6	
	Ans. 293·625 feet.

2. What will be the expense of paving a rectangular court-yard, whose length is 62 feet 7 inches, and breadth 44 feet 5 inches; and in which there is a foot-path, whose wh

length is 62 feet 7 inches, and breadth 5 feet 6 inches, this at 3*s*. per yard, and the rest at 2*s*. 6*d*. per yard?

Ans. £39 11*s*. $3\frac{1}{4}$*d*.

3. What is the expense of paving a court, at 3*s*. 2*d*. per yard; the length being 27 feet 10 inches, and the breadth 14 feet 9 inches? *Ans*. £7 4*s*. $5\frac{1}{4}$*d*.

4. What will the paving of a walk round a circular bowling-green come to, at 2*s*. 4*d*. per yard, the diameter of the bowling-green being 40 feet, and the breadth of the walk 5 feet? *Ans*. £9 3*s*. $3\frac{3}{25}$*d*.

How many yards of paving in an elliptical walk 4 feet broad, the longer diameter being 60 feet, and shorter 50?

Ans. 82·3797 yards.

VAULTED AND ARCHED ROOFS.

Arched roofs are either domes, vaults, saloons, or groins.

Domes are formed of arches springing from a circular or polygonal base, and meeting in a point directly over the centre of that base.

Saloons are made by arches connecting the side walls of a building to a flat roof or ceiling.

Groins are made by the intersection of vaulted roofs with each other.

Vaulted roofs are sometimes circular, sometimes elliptical, and sometimes Gothic.

Circular roofs are those of which the arch is a part of the circumference of the circle.

Elliptical roofs are those of which the arch is a part of the circumference of an ellipsis.

Gothic roofs are made by the meeting of two equal circular arches, exactly above the span of the arch.

Groins are generally measured like a paralleopipedon, and the content is found by multiplying the length and breadth of the base by the height.

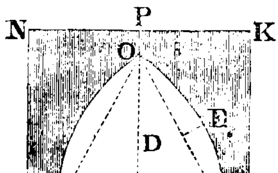
Sometimes one-tenth is deducted from the solidity thus found, and the remainder is reckoned as the solidity of the vacuity.

PROBLEM I.

To find the solidity of a circular, elliptical, or Gothic vaulted roof.

Rule. Find the erea of one end, by one of the foregoing rules, and multiply the area of the end by the length of the roof, or vault, and the product will be the content.

Note. When the arch is a segment of a circle, the area is found by Prob. XXVIII. Sec. II. When the arch is a segment of an ellipsis, multiply the span by the height, and that product by ·7854 for the area of the end. When it is a Gothic arch, find the area of an isosceles triangle, whose base is equal to the span of the arch, and its sides equal to the two chords of the circular segment of the arch; then add the areas of the two segments to the area of the triangle, and the sum will give the area of the end.



1. What is the content of a concavity of a semi-circular vaulted roof, the span being 30 feet, and the length of the vault 150 feet?

$30 \times 30 = 900$; then $900 \times \cdot 7854 = 706\cdot 86$, hence $706\cdot 86 \div 2 = 353\cdot 43$ the area of the end; then $353\cdot 43 \times 150 = 53014\cdot 5$ the content.

2. What is the solid content of the vacuity A O E B of a Gothic vault, whose span A B is 60 feet, the chord B O, or A O, of each arch 60 feet; the distance of each arch from the middle of the chords as D E = 12 feet, and the length of the vault 40 feet?

In this example, the triangle A B O equi-lateral, and its area is $\frac{1}{4}$ A $B^2 \sqrt{3} = 900 \sqrt{3} = 1557$. Again, $\frac{2}{3}$ (B O $\times$ D E) + $\frac{D E^3}{2 B O} = \frac{2}{3} (60 \times 12) + \frac{12^3}{60 \times 2} = 494\frac{2}{5}$ = area of segment O E B, and $494\frac{2}{5} \times 2 = 988\frac{4}{5}$ the areas of the two segments O E B and O H A; then $1557 + 988\frac{4}{5}) \times 40 = 101832$, the solidity required.

Let M L K L represent a perpendicular section of a vaulted roof (Gothic.) The span A B is 60 feet, the thickness of the wall M A, or B L, at the spring of the arch = 4 feet, the thickness O P at the crown of the arch = 3, and the length of the roof = 40 feet, the chord A O or O B = 60 feet, and the .versed sine D E 12 feet; required the solidity of the materials of the arch.

First, $\sqrt{(A O^2 - A C^2)} = \sqrt{(60^2 - 30^2)} = 51{\cdot}96$ = S O, the height of the vacuity of the arch, and S O + O P = $51{\cdot}96 + 3 = 54{\cdot}96$ = S P; again, A B + M A + B L = $60 + 4 + 4 = 68$ = M L, and M L $\times$ S P = the area of the rectangle M N K L; hence M L $\times$ S P $\times$ 40 — 1018832 (the solidity of the vacuity A O B by the last Problem), gives the solidity of the materials; that is
$63 \times 54{\cdot}96 \times 40 - 101832 = 47659{\cdot}2$ feet, the solidity required.

NOTE. When the arch A O B is an elliptical segment, its area multiplied by the length of the roof gives the solidity of the vacuity, and M L multiplied by S P, and the product by the length of the arch, gives the solidity of the cubic figure whose end is M N K L; and the difference of the two solidities is the solidity of the mixed solid whose section is A M N K L B E O H A. The materials of a bridge may be calculated after the same manner, by adding the solidities of T, T, and of the battlements, to the solidity as found in this Problem.

3. Required the capacity of the vacuity of an illiptical vault, whose span is 30 feet, and height 15 feet, the length of the vault being 90 feet. *Ans.* 31808·7 feet.

PROBLEM II.

To find the concave or convex surface of a circular, elliptical, or Gothic vaulted roof.

RULE. Multiply the length of the arch by the length of the vault, and the product will be the superficies.

NOTE. To find the length of the arch, make a line ply close to it, quite across from side to side.

1. What is the surface of a vaulted roof, the length of the arch being 45 feet, and the length of the vault 140 feet?

$$140 \times 45 = 6300 \text{ square feet.}$$

2. Required the surface of a vaulted roof, the length of the arch being 40 feet 6 inches, and the length of the vault 100 feet? *Ans.* 4050 feet.

3. What is the surface of a vaulted roof, the length of the arch being 40.5 feet, and the length of the vault 60 feet? *Ans.* 2430 feet.

PROBLEM III.

To find the solidity of a dome, having the height and the dimensions of its base given.

RULE. Multiply the area of the base by the height, and two-third sof the product will give the solid content.*

---

* This rule is correct only in one case, namely, when the dome is half a sphere, and in this case the height is equal to the radius of the circular base. It is a well-known property that the solidity of a sphere is two-thirds of that of a cylinder having the same base and height. But the solidity of a cylinder is found by multiplying the area of its base by the height. Hence the reason of the rule when applied to this particular case. No general rule can be given to answer every case, as some domes are circular, some elliptical, some polygonal, &c.; they are of various heights, and their sides of different curvature. When the height of the dome is equal to the radius of its base, (the curved sides being circular or elliptical quadrants), or to half the mean proportional between the two axes of its elliptical base, the above rule will answer pretty well; but with any other dimensions it ought not to be used.

1. What is the solid content of a dome, in the form of a hemisphere, the diameter of the circular base being 40 feet?

$40^2 \times \cdot 7854 = 1256 \cdot 64 =$ the area of base.
$\frac{2}{3}(1256 \cdot 64 \times 20) = \frac{2}{3}(25132 \cdot 8) = 16755 \cdot 2$, answer.

2. What is the solid content of an octagonal dome, each side of its base being 20 feet, and the height 21 feet?
Ans. 27039·1917 cubic feet.

3. Required the solidity of the stone-work of an elliptical dome, the two diameters of its base being 40 and 30 feet, the height 17·32 feet, and the stone-work in every part 4 feet thick. *Ans.* 9479·086848 cubic feet.

PROBLEM IV.

To find the superficial content of a dome, the height and dimensions of its base being given.

RULE. Multiply the square of the diameter of the base by 1·5708, and the product will be the superficial content.*

For an elliptical dome, multiply the two diameters of its base together, and the product resulting by 1·5708 for the superficial content, sufficiently correct for practical purposes.

1. The diameter of the base of a circular dome is 20 feet, and its height 10 feet; required its concave superficies?

$20^2 \times 1 \cdot 5708 = 628 \cdot 32$ feet, the answer.

2. The two diameters of an elliptical dome are 40 and 30 feet, and its height 17·32 feet; required the concave surface? *Ans.* 1884·96 square feet.

3. What is the superficies of a hexagonal spherical dome, each side of the base being 10 feet? *Ans.* 519·6152.

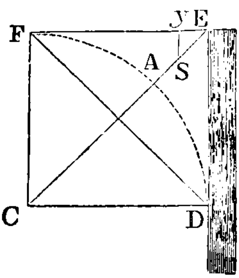
* This rule is correct only when the dome is circular, and its height equal to the radius of the base.—See Appendix, Demonstration 112.

PROBLEM V.

To find the solid content of a saloon.

RULE. Multiply the area of a transverse section by the compass or circumference of the solid part of the saloon, taken round the middle part. Subtract this product from the whole vacuity of the room, supposing the walls to go upright from the spring of the arch to the flat ceiling, and the difference will be the answer, as will appear evident from the following example.

1. What is the solid content of a saloon with a circular quadrantal arch of 2 feet radius, springing over a rectangular room of 20 feet long and 16 feet wide.



$2^2 \times \cdot7854 = 3\cdot1416$ = area of the quadrant C D A F. $2 \times 2 \div 2 = 2$ = area of the triangle C D F; then $3\cdot1416 - 2 = 1\cdot1416$ = area of the segment D A F. Now, $2 \times 2 = 4$ = area of the rectangle C D E F; then $4 - 3\cdot1416 = \cdot8584$ = area of the section D E F A D. $\sqrt{(2^2 + 2^2)} = \sqrt{8} = 2\cdot8284271$. $2 \times 16 + 2 \times 20 = 72$ = the compass within the walls. $\frac{1}{2}(2\cdot8284271 - 2) = \cdot4142136$ = ES and $2\cdot8284271 : \cdot4142136 :: 2 : \cdot2928932$ = Ey: hence $72 - (\cdot2928932 \times 8 = 69\cdot6568544$ = the circumference of the middle of the solid part of the saloon;

therefore 69·6568544 × ·8584 = 59·79344381696 = the content of the solid part of the saloon.

20 × 16 = 320 the area of the room floor, and 320 × 2 = 640 = the solidity of the upper part of the room; then 640 — 59·79344 = 580.20656 feet, the solidity of the saloon.

2. If the height D E of the saloon be 3·2 feet, the chord D F = 4·5 feet, and its versed sine = 9 inches; what is the solid content of the solid part, the mean compass being 50 feet.

Ans. 138·26489 feet.

PROBLEM VI.

To find the superficies of a saloon.

Rule. Find its breadth by applying a string close to it across the surface; find also its length by measuring along the middle of it, quite round the room; then multiply these two dimensions together for the superficial content.

1. The girt across the face of the saloon is 5 feet, and its mean compass 100 feet, what is its superficial content?

100 × 5 = 500, the answer.

2. The girt across the face of the saloon is 12 feet, and it mean compass 98; required its surface?

Ans. 1176 feet.

SECTION VIII.

SPECIFIC GRAVITY.

1. The specific gravity of a body is the relation which the weight of a given magnitude of that body has to the weight of an equal magnitude of a body of another kind.

In this sense a body is said to be specifically heavier than another, when under the same bulk it weighs less than that other. On the contrary, a body is said to be specifically lighter than another, when under the same bulk it weighs less than that other. Thus, if there be two equal spheres, each one foot or one inch in diameter, the one of lead and the other of wood, then since the leaden sphere is found to be heavier than the wooden one, it is said to be specifically, or in specie, heavier, and the wooden sphere specifically lighter.

2. If two bodies be equal in bulk, their specific gravities are to each other as their weight, or as their densities.

3. If two bodies be of the same specific gravity or density, their absolute weights will be as their magnitudes or bulks.

4. If two bodies be of the same weight, the specific gravities will be reciprocally as their bulks.

5. The specific gravities of all bodies are in a ratio compounded of the direct ratio of their weights, and the reciprocal ratio of their magnitude. Hence, again, the specific gravities are as the densities.

6. The absolute weights or gravities of bodies are in the compound ratio of their specific gravities and magnitudes or bulks.

7. The magnitudes of bodies are directly as their weights, and reciprocally as their specific gravities.

8. A body specifically heavier than a fluid, loses as much of its weight, when immersed in it, as is equal to the weight of a quantity of the fluid of the same bulk, or magnitude; if the body be of equal density with the fluid, it loses all its weight, and requires no force but the fluid to sustain it. If it be heavier, its weight in the fluid will be only the difference between its own weight and the weight of the same bulk of the fluid; and therefore it will require a force equal to this difference to sustain it. But if the body immersed be lighter than the fluid, it will require a force equal to the difference between its own weight and that of the same bulk of the fluid, to keep it from rising in the fluid.

9. In comparing the weights of bodies, it is necessary to consider some one as the standard with which all other bodies may be compared. Rain water is generally taken as the standard, it being found to be nearly alike in all places.

A cubic foot of rain water is found, by repeated experiments, to weigh $62\frac{1}{2}$ pounds avoirdupois, or 1000 ounces, and a cubic foot containing 1728 cubic inches, it follows that a cubic inch weighs ·03616898148 of a pound. Therefore if the specific gravity of any body be multiplied by ·03616898148, the product will be the weight of a cubic inch of that body in pounds avoirdupois; and if this weight be multiplied by 175, and the product be divided by 144, the quotient will be the weight of a cubic inch in pounds troy, 144 pounds avoirdupois being exactly equal to 175 pounds troy.

10. Since the specific gravities of bodies are as their absolute gravities under the same bulk; the specific gravity of a fluid will be to the specific gravity of any body immersed in it, as the part of the weight lost by the solid is to the whole weight. Hence the specific gravities of different fluids are as the weights lost by the same solid immersed in them.

PROBLEM I.

To find the specific gravity of a body.

CASE I. *When the body is heavier than water.*

Weigh the body first in water, and afterwards in the open air; the difference will give the weight lost in water; then say, as the weight lost in water is to the absolute weight of the body, so is the specific gravity of water to the specific gravity of the body.

CASE II. *When the body is lighter than the water.*

Fix another body to it, so heavy as that both may sink in water together, as a compound mass. Weigh the compound mass and the heavier body separately, both in the water and open air, and find how much each loses in water, by taking its weight in water from its weight in the open air. Then say, as the difference of these remainders is to the weight of the lighter body in air, so is the specific gravity of water to the specific gravity of the lighter body.

CASE III. *For a fluid of any kind.*

Weigh a body of known specific gravity both in the fluid and open air, and find the loss of weight by subtracting the weight in water from the weight out of it. Then say, as the whole, or absolute weight is to the loss of weight, so is the specific gravity of the solid to the specific gravity of the fluid.

The usual way of finding the specific gravities of bodies is the following, viz :—

On the arm of a balance suspend a globe of lead by a fine thread, and to the other arm of the balance fasten an equal weight sufficient to balance it in the open air ; immerse the globe into the fluid, and observe what weight balances it then, by which the lost weight is ascertained, which is proportional to the specific gravity.

Immerse the globe successively in all the fluids whose proportional specific gravity you require, observing the weight lost in each; then these weights lost in each will be the proportions of the fluids sought.

Examples.—Case I.

1. A piece of platina weighed 83·1886 pounds out of water, and in water, only 79·5717 pounds; what is its specific gravity, that of water being 1000?

83·1886 — 79·5717 = 3·6169 pounds, which is the weight lost in water; then 3·6169 : 83·1886 :: 1000 : 23000 the specific gravity, or the weight of a cubic foot of metal in ounces.

2. A piece of stone weighed 10 lbs. in the open air, but in water only $6\frac{3}{4}$ lbs.; what is its specific gravity?

Ans. 3077.

Examples.—Case II.

3. If a piece of elm weigh 15 lbs. in the open air, and that a piece of copper, which weighs 18 lbs. in open air, and 16 lbs. in water, is affixed to it, and that the compound weighs 6 lbs. in water; required the specific gravity of the elm?

Copper.	Compound.
18 in air.	33
16 in water.	6
—	—
2 loss.	27
	2
	—

As 25 : 15 :: 1000 : 600, the specific gravity of the elm.

4. A piece of cork weighs 20 lbs. in open air, and a piece of granite being affixed to it, which weighs 120 lbs. in air, and only 80 lbs. in water, the compound mass weighs $16\frac{2}{3}$ lbs. in water; required the specific gravity of the cork? *Ans.* 240.

Examples.—Case III.

5. A piece of cast iron weighed 259·1 ounces in a fluid, and 298·1 ounces out of it; required the specific gravity of the

fluid, allowing the specific gravity of the cast-iron to be 7645.

298·1 — 259·1 = 39, loss of weight in the iron; then 298·1 : 39 :: 7645 : 1000, the specific gravity of the fluid; showing the fluid to be water.*

6. A piece of lignum vitæ weighed $42\frac{3}{4}$ ounces in a fluid, and $166\frac{5}{8}$ out of it; what is the specific gravity of the fluid, that of the lignum vitæ being 1333?

Ans. 991 is the specific gravity of the fluid, which shows it to be liquid turpentine or Burgundy wine.

TABLE OF SPECIFIC GRAVITIES.

	Spec. Grav.	wt. cub. in. oz.
Platina	19500	11·285
Do. hammered	20336	11·777
Cast zinc	7190	4·161
Cast iron	7207	4·165
Cast tin	7291	4·219
Bar iron	7788	4·507
Hard steel	7816	4·523
Cast brass	8395	4·858
Cast copper	8788	5·085
Pure cast silver	10474	6·061
Cast lead	11352	6·569
Mercury	13568	7·872
Pure cast gold	19258	11·145
Amber	1078	wt. cub. ft.
Brick	2000	125·00
Sulphur	2033	127·06
Cast nichel	7807	4513
Cast cobalt	7811	4520
Paving stones	2416	151·00
Common stone	2520	157·50
Flint and spar	2594	162·12
Green glass	2642	
White glass	2892	
Pebble	2664	166·50

* In this manner may the species of a fluid or a solid be ascertained, by means of its specific gravity, and the above table. This table has been taken from Gregory's work for practical men.

	Spec. Grav.	wt. cub. ft. lbs.
Slate	2672	167 00
Pearl	2684	
Alabaster	2730	
Marble	2742	171·38
Chalk	2784	174 00
Limestone	3179	193·68
Wax	897	
Tallow	945	
Camphor	989	
Bees' Wax	965	
Honey	1456	
Bone of an ox	1659	
Ivory	1822	
Air at the earth's surface	14	
Liquid turpentine	991	
Olive oil	915	
Burgundy wine	991	
Distilled water	1·000	
Sea water	1·028	
Milk	1·030	
Beer	1·034	
Cork	240	15·00
Poplar	383	23·94
Larch	544	34 00
Elm and West India fir	556	34·75
Mahogany	560	35 00
Cedar	596	37·25
Pitch pine	660	41·25
Pear Tree	661	41·31
Walnut	671	41·94
Elder tree	695	43·44
Beech	696	43.50
Cherry tree	715	44·68
Mapel and Riga fir	750	46 87
Ash and Dantzic oak	760	47·50
Apple tree	793	49·56
Alder	800	50 00
Oak, Canadian	872	54·50
Box, French	912	57 00
Logwood	913	57·06
Oak, English	970	51·87
Oak, 60 years old	1170	73 12
Ebony	1331	83·18
Lignum vitæ	1333	83·31

PROBLEM II.

The specific gravity of a body, and its weight being given, to find its solidity.

RULE. Say, as the tabular specific gravity of the body is to its weight, in ounces avoirdupois, so is 1 cubic foot to the content.

1. What is the solidity of a block of marble that weighs 10 tons, its specific gravity being 2742?

First, 10 tons = 200 hundreds = 22400 pounds = 358400 ounces : then

$$2742 : 358400 :: 1$$
$$1$$

```
2742)358400(130 970/1371
     2742
     ----
      8420
      8226
      ----
      1940
   2)----(970/1371
      2742
```

2. How many cubic inches in an irregular block of marble which weighs 112 pounds, allowing its specific gravity to be 2520? *Ans.* $1228\frac{2016}{2520}$ cubic inches.

3. How many cubic inches of gunpowder are there in 1 pound weight, its specific gravity being 1745?

Ans. $15\frac{3}{4}$ nearly.

4. How many cubic feet are there in a ton weight of dry oak, its specific gravity being 925? *Ans.* $38\frac{138}{185}$.

PROBLEM III.

The linear dimensions, or magnitude of a body, being given, and also its specific gravity, to find its weight.

RULE. One cubic foot is to the solidity of the body, as the tabular specific gravity of the body is to the weight in avoirdupois ounces.

1. What is the weight of a piece of dry oak, in the form of a parallelopipedon, whose length is 56 inches, breadth 18 inches, and depth 12?

$56 \times 18 \times 12 = 12096$ cubic inches, the solid content. Then $1728 : 12096 :: 932 : 6524$ ounces $= 407\frac{3}{4}$ pounds, the weight required.

2. What is the weight of a block of dry oak, which measures 10 feet long, 3 feet broad, and $2\frac{1}{2}$ feet deep; its specific gravity being 925? *Ans.* $4335\frac{15}{16}$ lbs.

3. What is the weight of a block of marble, whose length is 63 feet, and its breadth and thickness, each 12 feet?

Ans. $694\frac{11}{100}$ tous.

PROBLEM IV.

To find the quantities of two ingredients in a given compound.

RULE. Take the difference of every pair of the three specific gravities, viz., of the compound and each ingredient; and multiply the difference of every two by the third.

Then as the greater product is to the whole weight of the compound, so is each of the other products to the weights of the two ingredients.*

1. A composition of 112lbs. being made of tin and copper, whose specific gravity is found to be 8784; what is the quantity of each ingredient, the specific gravity of tin being 7320, and of copper 9000?

9000	9000	8784
7320	8784	7320
1680	216	1464 diff.
8784	7320	9000
14757120	1581120	13176000. Then

$$14757120 : 112 :: \begin{cases} 13176000 : 100 \text{ lbs. copper.} \\ 1581120 : 12 \text{ lbs. tin.} \end{cases}$$

* For the reasen of this rule, see Alligation Total in the second book of Arithmetic, published by the Commissioners.

2. Hiero, king of Sicily, furnished a goldsmith with a quantity of gold, to make a crown. When it came home, he suspected that the goldsmith had used a greater quantity of silver than was necessary in the composition; and applied to the famous mathematician, Archimedes, a Syracusian, to discover the fraud, without defacing the crown.

To ascertain the quantity of gold and silver in the crown, he procured a mass of gold and another of silver, each exactly of the same weight with the crown; justly considering that if the crown were of pure gold, it would be of equal bulk, and therefore displace an equal quantity of water with the golden mass; and if of silver, it would be of equal bulk, and displace an equal quantity of water with the silver mass; but if of a mixture of the two, it would displace an intermediate quantity of water.

Now suppose that each of the three weighed 100 ounces; and that on immersing them severally in water, there were displaced 5 ounces of water by the golden mass, 9 ounces by the silver mass, and 6 ounces by the crown; what quantity of gold and silver did the crown contain?

Ans. { 75 ounces of gold.
25 ounces of silver.

NOTE. Questions relating to specific gravities may be wrought by the rules of Alligation in Arithmetic, as well as by any Algebraic process that might be employed.

PROBLEM V.

To find how many inches a floating body will sink in a fluid.

RULE. Find, by Problem III. the weight of the floating body from its solidity and specific gravity, and that will be the weight of the fluid which it will displace.

Then say, as the specific gravity of the fluid is to 1728 cubic inches, so is the weight of the body, in ounces, to the cubic inches immersed. The depth will be found from the given dimensions.

1. Suppose a piece of dry oak, in the form of a parallelopipedon, whose length is 56 inches, breadth 18, and depth 12, is to be floated upon common smooth water, on its broadest side; how many inches will it sink, its specific gravity being 932 ?

By Problem III., the weight of the piece of oak is 6524 ounces, which, by the preliminary part of this section, is the weight of the water displaced.

Then 1000 : 1728 :: 6524 : 11273·472 cubic inches of oak immersed. Therefore, $11273{\cdot}472 \div (56 \times 18) \times 11{\cdot}184$ inches the depth it will sink.

To find how far it will sink, allowing it to float on its narrower side, $11273{\cdot}472 \div (56 \times 12) = 16{\cdot}776$ inches.

2. How many inches will a cubic foot of dry oak sink in common water, allowing the specific gravity of the oak to be 970 ? *Ans.* 11·64.

PROBLEM VI.

To find what weight may be attached to a floating body, so that it may be just covered with a given fluid.

Rule. Multiply the cubic feet in the body by the difference between its specific gravity and that of the fluid, and the product will be the weight in ounces avoirdupois, just sufficient to immerse it in the fluid.

1. What weight must be attached to a piece of dry oak, 56 inches long, 18 inches broad, and 12 inches deep, to keep it from rising above the surface of a fresh-water lake; the specific gravity of the water being 1000, and that of the oak 932 ?

Here $56 \times 18 \times 12 = 12096$ cubic inches.

Then $12096 \div 1728 = 7$ feet.

Then $(1000 - 932) \times 7 = 68 \times 7 = 476$ ounces = 29 pounds 12 ounces.

2. What weight, fixed to a piece of dry oak, 9 inches long, 6 inches broad, and 3 inches deep, will keep it from rising above the surface of common water, the specific gravity of water being 1000, and that of the oak 970?

Ans. $2\frac{13}{16}$ ounces.

3. A sailor had half an anker of brandy, the specific gravity of the liquor was 927, the cask was oak, and contained 216 cubic inches, and its specific gravity was 932; to secure his prize from the custom-house officers, he fixed just as much lead to the cask as would keep it under water, and then threw it into the sea; what weight of lead was necessary for his purpose?

Ans. The cask of brandy contained 1371 cubic inches, the weight of sea-water of an equal bulk was 817·20486 ounces, the cask weighed 116·5 ounces, the brandy 619·609375, both together weighed 736.19375 ounces. The difference between the specific gravity of lead and sea-water is to this remainder, as the specific gravity of lead to its weight in ounces, which will be found to be 89·09495 ounces, or 5 pounds 9 ounces.

PROBLEM VII.

To find the solidity of a body, lighter than a fluid, which will be sufficient to prevent a body much heavier than the fluid, from sinking.

Rule. Find the solidity of the body to be floated; from its weight and specific gravity, by Problem II. Find also the weight of an equal bulk of the fluid by Problem III. Then say, as the difference between the specific gravity of the fluid, and that of the body lighter than the fluid, is to the difference between the weight of the body to be floated and the weight of an equal bulk of the fluid, so is 1728 to the solidity of the lighter body in cubic inches.

1. How many solid feet of yellow fir, whose specific gravity is 657, will be sufficient to keep a brass cannon, weighing 56

cwt., afloat at sea, the specific gravity of brass being 8396, and of sea water 1030?

First, 56 cwt. = 100352 ounces, weight of the body to be floated,

Then, 8396 : 100352 :: 1728 : 20653.675 cubic inches in the cannon.

And, 1728 : 20653·675 :: 1030 : 12310·9289, the weight of sea-water equal in bulk to that of the cannon.

Hence, 1030 — 657 : 100352 — 12310·9289 :: 1728 : 407868·5545 cubic inches = 236·036 feet, the answer.

2. The specific gravity of lead is 11325, of cork 240, and of sea-water 1030; now it is required to know how many cubic inches of cork will be sufficient to keep $49\frac{3}{8}$ pounds of lead afloat at sea? *Ans.* 1570·84 cubic inches.

TO FIND THE TONNAGE OF SHIPS.

1st.—VESSELS AGROUND.

By the Parliamentary Rule.

PROBLEM VIII.

For a ship or vessel, the length is to be measured on a straight line along the rabbet of the keel, from a perpendicular, let fall from the back of the main post, at the height of the wing-transom, to a perpendicular at the height of the upper deck (but the middle deck of three-decked ships), from the forepart of the stern; then from the length between these perpendiculars subtract three-fifths of the extreme breadth for the rake of the stern, and $2\frac{1}{2}$ inches for every foot of the height of the wing-transom above the lower part of the rabbet of the keel, for the rake abaft; and the remainder will be the length of the keel for tonnage.

The main breadth is to be taken from the outside of the outside plank, in the broadest part of the ship, either above or below the wales, deducting therefrom all that it exceeds the thickness of the plank of the bottom, which shall be accounted the main breadth; so that the moulding breadth, or the breadth of the frame, will then be less than the main breadth, so found, by double the thickness of the plank of the bottom.

Then multiply the length of the keel for tonnage by the main breadth, so taken, and the product by half the breadth, then divide the whole by 94, and the quotient will give the tonnage.

In cutters and brigs, where the rake of the stern-post exceeds $2\frac{1}{2}$ inches to every foot in height, the actual rake is generally subtracted instead of the $2\frac{1}{2}$ inches to every foot, as before mentioned.

1. Let us suppose the length from the fore-part of the stern, at the height of the upper deck, to the after-part of the stern-post, at the height of the wing-transom, to be 155 feet 8 inches, the breadth from out to outside 40 feet 6 inches, and the height of the wing-transom 21 feet 10 inches, what is the tonnage?

	ft.	
	40·6	breadth.
deduct	3	
	40·3	
	3	
	5)120·9	
	$24{\cdot}1\frac{4}{5}$	= 24·15
	21·10	height of wing-transom.
	$2\frac{1}{2}$	multiply.
	$12)54\frac{7}{12}$	

4·55 + 24·15 = 28·70

155·66 — 28·70 = 126.96 = length.

$$\frac{126{\cdot}96 \times 40{\cdot}25 + 20{\cdot}125}{94} = 1094, \text{ the answer.}$$

2. Suppose the length of the keel to be 50.5 feet, breadth of the midship-beam 20 feet; required the tonnage?

Ans. 107·4.

3. If the length of the keel be 100 feet, and the breadth of the beam 30 feet; what is the tonnage? *Ans.* 478.

2nd.—VESSELS AFLOAT.

Drop a plumb-line over the stern of the ship, and measure the distance between such line and the after-part of the stern-post, at the load water-mark: in a parallel direction with the water, to a perpendicular point immediately over the load water-mark, at the fore-part of the main-stern, substracting from such measurement the above distance, the remainder will be the ship's extreme length; from which is to be deducted three inches for every foot of the load draught of water for the rake abaft, and also three-fifths of the ship's breadth for the rake forward, the remainder shall be esteemed the just length of the keel to find the tonnage; and the breadth shall be taken from outside to outside of the plank, in the broadest part of the ship, either above or below the main-wales, exclusive of all manner of sheathing or doubling that may be wrought upon the sides of the ship; then multiply the length of the keel, taken as before directed, by the breadth, as before taught, and that product by half the said breadth, and dividing the product by 94, the quotient is the tonnage.

3rd.—STEAM VESSELS.

The length shall be taken on a straight line, along the rabbet of the keel, from the back of the main-stern post to a perpendicular line from the fore-part of the main-stem under the bowsprit; from which deducting the length of the engine-room, and subtracting three-fifths of the breadth, the remainder shall be esteemed the just length of the keel to find the tonnage; and the breadth shall be taken from the outside of the outside plank in the broadest place of the ship or vessel, be it either above or below the main-wales, exclusively of all manner of doubling planks that may be wrought upon the sides of the ship or vessel; then multiply the length and breadth so found together, and that product by half the same breadth, and dividing by 94, the quotient will be the tonnage, according to which all such vessels shall be measured.

NOTE. Under certain penalties nothing but the fuel can be stowed in the engine-room.

Some divide the last product by 100, to find the tonnage of king's ships, and by 95, to find that of merchant's ships.

FLOATING BODIES.

1. The buoyancy of casks, or the load which they will carry, without sinking, may be estimated by reckoning 10lbs. avoirdupois to the ale gallon, or $8\frac{1}{3}$lbs. to the wine gallon.

2. The buoyancy of pantoons may be estimated at about half a hundred weight, or 56lbs. for each cubic foot. Therefore a pantoon which contained 96 cubic feet, would sustain 48 hundred weight before it could sink.

N.B.—This is an approximation, in which the difference between $\frac{6}{11}$ and $\frac{1}{2}$ viz., $\frac{1}{22}$ of the whole weight is allowed for that of the pantoon itself.

3. The principles of buoyancy are very ingeniously applied in the self-acting flood-gate, which, in the case of common sluices to a mill-dam prevents inundation when a sudden flood occurs. By means of the same principle it is that a hollow ball attached to a metallic lever of about a foot long, is made to rise with the liquid in a water-cask, and thus to close the cock and stop the supply from the pipe, just before the time when the water would otherwise run over the top of the vessel.

The property of buoyancy has also been successfully employed in raising ships which had sunk under water, and in pulling up old piles in a river when the tide ebbs and flows. A large barge is brought over as pile a the water begins to rise; a strong chain which has been previously fixed to the pile by a ring, &c., is made to gird the barge, and is then firmly fastened; then, as the tide rises, the barge rises also, and by means of its buoyant force draws up the pile with it.

In a case which actually occurred, a barge of 50 feet long, 12 feet wide, 6 deep, and drawing two feet water was employed. Then $50 \times 12 \times (6-2) \times \frac{4}{7} = \frac{50 \times 12 \times 16}{7}$ $= 192 \times 7\frac{1}{7} = 1344 + 27\frac{3}{7} = 1371\frac{3}{7}$ cwt. $= 66\frac{1}{2}$ tons, nearly, which is the measure of the force with which the barge acted in pulling up the pile.

SECTION IX.

WEIGHT AND DIMENSIONS OF BALLS AND SHELLS.

The foregoing problems furnish rules for finding the weight and dimensions of balls and shells. But they may be found much easier by means of the experimental weight of a ball of a given size, and from the well-known geometrical property, that similar solids are as the cubes of their diameters.

PROBLEM I.

To find the weight of an iron ball from its diameter.

Rule. Nine times the cube of the diameter being divided by 64, will express the required weight in pounds.*

1. The diameter of an iron shot is 5 inches; required its weight?

$5 \times 5 \times 5 = 125 =$ cube of the ball's diameter.
Then $125 \times 9 \div 64 = 17\frac{37}{64}$lbs., the answer.

2. The diameter of an iron shot being 3 inches; required its weight? *Ans.* 3·8lbs.

3. The diameter of an iron shot is 5·54 inches; what is its weight? *Ans.* 24lbs.

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* See Appendix, Demonstration 113.

PROBLEM II.

To find the weight of a leaden ball, by having its diameter given.

RULE. Multiply the cube of its diameter by 2, and divide the product by 9, and the quotient will give the weight in pounds.*

1. What is the weight of a leaden ball of 5 inches diameter?

$$5 \times 5 \times 5 = 125 \text{ cube of ball's diameter.}$$

Then, $125 \times 2 \div 9 = 250 \div 9 = 27\frac{7}{9}$ lbs., answer.

2. What is the weight of a leaden ball whose diameter is 6·6 inches? *Ans.* 63·888 lbs.

3. What is the weight of a leaden ball, whose diameter is 3·5 inches? *Ans.* 9·53 lbs.

4. What is the weight of a leaden ball, whose diameter is 6 inches? *Ans.* 48 lbs.

PROBLEM III.

Having the weight of an iron ball, to determine its diameter.

RULE. Multiply the weight by $7\frac{1}{9}$, then take the cube root of the product for the diameter.†

1. What is the diameter of an iron ball, whose weight is 42 lbs.

$$42 \times 7\tfrac{1}{9} = 298\tfrac{2}{3}.$$

Then, $\sqrt[3]{298} = 6·685$ inches, the answer.

2. Required the diameter of an iron ball, whose weight is 24 lbs.? *Ans.* 5·54 inches.

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* See Appendix. Demonstration 114.

† This rule is obvious from Problem I., being the converse thereof.

3. What is the diameter of an iron ball, whose weight is 3·8 lbs.? *Ans.* 3 inches.

PROBLEM IV.

Having the weight of a leaden ball, to determine its diameter.

RULE. Multiply the weight by 9, and divide the product by 2; and the cube root of the quotient will express the diameter.*

1. What is the diameter of a leaden ball, whose weight is 64 lbs.?

$$64 \times 9 = 576.$$
$$\text{Then, } 576 \div 2 = 288.$$

Hence, $\sqrt[3]{288} = 6·6$ inches, the answer.

2. Required the diameter of a leaden ball, whose weight is $27\frac{7}{9}$ lbs.? *Ans.* 5 inches.

3. What is the diameter of a leaden ball, whose weight is 63·888 lbs.? *Ans.* 6·6 inches.

PROBLEM V.

Having given the external and internal diameter of an iron shell, to find its weight.

RULE. Find the difference between the cubes of the two diameters, and multiply it by 9; divide the product by 64, and the quotient will express the weight in pounds.†

1. What is the weight of an 18-inch iron bomb-shell, whose mean thickness is $1\frac{1}{4}$ inches?

$18 - 2\frac{1}{2} = 15\frac{1}{2}$ = internal diameter.
Then, $18^3 = 5832$ the cube of external diameter.
$(15.5)^3 = 3723·875$ the cube of internal diameter.
And, $5832 - 3723·875 = 2108·125$ = difference of cubes.
Hence, $2108·125 \times 9 \div 64 = 296·45$ lbs., the answer.

* This rule is manifest from Problem III., being its converse.
† See Appendix, Demonstration 115.

2. What is the weight of a 9-inch iron bomb-shell, whose mean thickness is $1\frac{1}{2}$ inch. *Ans.* 72·14 lbs.

3. What is the weight of an iron bomb-shell, whose external diameter is 9·8 inches, and internal diameter 7 inches? *Ans.* $84\frac{1}{8}$ lbs.

PROBLEM VI.

To find how much powder will fill a shell of given dimensions.

RULE. Divide the cube of the internal diameter in inches, by 57·3, and the quotient will express the answer.*

1. What quantity of powder will fill a shell, whose internal diameter is 10 inches?

First, $10 \times 10 \times 10 = 1000 =$ cube of diameter.

```
57·3)1000(17·45 lbs., answer.
     573
     ----
     4270
     4011
     ----
      2590
      2292
      ----
       2980
       2865
       ----
        115, &c.
```

NOTE. In some recent works, the cube of the diameter is divided by 59·32, for the weight of powder in pounds.

2. How many pounds of gunpowder are required to fill a hollow shell, whose internal diameter is 13 inches?

Ans. 37 lbs., according to the note.

3. Required the number of pounds of powder that will fill a shell, whose internal diameter is 7 inches?

Ans. 6 lbs. by the rule in the text.

* See Appendix, Demonstration 116.

PROBLEM VII.

To find how much powder will fill a rectangular box of given dimensions.

Rule. Multiply the length, breadth, and depth together in inches, and the last result by ·0322, and the last product will give the weight in pounds.*

1. How many pounds of powder will fill a rectangular box, whose length is 16 inches, breadth 12 inches, and depth 6 inches?

$16 \times 12 \times 6 = 1152$ = content of the box.
Then, $1152 \times \cdot 0322 = 37\cdot 0944$, the answer.

2. How many pounds of powder will fill a rectangular box, whose length is 10 inches, breadth 5 inches, and depth 2 inches? *Ans.* 3·22 lbs.

3. How many pounds of powder will fill a rectangular box, whose length is 5 inches, breadth 2 inches, and depth 10 inches? *Ans.* 3·22 lbs.

PROBLEM VIII.

Having the length and diameter of a cylinder, to determine how many pounds of gunpowder will fill it.

Rule. Multiply the square of the diameter by the length, and divide the product by 40, for the weight in pounds.†

1. The diameter of a hollow cylinder is 10 inches, and the length 14 inches; how many pounds will it hold?

$10 \times 10 = 100$ = square of diameter.
Then, $100 \times 14 = 1400$.
Hence, $1400 \div 40 = 35$ lbs., the answer.

---

* See Appendix, Demonstration 117.
† See Appendix, Demonstration 118.

2. The diameter of a hollow cylinder is 5 inches, and its length 40 inches; how much powder will it hold?

Ans. 25 lbs.

3. The diameter of a hollow cylinder is 5 inches, and the length 12 inches; how many pounds will it hold?

Ans. 7·5 lbs.

PROBLEM IX.

To find what portion of a cylinder will be occupied by a given quantity of powder, the diameter of the cylinder being given.

Rule. Multiply the given weight of powder by 40, and divide the product by the square of the diameter of the cylinder, and the quotient will be the pounds required.*

1. The diameter of a hollow cylinder is 10 inches; how much of it will hold 50 lbs. of powder?

$$50 \times 40 = 2000.$$

Then, $2000 \div 100 = 20$ inches, the answer.

2. How much of a cylinder of 14 inches diameter, will hold 10 lbs. of powder? *Ans.* 2·05.

3. How much of a cylinder, 12 inches in diameter, will hold 144 lbs. of powder? *Ans.* 40 inches.

PILING OF BALLS AND SHELLS.

Iron-shot and shells are usually piled in horizontal courses, either in a pyramidical or in a wedge-like form; the base being either an equi-lateral triangle, a square, or a rectangle

† See Appendix, Demonstration 119.

Those piles whose bases are triangles or squares, terminate in one ball at the top: but piles whose bases are rectangles terminate in a single row of balls.

In triangular and square piles, the number of horizontal rows of courses, is always equal to the number of balls in one side of the bottom row.

And in rectangular piles, the number of rows is equal to the number of balls in the breadth of the bottom.

Also the number in the top row or edge, is one more than the difference between the length and breadth of the bottom row.

PROBLEM I.

To find the number of balls in a rectangular pile.

Rule. Multiply the number in one side of the bottom row, by that number increased by 1, and the result by the same number increased by 2; then the one-sixth of the last product will give the number of balls required.*

1. Required the number of shot in a complete triangular pile, one of whose sides contains 22 balls?

```
   22 = the number in one side of base.
   23 = the number + 1.
   --
   66
  44
 ---
  506
   24 = the number + 2.
 ----
 2024
1012
-----
6)12144
 -----
 2024 = the number of shot in the pile.
```

* See Appendix, Demonstration 120.

2. Required the number of shot in a complete triangular pile, one side of whose base contains 15 bails?

Ans. 680 balls.

3. Required the number of balls in a triangular pile, each side of the base containing 30 balls? *Ans.* 4960.

PROBLEM II.

To find the number of balls in a square pile.

Rule. Multiply continually together the number in one side of the bottom course, that number increased by 1, and double the same number increased by 1; then one-sixth of the last product will be the answer.*

1. How many balls are in a square pile of 30 rows?

30 = number in one side.
31 = number in one side + 1.

930
61 = twice the number in one side + 1.

6)56730

9455 answer.

2. Required the number of shot in a complete square pile, one side of whose base contains 19? *Ans.* 2470.

3. How many shot in a finished square pile, when a side of the base contains 21 shot? *Ans.* 3311.

PROBLEM III.

To find the number of shot in a finished rectangular pile.

Rule. Add 1 to three times the number of shot contained in the length of the base, subtract the number of shot in the

* See Appendix, Demonstration 121.

breadth of the base, multiply the remainder by the said number increased by 1, and this result again by the number in the breadth; then one-sixth of the last result will give the number of shot in the rectangular pile.*

1. Required the number of shot in a finished rectangular pile, the length of the base containing 59, and its breadth containing 20 balls?

$$\begin{array}{l} 59 = \text{the number of shot in the length.} \\ \underline{3} \\ 177; \text{ then } 177 + 1 = 178, \text{ and } 178 - 20 = 158. \\ 158 \times 21 = 3318, \text{ and } 3318 \times 20 = 66360. \text{ Hence} \\ 66360 \div 6 = 11060, \text{ the answer.} \end{array}$$

2. How many balls are in a rectangular complete pile, the length of the bottom course being 46, and its breadth 15?

Ans. 4960.

PROBLEM IV.

To determine the number of balls contained in a pile which is not finished, the highest course being complete, and the number of balls in each side thereof being given.

RULE. Find the number of shot which would be contained in the pile if it were complete. Find also the number in that complete pile, each side of whose base contains one shot fewer than the corresponding side of the uppermost course of the unfinished pile, and the difference between these results will evidently give the number of balls in the unfinished pile.

1. How many shot are there in an unfinished triangular pile, a side of whose base contains 23, and a side of the uppermost course 7 shot?

---

* See Appendix, Demonstration 122.

```
   23 = number of balls in the base.
   24 = number of balls in the base + 1.
  ----
  552
   25
  ----
6)13800
  ----
 2300 = number of the pile when complete.
  ----
    6
    7
  ----
   42
    8
  ----
 6)336
  ----
   56 number of balls in the imaginary pile.
```

Therefore, 2300 — 56 = 2244, the answer.

2. How many balls in an incomplete square pile, the side of the base being 24, and of the top 8 ? *Ans.* 4760.

3. How many balls are there in the incomplete rectangular pile of 12 courses, the length and breadth of the base being 40 and 20 ? *Ans.* 6146.

DETERMINING DISTANCES BY SOUND.

The velocity of sound, or the space through which it is propagated in a given time, has been very differently estimated by philosophers who have written on this subject. We shall, however, take it to be 1142 feet in a second.

From repeated experiments it has been ascertained that sound moves uniformly, or, to speak more philosophically, that the pulses of air which excite it move uniformly. The velocity of sound is the same with that of the ærial waves, and does not vary much whether it go with the wind or against it. By the wind, no doubt, a certain quantity of air is carried from one place to another, and the sound is somewhat accelerated while its waves move through that part of the air, if their direction be the same as that of the wind. But as the velocity of sound is vastly swifter than the wind, the acceleration it will thereby receive is but inconsiderable, being at most but $\frac{1}{20}$ of the whole velocity.

The chief effect perceptible from the wind is, that it increases and diminishes the space through which sound is propagated. The utmost distance at which sound has been heard is about 200 miles. It is said that the unassisted human voice has been heard from Old to New Gibraltar, a distance of about 12 miles. Dr. Derham, placing cannon at different distances, and causing them to be fired off, observed the intervals between the flash and report, by means of which he found the velocity of sound to be as above stated.

1. Having observed the flash of a cannon, I noticed by my watch that 5 seconds elapsed previous to my hearing the report; determine my distance from the gun.

$$\begin{array}{r} 1142 \\ 5 \\ \hline 5710 \end{array}$$ feet, the answer.

2. Being at sea, I saw the flash of a cannon, and counted 8 seconds between the flash and the report; required the distance? *Ans.* 1 $\frac{7}{10}$ mile.

SECTION X.

GAUGING.

Gauging is the art of measuring the capacities of vessels, such as casks, vats, &c.

The business of guaging is generally performed by means of two instruments, namely, the gauging or sliding rule, and the gauging or diagonal rod.

1. OF THE GAUGING RULE.—LEADBETTER'S.

By this instrument is computed the contents of casks, &c., after the dimensions have been taken. It is a square rule, having various logarithmic lines on its four faces, and three sliding pieces capable of being moved through grooves in which they fit, in three of these faces.

On the first face are delineated three lines, namely, two marked A B, on which multiplication and division are performed; and the third marked M D, signifies malt depth, and serves to guage malt. The middle one B is on the slider, and is a kind of double line, being marked at both edges of the slider, for applying it to both the lines A and M D. These three lines are all of the same radius, or distance from 1 to 10, each containing twice the length of the radius. A and B are numbered and placed exactly alike, each commencing at 1, which may be either 1, or 10, 100, &c., or 1, or ·01, ·001, &c. Whatever the 1 at the beginning is estimated at, the middle division, 10, will be 10 times as much, and the last division 100 times as much, But 1 on the line M D is opposite 2220, or more exactly 2218·2 on the other lines, which number 2218·2 denotes the cubic inch in an imperial malt bushel; and its divisions numbered retrograde to those of A and B. On these two lines are also several other marks and letters; thus on the line A and M B, or sometimes only B, for malt bushel, at the number 2218·2, and A for ale, at 282, the cubic inches in an old ale gallon; and on the line B, is W, for wine, at 231, the cubic inches in an old wine gallon.

These marks are now usually omitted upon the rule, since the late new Act of Parliament for uniformity of weights and measures, and G for gallon is put at 277·274 the inches in an imperial gallon,* whether for ale, wine, or spirits.

On many sliding rules are also found *s i*, for square inscribed at ·707, the side of a square inscribed in a circle, whose diameter is 1; *s e*, for square equal at ·886, the side of a square which is equal to the same circle; and *c* for circumference, at 3·1416, the circumference of the same circle.

On the second face, or that opposite the first, are a slider and four lines marked D, C, D, E, at one end, and root square, root cube at the other end; the lines C and D containing, respectively, the squares and cubes of the opposite numbers on the lines D, D; the radius of D being double to that of A, B, C, and triple to that of E; therefore whatever the first 1 on D denotes, the first on C is its square, and the first on E its cube; that is, if D begin with 1, C and E will begin with 1; but if D begin with 10, C will begin with 100, and E with 1000; and so on.

On the line C are marked *o c* at ·0796, for the area of the

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* Until 5 George IV., in which a uniform System of weights and measures was established under the denomination of IMPERIAL WEIGHTS AND MEASURES there were, amongst other sources of inconvenience, different measures, though of the same name, for ale and wine. A gallon of ale contained 282 cubic inches, and a gallon of wine 231; a bushel of malt contained 2150·42 cubic inches.

To reduce old measure into new, say, as the number of cubic inches in the imperial standard is to the number of cubic inches in the old standard, so is the number of gallons or bushels, &c., old measure, to the number of gallons, &c., imperial measure.

When great accuracy is not required, old wine gallons may be reduced to imperial gallons by dividing by 1·2; and the old ale gallons may be reduced to imperial gallons by multiplying by 60, and dividing the product by 59; and old or Winchester bushels may be reduced to imperial bushels by multiplying by 31, and dividing the product by 32.

circle whose circumference is 1; and *o d*; at ·7854, for the area of the circle whose diameter is 1.

On the line D are marked G S, for gallon square at 16·65, and G R for gallon round at 18·789; also M S for malt square at 47·097, and M R for malt round at 53·144.

These are the respective gauge-points for gallons and bushels. The first 16·65 is the side of a square, which at an inch depth holds a gallon; the second 18·789, the diameter of a circle, which at an inch depth holds a gallon; the third 47·097 the side of a square, which at an inch depth holds a bushel; the fourth, 53·144, the diameter of a circle, which at an inch depth holds a bushel.

On the third face are three lines: one on a slider, marked N; and two on the stock, marked S S and S L, for segment standing and segment lying, which serve ullaging, standing and lying casks.

And on the fourth side, or opposite face, are a scale of inches, and three other scales, marked spheroid, or 1st variety, 2nd variety, 3rd variety; the scale for the fourth or conic variety, being on the inside of the slider in the third face. The use of these lines is, to find the mean diameter of casks. On the inside of the two first sliders, besides all those already described, are two other lines, being continued from one slider to the other.

The one of these is a scale of inches, from $2\frac{1}{2}$ to 36, and the other is a scale of ale gallons, between the corresponding number 435 and 3·61; which form a table, to show, in ale gallons, the contents of all cylinders whose diameters are from $12\frac{1}{2}$ to 36 inches, their common altitude being 1 inch.

VERIE'S SLIDING RULE.

This rule is in the form of a parallelopipedon, and is generally made of box.

1. The line marked A, on the face of this rule, is called Gunter's line, and is numbered, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. At 2218·192 is fixed a brass pin, marked IM, B, signifying the cubic inches in a imperial bushel; at 277·274 is fixed

another brass pin marked IM, G, denoting the number of cubic inches in an imperial gallon.

2. The line marked B is on the slide, and is divided exactly like that maked A. There is another slide B, on the opposite side, which is used along with this. The slide on the first face is called the *second radius*, and that on the opposite face, the *first radius*. The two brass ends, when placed together, make a double radius, numbered from the left-hand towards the right. At 277·274, on the second radius, is a fixed brass pin, marked IM, G, denoting the cubic inches in an imperial gallon; at 314 is fixed another brass pin, marked C, signifying the circumference of a circle whose diameter is 1. These lines are used and read exactly as the lines A and B, on the carpenter's rule, which have been already described.

3. The back of one slide or radius, marked B, has the dimensions for imperial gallons, and bushels, green starch, dry starch, hard soap hot, hard soap cold, green soft soap, white soft soap, flint glass, &c., &c., as in table, page 206.

The back of the other slide or radius, marked B, contains the gauge-points corresponding to these divisors, where S denotes squares, and C circles.

4. The line M D on the rule, denoting malt depth, is a line of numbers commencing at 2281·192, and is numbered from the left to the right-hand 2, 10, 9, 8, 7, 6, 5, 4, 3. This rule is used in malt gauging.

5. The two slides B, just described, are always used together, either with the line A, M D, or the line D, which is on the opposite face of the rule to that already described. The line D is numbered from the left-hand towards the right, 1, 2, 3, 31, to 32, which is at the right-hand end; it is then continued from the left hand end of the other edge of the rule, 32, 4, 5, 6, 7, 8, 9, 10. At 16·651 is a brass pin G S, signifying a *guage square*, being the square guage-point for imperial gallons. At 18·789 is fixed a brass pin, marked G R, denoting *gauge round*, or circular gauge point for imperial gallons. At 47·097, M S signifies *malt square*;

the square gauge-point for malt bushels. At 53·144, M R denotes *malt round*, the round or circular gauge-point for malt bushels. The line D on this rule is of the same nature as the line marked D on the carpenter's rule, which has been already described. The line A and the two slides B, are used together, for performing multiplication, division, simple proportion, &c.; and the line D, and the same slides B, are used together for extracting the square and cube roots.

6. The other two slides belonging to this rule are marked C, and are divided in the same manner, and used together, like the slides B.

The back of the first slide or radius, marked C, is divided, next the edge, into inches, and numbered from the left-hand towards the right, 1, 2, 3, 4, 5, &c., and these inches are again subdivided into 10 equal parts. The second line is marked spheroid, and is numbered from the left hand towards the right 1, 2, 3, 4, 5, 6, 7, 8. The third line is marked second variety, and is numbered 1, 2, 3, 4, 5, 6. These lines are used, with the scale of inches, for finding a mean diameter.

The back of the second slide or radius, marked C, has several factors for reducing goods of one denomination to others of equivalent values. Thus | X. to VI. 6. | signifies that to reduce strong beer at 8*s*. per barrel, to small beer at 1*s*. 4*d*. you are to multiply by 6. | VI. to X. 17. | signifies that to reduce small beer at 1*s*. 4*d*. per barrel to strong beer at 8*s*. per barrel, you are to multiply by ·17. | C 4 ∞ to X. 27. | signifies that 27 is the multiplier for reducing cider at 4*s*. per barrel to another at 8*s*., &c.

7. The two slides C, just described, are always used together, with the lines on the rule marked Seg. St., or S S, segments standing; and Seg. L *y* or S L, segments lying; for ullaging casks. The former of these lines is numbered 1, 2, 3, 4, 5, 6, 7, 8, which stands at the right hand end; it then goes on from the left-hand on the other edge 8, 9, 10, &c., to 100, the latter is numbered in the same manner 1, 2, 3, 4, which stands at the right hand end; it then goes on from the left-hand on the other edge, 4, 5, 6, 7, &c., to 100.

PROBLEM I.

To find the several multipliers, divisors, and gauge-points belonging to the several measures now used

MULTIPLIERS FOR SQUARES.

As 277·274 solid inches are contained in one imperial gallon, and 2218·192 solid inches in an imperial bushel; then it is obvious that if 1 be divided by 277·274, and 2218·192, respectively, the quotients will be the multipliers for imperial gallons and bushels respectively.

Hence the method of finding the following multipliers is obvious :—

277·274)1·00000(·0036065 multiplier for imperial gallons.
2218·192)1·00000(·0004508 multiplier for imperial bushels.

Now it is manifest that if the solid inches contained in any vessel be multiplied by the first of these multipliers, the product will be the imperial gallons that vessel will contain; and if multiplied by the other, the product will be the imperial bushels.

MULTIPLIERS AND DIVISORS FOR CIRCLES.

It has been shown that when the diameter of a circle is 1, the area of that circle is ·785398, &c., ·7854, nearly; then by dividing the solid capacity of any figure by ·7854, the quotient will be the proper divisor for the square of the diameter of a circular figure. Then to reduce the area at one inch deep into gallons, divide ·7854, or ·785398, &c., by 277·274, and 2218·192, and the quotients will give the multipliers for imperial gallons and bushels respectively; and ·7854 divided into 277·274 and 2218·192, will give the divisors for the imperial gallons and bushels.

277·274)·785398(·002832 multiplier for imperial gallons.
2218·192)·785398(·00354 multiplier for imperial bushels.
·785398)277·274(350·0362 divisor for imperial gallons.
·785398)2218·192(2824·2897 divisor for imperial bushels.

The gauge-points are found by extracting the square root of the divisors.

GAUGE-POINTS FOR SQUARES.

$\sqrt{277 \cdot 274} = 16 \cdot 651$ imperial gallons.
$\sqrt{2218 \cdot 192} = 47 \cdot 097$ imperial bushels.

GAUGE-POINTS FOR CIRCLES.

$\sqrt{353 \cdot 0362} = 18 \cdot 789$ imperial gallons.
$\sqrt{2824 \cdot 2897} = 53 \cdot 144$ imperial bushels.

In this manner the numbers in the following table were calculated.

A TABLE

Of Multipliers, Divisors, and Gauge-points, for Squares and Circles.

NOTE. The Areas, &c., are all in inches	Multipliers for		Divisors for		Gauge-points for	
	Squares.	Circles.	Squares.	Circles.	Squares.	Circles.
The side of a diameter 1	1	·785398	1·	1·27324	1·	1·128
A superficial foot	·006944	·005454	144·	183·34	12·	13·54
A solid foot	·000578	·000454	1728·	2200·16	41·57	46·91
Imperial gallon	·003606	·002832	277·274	353·03	16·65	18·79
Imperial bushel	·000450	·000354	2218·192	2824·29	47·09	53·14
A pound of hard soap	·036845	·028939	27·14	34·56	5·21	5·88
A pound of hot soap	·035714	·028050	28·	35·65	5·29	5·97
A pound of green soap	·038956	·0306	25·67	32·68	5·06	5·72
A pound of white soft soap	·039123	·03073	25·56	32·54	5·05	5·7
A pound of tallow net	·031847	·025172	31·4	39·98	5·6	6·32
A pound of green starch	·028736	·022569	34·8	44·31	5·9	6·66
A pound of dry starch	·024813	·019377	40·3	51·31	6·35	7·16
A pound of flint glass	·094697	·074374	10·56	13·44	3·25	3·66
A pound of white glass	·071124	·05586	14·06	17·9	3·75	4·23
A pound of green glass	·082102	·064826	12·18	15·5	3·48	3·94

NOTE. It very often happens in the practice of gauging, that when the one given number is set to the gauge-point on the sliding rule, the other given number will fall off the rule; hence in many cases it will be necessary to find a second or new gauge-point. The second gauge-points are the square roots of ten times the divisors in the above table. Thus, for squares, the new gauge-point for imperial gallons is 52·65, for bushels 148·93; and for circles, the new gauge-point for gallons is 59·42, for malt bushels 168·05.

PROBLEM II.

To find the area, in imperial gallons, of any rectilineal plane figure.

Rule. By the rules given in Mensuration of Superficies, find the area of the figure in inches, which being divided by 277·274, or multiplied by ·0036065, will give the area in gallons.*

1. Suppose a back or cooler in the form of a parallelogram to be 100 inches in length, and 40 in breadth; required the area in imperial gallons.

100 × 40 = 4000 the area in inches, which divided by 277·274 the quotient 14·426 = the number of imperial gallons; or if we multiply 4000 by ·0036065, the product 14·426 is the number of imperial gallons as before.

BY THE SLIDING RULE.

	On A		On B		On A		On B
As	277·274	:	40	::	100	:	14·4, nearly.

2. If the side of a square be 40 inches, what is the area in imperial gallons? *Ans.* 5·77 gallons.

3. If the side of a rhombus be 40 inches, and its perpendicular breadth 37 inches; required its area in wine gallons. *Ans.* 5·41.

4. What is the area of a square cooler, in imperial gallons, the side being 144 inches? *Ans.* 74·785.

5. Allowing the side of a hexagon to be 64 inches, and the perpendicular from the centre to the middle of one of

* The areas of plane figures, in gauging, are expressed in gallons, or bushels For there will be as many solid inches in any vessel of one inch deep, as there are superficial inches in its base. What is called in gauging a surface or area is in reality a surface of one inch deep, which, multiplied by the height, will give the whole content in gallons or bushels.

the sides 55·42 inches; required its area in imperial gallons and malt bushels?

Ans. { 38·38 imperial gallons. 4·8 malt bushels.

PROBLEM III.

The diameter of a circular vessel being given in inches, to find its area in imperial gallons.

Rule. Multiply the square of the diameter by ·002832; or divide the square of the diameter by 353·036, the product or quotient will give the area in imperial gallons.

When it is required to find the area in any other denomination than imperial gallons, use the proper multiplier or divisor for the required denomination, as given in the table, page 206.

1. The diameter of a circular vessel is 32·6 inches; required the area in imperial gallons?

$(32·6)^2 = 1062·76$. Then,

$1062·76 \times ·002832 = 3·01$ gallons.

Or, $1062·76 \div 353·036 = 3·01$.

BY THE SLIDING RULE.

As 18·78 is the circular gauge-point for imperial gallons, say

On D		On B		On D		On B
As 18·78	:	1	::	32·6	:	3

2. If the diameter of a circular vessel be 10 inches, what is the area in imperial gallons? *Ans.* ·283.

3. Suppose the diameter of a circular vessel is 30 inches, what is its area in imperial gallons? *Ans.* 2·548.

4. What is the area in imperial gallons of a round vessel, whose diameter is 24 inches? *Ans.* 1·631.

PROBLEM IV.

Given the transverse and conjugate diameter of an elliptical vessel, to find its area in imperal measure.

Rule. Multiply the product of the two diameters by ·002832; or divide the product of the two diameters by 353·036; the product or quotient will give the imperial gallons required

When any other denomination is required, the proper multiplier or divisor in the table is to be employed.

1. Suppose the longer diameter of an elliptical vessel is 10, and the shorter diameter 6, required the area in ale and wine gallons.

Here, $10 \times 6 = 60$.
Then, $60 \times \cdot002832 = \cdot17$ of a gallon.

2. The transverse or longer diameter of an elliptical vessel is 20, and the conjugate or shorter diameter 10 inches; what is the area in imperial measure? *Ans.* ·566 of a gallon.

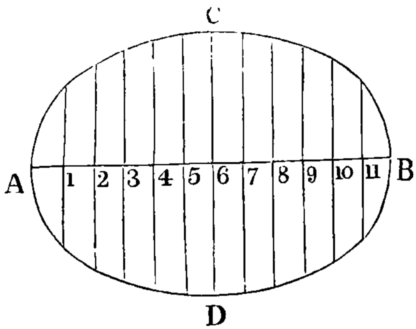
On A On B On A On B
As 353 : 20 :: 10 : ·566 of a gallon.

3. Suppose the transverse diameter of an elliptical vessel is 70 inches, conjugate 50 inches; required its area in imperial gallons and malt bushels?

Ans. { 9·914 gallons.
1·24 malt bushels.

Note. As vessels are seldom or never made truly elliptical, being generally ovals, the area found by the above rule is not correct, except the vessel be a truly mathematical ellipsis; when the vessel is of an oval form, the area is best found by the method of equi-distant ordinates.

Let A B C D be the oval vessel whose area is required, and let A B and C D be the transverse and conjugate diameters, at right angles to each other, the former being 102·2



inches. Divide this transverse (102·8) by some even number which will leave a small remainder, the quotient will be the distance of the ordinates; which distance may be laid off on both sides of the conjugate diameter a number of times equal to half the even number by which the transverse was divided, then with chalk and a parallel ruler, draw the ordinates through the points 1, 2, 3, 4, &c. Then, by Problem XXI., Sec. III., the area may be found, which being multiplied or divided by the proper tabular numbers, will give the area in gallons, &c. Or,

1st. Add together the first and last ordinates.

2nd. Add together the even ordinates, that is, the 2, 4, 6, 8, 10, &c., and multiply the sum by 4.

3rd. Add together the odd ordinates, except the first and last; that is, add the ordinates 3, 5, 7, 9, &c., and multiply the sum by 2.

4th. Multiply the sum of the extreme ordinates by their distance from the curve.

5th. Add the three first found sums together, and multiply the sum by the common distance of the ordinates, and to the product add the fourth found sum, and divide the total by 3, and the quotient resulting by 277·274, or 2218·192, for the area in imperial gallons, or malt bushels, respectively.

First, $102 \cdot 8 \div 10 = 10$ the distance of the ordinates asunder, and the remainder 2·8 is double the distance of the extreme ordinates from the curve; that is, $1 \cdot 4 = $ A 1, or B 11.

Now let us suppose the lengths of the ordinates to be 20, 40·2, 57, 66·6, 73, 75, 73, 66·6, 57, 40·2, 20, respectively beginning at 1, and proceeding to 11.

$$\text{1st.}\ \begin{cases} 1 = 20 \\ 11 = 20 \end{cases}$$

40 inches, sum of the first and last.
1·4

56

$$\text{2nd.}\ \begin{cases} 2 = 40 \cdot 2 \\ 4 = 66 \cdot 6 \\ 6 = 75 \cdot 0 \\ 8 = 66 \cdot 6 \\ 10 = 40 \cdot 2 \end{cases}$$

$$288 \cdot 6 \times 4 = 1154 \cdot 4$$

$$\text{3rd.}\ \begin{cases} 3 = 57 \\ 5 = 73 \\ 7 = 73 \\ 9 = 57 \end{cases}$$

$$260 \times 2 = 520$$

Then, $40 + 1154 \cdot 4 + 520 = 1714 \cdot 4$ sum of first three sums.
10

17144
56

3)17200

5733·3 ; then,

$5733 \cdot 3 \div 277 \cdot 274 = 20 \cdot 64$ gallons.

$5733 \cdot 3 \div 2218 \cdot 192 = 2 \cdot 58$ malt bushels.

When the vessel is not circular, or elliptical, it is best to measure the equi-distant ordinates, which, though ever so unequal, will, by proceeding as above, serve to find the area of the base. Whenever the vessel is an irregular curved figure, the area should be invariably found by the method of equi-distant ordinates, as the true result cannot be found by any other method.

4. What is the area, in imperial measure, of an ellipse, whose transverse axis is 24, and conjugate 18?

Ans. 1·2234 gallons.

PROBLEM V.

To find the content of a prism, in imperial gallons.

Rule. Find the area of the base, by Problem II., in gauging, which, being multiplied by the depth within, will give the content in gallons.

Or, find the solid content by mensuration, and divide that content by 277·274 for imperial gallons.

A vessel, whose base is a right-angled parallelogram, is 49·3 inches in length, the breadth 36·5 inches, and the depth 42·6 inches; required its content in imperial gallons?

Here, $49·3 \times 36·5 \times 42·6 = 76656·57$.
Then, $76656·57 \div 277·274 = 276·465$ gallons.
And $76656·57 \div 2218·192 = 34·558$ malt bushels.

BY THE SLIDING RULE.

On B		On D		On B		
49·3	:	49·3	::	36·5	:	42·42.

On D		On B		On D	
16·65 } 46·37 }	:	42·6	::	42·42	{ 27·6 gallons. { 34·5 malt bushels.

2. Each side of the square base of a vessel is 20 inches, and its depth 10 inches, what is the content in old ale gallons? *Ans.* 14.28 gallons.

3. The side of a vessel in the form of a rhombus is 20 inches, breadth 15 inches, and depth 10 inches; required the content in old ale gallons? *Ans.* 10·638 gallons.

4. What is the content in old wine gallons of a vessel in the form of a rhomboid, whose longest side is 20 inches, breadth from side to side 8 inches, and depth 10 inches? *Ans.* 6·88 wine gallons.

PROBLEM VI.

To find the content of any vessel, whose ends are squares or rectangles of any dimensions.

Rule. Multiply the sum of the lengths of the two ends, by the sum of their breadths, to which add the areas of the two ends; this sum, multiplied by one-sixth of the depth, will give the solidity in cubic inches; then divide by 277·274, or 2218·192 for the content in imperial gallons, or malt bushels.

1. Suppose the top and bottom of a vessel are parallelograms, the length of the top is 40 inches, and its breadth 30 inches; the length of the bottom is 30 inches, and its breadth 20; and the depth 60 inches; required the contents in imperial gallons?

$40 + 30 = 70$ sum of the lengths.
$30 + 20 = 50$ sum of the breadhts.

3500 product.
$40 \times 30 = 1200$ area of the greater base.
$30 \times 20 = 600$ area of the lesser base.

5300
10 one-sixth of the depth.

53000 solidity in cubic inches.
Then $53000 \div 277{\cdot}274 = 191{\cdot}146$.

BY THE SLIDING RULE.

Find a mean proportional ($\sqrt{(40 \times 30)} = 34 \cdot 64$), between the length and breadth at the top, and a mean proportional ($\sqrt{(30 \times 20)} = 24.49$), between the length and breath at the bottom; the sum of these is 59·13, twice a mean proportional between the length and breadth in the middle. Then,

On D	On B		On D		On B	
			34·64 :	———		
16·65 :	$\frac{60}{6}$	::	24·49 :	———		sum 191·146 imperial gallons.
			59·13 :	———		

2. Suppose the top and bottom of a vessel are parallelograms, the length of the top is 100 inches, and its breadth 70 inches; the length of the bottom 80, and its breadth 56, and the depth 42 inches; what is its content in imperial gallons? *Ans.* 862·59 imperial gallons.

THE GAUGING OR DIAGONAL ROD.

The diagonal rod is a square rule, having four faces, and is generally 4 feet long. It folds together by joints. This instrument is employed both for gauging and measuring casks, and computing their contents; and that from one dimension only, namely, the diagonal of the cask, or the length from the middle of the bung-hole to the meeting of the cask with the stave opposite the bung; being the longest line that can be drawn from the middle of the bung-hole to any part within the cask.

On one face of the rule is a scale of inches for measuring this diagonal; to which are placed the areas, in ale gallons, of circles to the corresponding diameters, in like manner as the lines on the under sides of the three slides in the sliding rule.

On the opposite face, there are two scales of ale and wine gallons, expressing the contents of casks having the corresponding diagonals.

All the other lines on the instrument are similar to those on the sliding rule, and are used in the same manner.

Example. The diagonal, or distance between the middle of the bung-hole to the most distant part of the cask, as found by the diagonal rod, is 34·4 inches: what is the content in gallons?

To 34·4 inches correspond, on the rod, $90\frac{3}{4}$ ale gallons, or 111 wine gallons, $92\frac{1}{2}$ imperial gallons, the content required?

NOTE. The contents shown by the rod answer to the most common form of casks, and fall in between the 2nd and 3rd varieties following.

OF CASKS, AS DIVIDED INTO VARIETIES.

Casks are usually divided into four varieties, which are easily distinguished by the curvature of their sides.

1. The middle frustum of a spheroid belongs to the first variety.

2. The middle frustum of a parabolic spindle belongs to the second variety.

3. The two equal frustums of a paraboloid belong to the third variety.

4. And the two equal frustums of a cone belong to the fourth variety.

If the content of any of these be found in inches by their proper rules, and this divided by 277·274, or 2218·2, the quotient will be the content in imperial gallons, or bushels, respectively.

PROBLEM VII.

To find the content of a vessel, in the form of the frustum of a cone.

RULE. To three times the product of the two diameters add the square of their difference: multiply the sum by one-third of the depth, and divide the product by 353·0362 for imperial gallons, and by 2824·289 for malt bushels.

1. What is the content of a cone's frustum, whose greater diameter is 20 inches, least diameter 15 inches, and depth 21 inches?

$$20 \times 15 \times 3 = 900$$
$$20 - 15 = 5 \ \& \ 5^2 = 25$$
$$925 \times 7 = 6475. \text{ Then,}$$
353·0362)6475(18·34 imperial gallons.
294·12)6475(22·01 wine gallons.

2. The greater diameter of a conical frustum is 38 inches, the less diameter 20·2, and depth 21 inches; what is the content in old ale gallons? *Ans.* 51·07 gallons.

PROBLEM VIII.

To find the content of the frustum of a square pyramid.

RULE. To three times the product of the top and bottom sides, add the square of their difference, multiply their sum by one-third of the depth, and divide the product by 282 and 231, for old ale and wine gallons, respectively; and by 277·274, for imperial gallons.

1. Suppose the greater base is 20 inches, the less base 15 inches, and depth 21 inches; required the content in old wine measure?

$$20 \times 15 \times 3 = 900$$
$$20 - 15 = 5$$
$$\text{Then, } 5 \times 5 = 25$$
$$925 \times 7 \div 231 = 27·8 \text{ gallons.}$$

NOTE. The content of the frustum of a pyramid is found just like that of a cone, with the exception of the tabular divisor, or multiplier, the cone requiring the circular factor, and the pyramid the square one.

PROBLEM IX.

To find the content of a globe.

Rule. Multiply the diameter of the globe by its circumference, and the resulting product by one-sixth of the diameter; then the last product multipled or divided by the circular factor, will give the content in gallons.

1. Let the diameter be 34 inches, what is its content?
   $34 \times 34 \times 34 \times \cdot 5236 = 20579 \cdot 5744$.
   Then, $20579 \cdot 5744 \div 282 = 72 \cdot 9772$ old ale gallons.
   And, $20579 \cdot 5744 \div 231 = 89 \cdot 08$ old wine gallons.

Rule II. Or cube the diameter of the globe, which multiply by ·001888 ($\frac{2}{3}$ of ·002832) for the content in imperial gallons.
$34^3 = 39304$; then $39304 \times \cdot 001888 = 74 \cdot 2$ imperial gallons.

2. What is the content of a globe in old ale and wine measure, the diameter being 20 inches?
   Ans. { 14·848 old ale gallons. 18·128 old wine gallons.

3. Required the content of a globular vessel, whose diameter is 100 inches?
   Ans. $1888\frac{1}{3}$ imperial gallons.

PROBLEM X.

To find the content of the segment of a sphere, as the rising crown of a copper still, &c.

Rule. Measure the diameter, or chord of the segment, and the altitude just in the middle. Multiply the square of half the diameter by 3; to the product add the square of the altitude; multiply this sum by the altitude, and the product again by ·001856, or ·002266, for old ale or wine measure, respectively, and by ·001888 for imperial gallons.

1. The diameter of the crown of a copper still is 27·6, its depth 9·2; required its content?

Here, 27·6 ÷ 2 = 13·8.

Then, 13·8 × 13·8 × 3 = 571·32

9·2 × 9·2 = 84·64

655·96 sum.

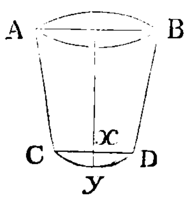
9·2 depth.

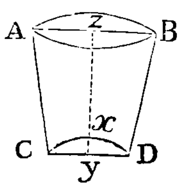
6034·832 × ·001888 = 13·39 imperial gallons.

PROBLEM XI.

*To gauge a copper having either a concave or convex **bottom**; or what is called a falling bottom, or rising crown.*

Rule. If the side of the vessel be straight with a falling bottom, find the content of the segment C y D, by Prob. X.; find also the content of the upper part A B D C, by Prob. VII.; the sum of both will give the content of the copper.





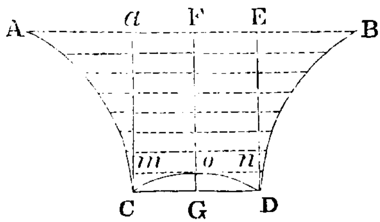
When the copper has a rising crown, find the content of A B C D, by Prob. VII., from which deduct the content of the segment C x D, and the remainder will be the content of the vessel A B D x C.

PROBLEM XII.

To gauge a vessel whose side is curved from top to bottom.

Take the diameters at equal distances of 2, 3, 4, or 5 inches, according as the case may require; if the side of the vessel be considerably curved, the number of diameters that will be required will be considerable; the less the curvature of the side, the less the number of diameters that will be required.

To gauge the vessel, or copper, A B D C, fasten a piece of pack-thread to A and B, as A F B; then with some con-



venient instrument find the distance *a* C of the deepest part of the copper, which let us suppose to be 47 inches.

By means of the same instrument measure the distance *o* F from the top of the crown F the middle of A B; which let us suppose to be 42 inches, this deducted from *a* C, 47, will leave 5 (= *o* G) the height of the crown.

To find the diameter C D, *of the bottom of the crown.*

Measure the top diameter A B, which suppose to be 99 inches: then hold a thread, so that a plummet attached to the end thereof, may hang just over C, and measure A *a* = B E, each of which let us admit to be 17·5 inches; add these together, and deduct their sum (35) from 99, and the remainder (64) will evidently be equal to C D, the diameter at the bottom of the crown. Measure the diameter *m o n*, which touches the top of the crown, which suppose is 65 inches.

Now, as this copper is not considerably curved, the diameters may be taken in the middle of every 6 inches of the depth, which suppose to be as in the second column of the following table; to each diameter find the area in imperial gallons, by Prob. III., which write in the third column; find also the content of every 6 inches, corresponding to these diameters, which write in the fourth column of the table; lastly find the content of the crown by Prob. X., and subtract it from the content of A B D G C, the remainder will give the capacity of the copper.

Or thus, C D being 64 inches, the area answering to it is 11·6022, this multiplied by half the altitude of the crown, viz., by 2·5, gives 29·0055 gallons, the content of the crown. The content of the part *m n* D C is 58·9222 gallons, from which the content of the crown being deducted, the remainder (29·9167 gallons) is the quantity of liquor which covers the crown.

Parts of the depth.	Diameters.	Areas.	Content of every 6 inches.
6	95·3	25·7257	154·3542
6	90·1	22·9948	137·9688
6	85·	20·4653	122·7918
6	80·	18·1284	108·7704
6	75·2	16·0183	96·1098
6	70·5	14·0786	84·4716
6	66·	12·3387	74·0322
	The sum . .		778·4988
	To cover crown .		29·9167
	The whole content .		808·4155

PROBLEM XIII.

To find the content of any close cask.

Whatever be the form of the cask, the following dimensions must be taken; that is,

The bung diameter,
The head diameter, } within.
The length of the cask,

On account of the difficulty in ascertaining the figure of the cask, it is not, in many cases, easy to find the exact contents of casks.

In taking the dimensions of a cask, it is essential that the bung-hole be in the middle of the cask, and also that the bung-stave, and the stave opposite to it, are both regular and even within.

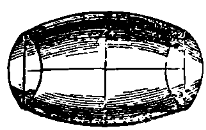
It is likewise essential that the heads of casks are equal and truly circular; and if so, the distance between the inside of the chimb to the outside of the opposite stave, will be the head diameter within the cask, nearly.

From the variety in the forms of casks, no general rule could be given to answer every form; two casks may have equal head diameters, equal bung diameters, and equal lengths, and yet their contents may be very unequal.

PROBLEM XIV.

To find the content of a cask of the first variety.

Rule. To the square of the head diameter add double the square of the bung diameter, and multiply the sum by the length of the cask. Then multiply the last product by $\cdot 0009\frac{4}{9}$, or divide by 1059·1, the product or quotient will be the content in imperial gallons.



1. What is the content of a spheroidal cask, whose length is 40 inches, bung diameter 32 inches, and head diameter 24 inches?

$$
\begin{array}{r}
24 \times 24 = 576 \\
32 \times 32 = 1024 \\
2 \\
\hline
2048 \\
576 \\
\hline
2624 \times 40 = 104960 \\
\cdot 0009\frac{4}{9} \\
\hline
944640 \\
34987 \\
11662 \\
\hline
99 \cdot 1289 \text{ imperial gallons.}
\end{array}
$$

BY THE GAUGING RULE.

Set 40 on C, to the G R 18·79 on D, against
24 on D, stands 64·99 on C,
32 on D, stands 116·2 on C.
+ 116·2

3)297·39

99·13 gallons.

2. What is the content of a spheroidal cask, whose length is 20 inches, bung diameter 16 inches, and head diameter 12 inches?

Ans. { 12·36 old ale gallons.
14·869 old wine gallons. }

To find the content of a cask by the mean diameter.

RULE. Multiply the difference of the head and bung diameters by ·68 for the first variety; by ·62 for the second variety; by ·55 for the third; and by ·5 for the fourth, when the difference between the head and bung diameters is less than 6 inches; but when the difference between these exceeds 6 inches, multiply that difference by ·7 for the first variety;

by ·64 for the second; by ·57 for the third; and by ·52 for the fourth. Add this product to the head diameter, and the sum will be a mean diameter. Square this mean diameter, and multiply the square by the length of the cask; this product multiplied, or divided, by the proper multiplier or divisor, will give the content.

By resuming the last example but one, we have

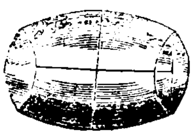
Bung diameter	32	29·6	mean diameter.
Head diameter	24	29·6	
	8	876·16	square.
	·7	40	length.
	5·6	359·5)35046·40	
	24		
		97·6	gallons.
mean diameter	29·6		

In the same manner the content for the second variety will be 94·46 ale gallons; for the third variety 90·87 ale gallons; and for the fourth variety 83·34 gallons.

PROBLEM XV.

To find the content of a cask of the second variety.

RULE. To the square of the head diameter add double the square of the bung diameter, and from the sum deduct two-fifths of the square of the difference of the diameters; multiply the remainder by the length, and the product again by ·0009$\frac{4}{9}$ for the content in imperial gallons.



1. What is the content of a cask, whose length is 40 inches, bung diameter 32 inches, and head diameter 24 inches?

$32 - 24 = 8$; then $8^2 = 64$, and $\frac{2}{5}$ of $64 = 25{\cdot}6$
$24^2 = 576$, and $32^2 = 1024$, then $1024 \times 2 = 2048$
$2048 + 576 = 2624$, and $2624 - 25{\cdot}6 = 2598{\cdot}4$

$$\begin{array}{r} 2598{\cdot}4 \\ 40 \\ \hline 103936 \end{array}$$

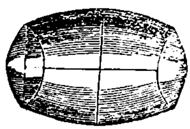
$103936 \times {\cdot}0009\frac{4}{9} = 98{\cdot}1617$ gallons.

PROBLEM XVI.

To find the content of a cask of the third variety.

Rule. To the square of the bung diameter add the square of the head diameter; multiply the sum by the length, and the last product by ·001416 for the answer in imperial gallons.

Let us resume the last example: thus



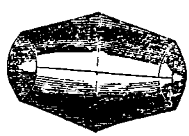
$$\begin{array}{r} 32^2 = 1024 \\ 24^2 = 576 \\ \hline 1600 \times 40 = 64000 \\ {\cdot}001416 \\ \hline 90{\cdot}624 \end{array}$$

90·624 imperial gallons.

PROBLEM XVII.

To find the content of a cask of the fourth variety.

Rule. Add the square of the difference of the diameters to 3 times the square of their sum; multiply the sum by the length, and the last product by ·000236 for the content in gallons.



Resuming still the last example $32 + 24 = 56$, and $56^2 \times 3 = 3136 \times 3 = 9408$, and $8^2 = 64$, then $9408 + 64 = 9472$; then $9472 \times 40 = 378880$, and $378880 \times {\cdot}000236 = 89{\cdot}41668$ imperial gallons.

PROBLEM XVIII.

To find the content of any cask by Doctor Hutton's general rule.

RULE. Add into one sum, 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of the two diameters; then multiply the sum by the length, and the product again by ·00031$\frac{4}{9}$ for the content in gallons.

1. What is the content of a cask, whose length is 40 inches, and the bung and head diameters 32 and 24 ?

$32^2 = 1024$	$24^2 = 576$	$32 \times 24 = 768$
39	25	26
39936	14400	19968

39936
14400
19968

$74304 \times 40 = 2972160$
·00031$\frac{4}{9}$

93·4579 gallons.

ULLAGING.

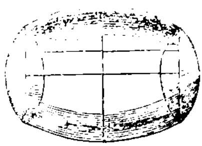
PROBLEM XIX.

To ullage a lying cask.

This is the finding what quantity of liquor is contained in a cask when partly empty.

To ullage a lying cask, the wet and dry inches must be known, as also the content of the cask and bung diameter.

RULE. Take the wet inches, and divide them by the bung diameter; find the quotient in the column of versed sines, in the table at the end of the pratical part of this book, and take out its corresponding segment; multiply this segment by the whole content of the cask, and the product arising by $1\frac{1}{4}$ for the ullage required, nearly.



1. Find the ullage for 8 wet inches, the bung diameter being 32 inches, and the content 92 ale gallons?

32)8(·25, whose tabular segment is ·153546.

Then, $·153546 \times 92 = 14·126232$.

And $14·126232 \times 1\frac{1}{4} = 17·65779$ gallons.

PROBLEM XX.

To ullage a standing cask.

RULE. Add together the square of the diameter at the surface of the liquor, the square of the diameter of the nearest end, and the square of double the diameter taken in the middle between the other two; multiply the sum by the length between the surface and nearest end, and the product arising by ·000472 for the gallons in the less part of the cask whether empty or filled.

1. What is the ullage for 10 wet inches, the three diameters being 24, 27, and 29 inches?

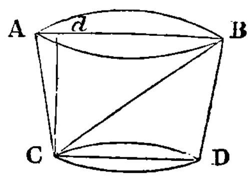
$24^2 = 576$	43330
$29^2 = 841$	·000472
$(2 \times 27)^2 = 2916$	86660
4333	303310
10	173320
43330	20·45176 gallons.

PROBLEM XXI.

To find the content of an ungula, or hoof, of the frustum of a cone.

RULE. For the less hoof, multiply the product of the less diameter and height, by the product of the greater diameter multiplied by a mean proportional between both diameters, less the square of the less diameter, and this last divided by three times the circular factor multiplied by the difference of the diameters, gives the content of the less hoof.

1. C D = 30, A B = 40, C d = 20, required the content of the less hoof.



40 × 30 = 1200, and $\sqrt{1200}$ = 34·6 mean.
30 × 20 = 600, 1st product.
40 × 34·6 = 1384, 2nd product.
30 × 30 = 900

484 remainder.
484 × 600 = 290400
40 — 30 = 10, then 359 × 3 × 10 = 10770, and
290400 ÷ 10770 = 26·96 gallons.

RULE. For the greater hoof multiply the product of the greater diameter and the height of the frustum, by the square of the greater diameter made less by the product of the less diameter multipled by a mean proportional between those diameters: this remainder, divided by three times the circular divisor multiplied by the difference of the diameters, gives the content of the greater hoof.

Resuming the last example we have
40 × 40 = 1600
20 × 40 = 800, 1st product.
40 × 30 = 1200, and $\sqrt{1200}$ = 34·6
34·6 × 30 = 1038, 2nd product.
40 — 30 = 10.

Then 1600 — 1038 = 562
800

359 × 3 × 10 = 10770)449600, last product.

41·74 old ale gallons

PROBLEM XXII.

To guage a still.

Fill the still with water, and draw it off in another vessel of some regular form, whose content is easily computed. This is by far the most accurate method that can be employed.

Or gauge the shoulder by itself, and gauge the body by taking a greater number of diameters at near and equal distances throughout, first covering the bottom, if there be any cavity, with water, the quantity of which is known.

SECTION XI.

LAND SURVEYING.

Land surveying is that art which enables us to give a true plan or representation of any field or parcel of land, and to determine the superficial content thereof.

In measuring land, the area or superficial content is always expressed in acres, or in acres, roods, and perches; each acre containing 4 roods, and each rood 40 perches.

Land is measured with a chain, called Gunter's chain, of 4 poles, or 22 yards in length, which consists of 100 equal links, each link being $\frac{22}{100}$ of a yard long, or $\frac{66}{100}$ of a foot, or 7·92 inches. 10 square chains, or 10 chains in length and 1 in breadth, make an acre; or 4840 square yards, 160 square poles, or 100,000 square links make an acre. The length of lines measured with a chain, are generally set down in links as integers; every chain being 100 links in length. Therefore, after the content is found, it will be in square links, and as 100,000 square links make an acre, it will be necessary to cut off five of the figures on the right hand for decimals, and the rest will be acres. The decimals are reduced to roods by multiplying by 4, and cutting off five figures as before for decimals, which decimal part is reduced to perches by multiplying by 40, and cutting off five figures from the product. As an example:

Suppose the length of a rectangular piece of ground to be 792 links, and its breadth 385; required the number of acres, roods, and perches it contains?

792	3·04920
385	4
3960	·19680
6336	40
2376	7·87200
304920	

A. R. P.
Ans. 3. 0. 7.

The statute perch is $5\frac{1}{2}$ yards, but the Irish plantation perch is 7 yards; hence the length of a plantation link is 10·08 inches.

PROBLEM I.

To measure a line or distance on the ground, two persons are employed; the foremost, for the sake of distinction, is called the leader, the hindermost, the follower.

Ten small arrows or rods, to stick in the ground at the end of each chain, are provided; also some station-staves, or long poles with coloured flags, to set up in the direction of the line to be measured, if there do not appear some marks naturally in that direction.

The leader takes the 10 arrows in one hand, and one end of the chain by the ring, in the other; the follower stands at the beginning of the line, holding the ring at the end of the chain in his hand, while the leader drags forward the chain by the other end of it, till it is stretched straight and the leader, directed by the follower, by moving his hand to the right or left, till the follower see him exactly in a line with the mark or direction to be measured to; then both of them holding the chain level and stretched, the leader sticks an arrow upright in the ground, as a mark for the follower to come to, and advances another chain forward, being directed in his position by the follower standing at the arrow, as before, as also by himself, now and at every succeeding chain's length, by moving himself from side to side, till the follower and back-mark be in a direct line.

Having then stretched the chain, and stuck down an arrow, as before, the follower takes up the arrow, and thus they proceed till the 10 arrows are employed, or in the hands of the follower, and the leader, without an arrow, is arrived at the end of the eleventh chain length. The follower then sends or brings the 10 arrows to the leader, who puts one of them down at the end of his chain, and advances with his chain, as before. And thus the arrows are changed from one to the other at every 10 chains' length, till the whole line is finished, if it exceed 10 chains; and the number of changes shows how many times 10 chains the line contains, to which the follower adds the arrows he holds in his hand, and the number of links of another chain over to the mark or end of the line. Thus, if the whole line measure 36 chains 45 links, or 3645 links, the arrows have been changed three times, the follower will have 5 arrows in his hand, the leader 4, and it will be 45 links from the last arrow, to be taken up by the follower, to the end of the line.

In works on Surveying, it is usual to describe the various instruments used in the art. The pupil, however, will best learn the use of these instruments when actually engaged in the practice. The chief instruments employed are the chain, the plain table, the theodolite, the cross, the circumferentor, the offset staff, the perambulator, used in measuring roads, and other great distances.

Levels, with telescopic or other sights, are used to find the levels between two or more places, or how much one place is higher or lower than the other.

Besides all these, various scales are used in protracting and measuring on paper; such as plane scales, line of chords, protractor, compasses, reducing scales, parallel and perpendicular rulers, &c.

THE FIELD-BOOK.

In surveying with the plain table, a field-book is not required, as everything is drawn on the table immediately when it is measured. But when the theodolite, or any other instrument is used, some sort of a field-book is used in order to register all that is done relative to the survey in hand. This book every one contrives and rules as he thinks fit. It is, however, usually divided into three columns. The middle column contains the different distances on the chain-line, angles, bearings, &c., and the columns on the right and left are for the offsets on the right and left, which are set against their corresponding distances in the middle column; as also for such remarks as may occur, and may be proper to note in drawing the plan; such as houses, ponds, castles, churches, rivers, trees, &c., &c.

But in smaller surveys, an excellent way of setting down the work is, to draw by the eye, on a piece of paper, a figure resembling that which is to be measured; and then write the dimensions, as they are found, against the corresponding parts of the figure. This method may be practised even in larger surveys, and is far superior to any other at present practised. A specimen of this plan will be seen further on.

FORM OF THE FIELD-BOOK.

Offsets and remarks on the left.	Stations. Bearings, and Distances.	Offsets and remarks on the right.
	□ 1	
	104° 25′	
	00	
Cross a hedge 24	67	Brown's barn.
a brook 30	120	
	734	
	954	Tree.
	736	67 Stile.
	82	
	62° 25′	
	00	
House corner 61	40	
	67	
Footh-path 15	84	44
	95	
	467	14 Spring.
	976	
	□ 3	
	54° 17	
	62	20 Pond.
	124	
Clayton's hedge 24	630	
	767	
	767	30 Stile.
	305	
	760	

In this form of a field-book □ 1 is the first station, where the angle or bearing is 104° 25′. On the left, at 67 links in the distance or principal line, is an offset of 24; and at 120 an offset of 30 to a brook on the right: at 67 Brown's barn is situated; at 954 is an offset of 20 to a tree, and at 736 an offset to a stile.

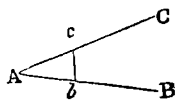
And so on for the other stations.

A line is drawn under the work, at the end of every station, to prevent confusion.

PROBLEM II.

To make angles and bearings.

Let it be required to take the bearings of the two objects B, C, from the station A.



In this problem it is required to measure the angle at A, formed by two lines, passing from the station A, through two objects B and C.

1. *By measurement with the chain, &c.*

Measure with the chain any distance along the two lines A B, A C, as A *b*, A *c*; then measure the distance *b c*; and this being done, transfer the three sides of the triangle A *b c* to paper, on which measure the angle *c* A *b*, as in Problem XV., Practical Geometry.

2. *With the magnetic needle and compass.*

Turn the instrument, or compass, so that the north end of the needle may point to the flower-de-luce. Then direct the sights to a mark at B, noting the degress cut by the needle. Next direct the sights to another mark at C, noting the degrees cut by the needle as before. Then their sum or difference, as the case may be, will give the number of degrees in the angle C A B.

3. *With the theodolite, &c.*

Direct the fixed sights along the line A B, by turning the instrument about till you see the mark B through these sights, and in that position screw the instrument fast. Then turn the moveable index about till, through its sights, you see the other mark C. Then the degrees cut by the index, on the graduated limb or ring of the instrument, show the number of degrees in the angle C A B.

4. *With the plain table.*

Having covered the table with paper, and fixed it on its stand, plant it at the station A, and fix a fine pin, or a point of the compass in a proper point of the paper, to represent the station A. Close by the side of this pin, lay the fiducial edge of the index, and turn it about, still touching the pin, till one object B can be seen through the sights; then by the fiducial edge of the index draw a line. By a similar process draw another line in the direction of the object C. And it is done.

PROBLEM III.

To measure the offsets.

Let A *b c d e f g* be a crooked hedge, river, or brook, &c., and A G a base line.

Begin at the point A, and measure towards G; and when you come opposite to any of the corners *b c d*, &c., which is

ascertained by means of the cross-staff, measure the offsets B *b*, C *c*, D *d*, &c., with the chain, and register the dimension, as in the annexed field-book.

FIELD BOOK.

91	785 = A G.	——
57	634	——
98	510	——
70	340	——
84	220	——
62	45	——
—	☐ A go North.	——
Offsets Left.	Base line A G, or ☐ Station.	Offsets Right.

To lay down the plan.

Draw the line A G of an indefinite length; then by a diagonal scale, set off A B equal to 45 links; at B erect the perpendicular B *b* equal to 62 links taken from the same scale. Next set off A C equal to 220 links, or 2 chains 20 links, and at C erect the perpendicular C *c*, equal to 84 links; in the same way set off A C equal to 340 links, or 3 chains. 40 links, and at D erect the perpendicular D *d* equal to 70 links. Proceed in a similar manner with the remaining offsets, and straight lines joining the points A *b c d*, *e*, &c., will complete the figure.

To find the content.

Some authors direct to add up all the perpendiculars B *b*, C *c*, &c., and divide their sum by the number of them, then multiply the quotient by the length A G. This method, however, should never be used, except when the offsets B *b*, C *c*, &c., are equally distant from each other.

When the offsets are not equally distant from each other, which indeed is generally the case, this method is erroneous; therefore the following method ought to be employed.

Find the content of the space A B *b* as a triangle, by Problem V., Section II. Find the contents of the figures B C *c b*, C D *d c*, &c., as trapezoids, by Problem XIII., Section II., the sum of all these separate results will be the content of the figure A G *g f e d c b* A.

The actual calculation is as follows:

CALCULATION.

AB = 45	AC = 220	AD = 340	AE = 510	AF = 634	AG = 784
B*b* = 62	AB = 45	AC = 220	AD = 340	AE = 510	AF = 634
90	BC = 175	CD = 120	DE = 170	EF = 124	GF = 151
270					
	B*b* = 62	C*c* = 84	D*d* = 70	E*e* = 98	F*f* = 57
2790	C*c* = 84	D*d* = 70	E*e* = 98	E*f* = 57	G*g* = 91
	Sum 146	Sum 154	Sum 168	Sum 155	Sum 148
	BC = 175	CD = 120	DE = 170	EF = 124	FG = 151
	Prod. 25550	18480	28560	19220	22348

These respective products are evidently double the true contents of the respective figures A B *b*, B C *c b*, C D *d c*. &c., that is,

2790 = double area of A B *b*.
25550 = double area of B C *c b*.
18480 = double area of C D *d c*.
28560 = double area of D E *e d*.
19220 = double area of E F *f e*.
22348 = double area of E G *g f*.

2)116948 = double area of the whole in square links.

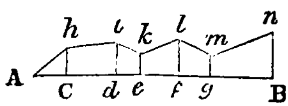
58474 = area in square links.

·58474 = area in acres = 0A., 2R., 13·5584P.

2. Required the plan and content of part of a field, from the following field-book :

AC	45	62	C	*h*
A *d*	220	84	*d*	*i*
A *e*	340	70	*e*	*k*
A *f*	510	88	*f*	*l*
A *g*	634	57	*g*	*m*
AB	785	91	B	*n*

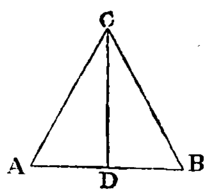
Ans. 0A., 2R., 12P.



PROBLEM IV.

To measure a field of a triangular form.

1. *By the chain.*



Set up marks at the three corners A, B, C, and measure with the chain, the distance A D, D being the point at which a perpendicular demitted from C, would meet the line A B; measure also the distance D B; hence you have the measure of A B. Next measure the perpendicular D C; then from the two dimensions A B and D C, the content may be found by Problem IV., Section II.

Let A D = 794, A B = 1321, D C = 826 links.
1321 × 826 ÷ 2 = 545573 links.
Then, 545573 ÷ 100000 = 5·45573 acres.
45573 × 4 = 1·82292 roods.
82292 × 40 = 32·91680 perches.
Hence the answer 5A., 1R., 33P., nearly.

2. What is the area of a trianglar field, whose base is 12·25 chains, and height 8·5 chains. *Ans.* 5A., 0R., 33P.

2. *By taking one or more of the angles.*

Measure two sides A B, A C, and the angle A, included between them; then half the continual product of the two sides, and the natural sine of the contained angle will give the area.*

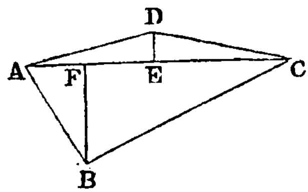
Or, measure the two angles A and B, and the adjacent side A B, from which the figure may be planned, and the perpendicular C D found, which perpendicular being multiplied by half the base A B, will give the area. Or by measuring the three sides of the triangle, its area may be found by Problem V., Section II.

PROBLEM V.

1. *By the chain.*

To survey a four-sided field.

Measure the diagonal A C, and, as before directed, measure the perpendiculars D E and B F; then the area of each



* See Appendix. Demonstration 11.

of the triangles A B C, A D C may be found, as in the last problem, and both areas being added together, will give the content of the four-sided figure A B C D.

1. Let A C = 592, D E = 210, B F = 306 links.

592 × 210 = 124320 double area of A B C.
592 × 306 = 181152 double area of A B C.

2)305472 double area of A B C D.

1·52736 = area of A B C D.
4

2·10944
40

4·37760

Hence 1A., 2R., 4 P., the answer.

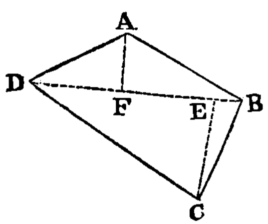
2. *By taking one or more of the angles.*

Measure the diagonal A C, also the sides A D and A B. Next measure the angles D A C and B A C: then the area of each of the triangles A B C and A D C may be found by case 2, last problem.

2. Required the plan and content of a field by the following field-book:

FIELD-BOOK.

——	1360 = A B.	——
——	1190	625
342	600	——
——	□ D go East.	——
Offsets Left.	Station □, or base line.	Offsets Right.



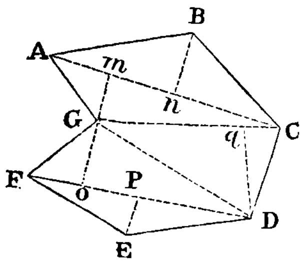
Ans. 6A., 2R., 12P.

How many acres are there in a four-sided field, whose diagonal is 4·75 chains, and the two perpendiculars falling on it, from its opposite angles, 2·25 and 3·6 chains, respectively? *Ans.* 1A., 1R. 22·3P.

PROBLEM VI.

To survey a field of many sides by the chain only.

Let A B C D E F G be the field whose content is required. Set up marks at the corners of the field, if there be none there naturally. Consider how the field may be best divided



into trapeziums and triangles; measure them separately, as in the two last problems: and the sum of all the separate results will give the area of the whole field.

In this way of measuring with the chain, the field should be divided into trapeziums and triangles, by drawing diagonals from corner to corner, so that all the perpendiculars may be within the figure.

The last figure is divided into two trapeziums A B C G, G D E F, and the triangle G C D. In the first trapezium measure the diagonal A C, and the two perpendiculars G *m* and B *n*. In the trangle G C D, measure the base G C, and the perpendicular D *q*. Finally, measure the diagonal

F D, and the two perpendiculars G *o* and E *p*. Having drawn a rough figure resembling the field, set all these measures against the corresponding parts of the figure. Or set them down thus:

CALCULATION.

A *m* 135 A *n* 415 A C 550	130 *m* G 180 *n* B
C *q* 152 C G 440	230 *q* D
F *o* 206 F *p* 288 F D 520	120 *o* G 80 *p* E

130 + 180 = 310, 550 ÷ 2 = 275, 275 × 310 = 85250 = A B C G.

440 × 230 ÷ 2 = 50600 = C G D.

120 + 80 = 200, 520 ÷ 2 = 260, 260 × 200 = 42000 = D E F G.

$$
\begin{array}{r}
1{\cdot}878502 = \text{ABCDEFG.} \\
4 \\
\hline
3{\cdot}51400 \\
40 \\
\hline
20{\cdot}56000
\end{array}
$$

1A., 3R., 20·56P., answer.

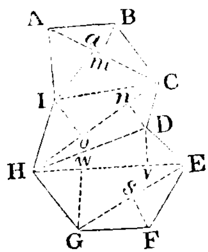
Other methods will naturally present themselves to an ingenious practitioner who has read the preceding part of this work, or who has been previously acquainted with the principles of Mathematics. Every surveyor ought to be well acquainted with Plane Geometry at least. This, with a knowledge of Trigonometry, would be sufficient for the purpose of most surveyors.

The content of the last figure may be found by measuring the sides A B, B C, C D, D E, E F, F G, G A; and the diagonals A C, C G, G D, D F, by which the figure is divided into triangles, the content of each of which may be found by Problem V., Section II.

2. *Required the plan and content of a field of an irregular form from the following.*

FIELD-BOOK.

Offsets Left.	Stations, □, or Base Lines.	Offsets Right.
——	900 = EG	——
268	550	——
——	□ F, go S.W.	——
——	1100 = HE	——
280	790	——
——	350	410
——	□ H, go East.	——
——	1180 = CH	——
——	710	280
140	350	——
——	□ C, go S.W.	——
——	900 = AC	——
200	430	——
——	300	450
——	□ A, go S.E.	——



Ans. 10A., 1R., 24·64P.

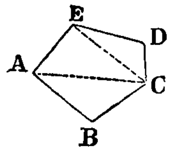
PROBLEM VII.

To survey a field with the theodolite, &c.

1. *From one point or station.*

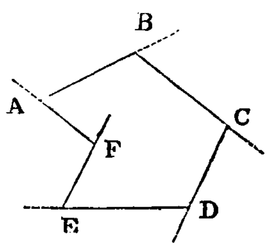
When all the angles can be seen from one point, as suppose C.

Having placed the instrument at C, turn it about till, through the fixed sights, the mark B may be seen. Fixing the instrument in this position, turn the moveable index about, till the mark A is seen through the sights, and note the degrees on the instrument. In the same manner, turn the index successively to the angles E and D, taking care to note the degrees cut off at each; by which you have all the angles, viz., B C A, B C E, B C D. Now, having obtained the angles, measure the lines C B, C A, C E, C D; entering the respective measures against the corresponding part of a rough figure, drawn to resemble the figure.



2. *By going round the field.*

Set up marks at B, C, D, &c. Place the instrument at the point A, and turn it about till the fixed index be in the direction A B, and then screw it fast: turn the moveable index in the direction A F, and the degrees cut off will be



the angle A; next measure A B, and planting the instrument at B, measure, as before, the angle B; measure the line B C, and the angle C: and so proceed round the figure, always measuring the side as you go along, as also the angles.

The 32d Proposition of the 1st Book of Euclid affords an easy method of proving the work: thus, add all the internal angles, A, B, C, &c., of the figure together, and their sum must be equal to twice as many right angles as the figure has sides, wanting four right angles. But when the figure has a re-enterant angle as F, measure the external angle, which is less than two right angles, and deduct it from four right angles, or 360 degrees, the remainder will give the internal angle (if such it may be called), which is greater than 180 degrees.

When the field is surveyed from one station, as in the first case shown above, the content of the figure is found as in the second case of Prob. IV., since we have two sides and the angle included between them in each triangle of the figure.

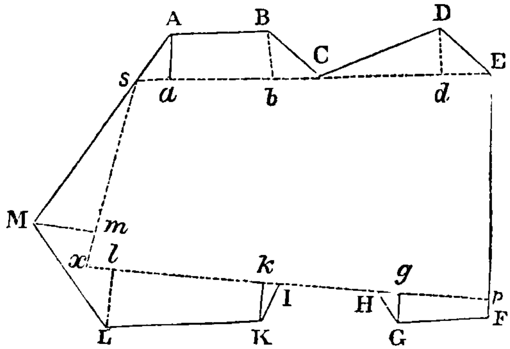
PROBLEM VIII.

To survey a field with crooked hedges.

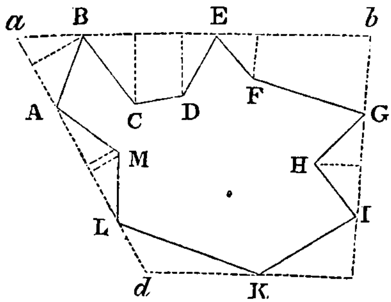
Measure the lengths and positions of lines running as near the sides of the field as you can; and, in proceeding along these lines, measure the offsets to the different corners, as before taught, and join the ends of the offsets; these connecting lines will represent the required figure. When the plane table is used, the plan will be truly represented on the paper which covers it. But when the survey is made with the theodolite, or other instrument, the different measures are to be noted in the field-book, from which the sides and angles are laid down on a map, after returning from the field.

In surveying the piece A B C D E F G H I K L M, set up marks at *s* E F *x*. Begin at the station *s*, and measure the lines *s* E, E F, F *x*, *x s*, as also their positions, or the angles E *s x*, *s* E F, E F *x*, and F *x s*; and in going along the four-sided figure *s* E F *x*, measure the offsets at *a*, *b*, *d*, *g*, *k*, *l*, *m*, as before taught. By means of the figure *s* E F *x*, and of the offsets, the ground is easily planned.

When the principal lines are taken within the figure, as in the above case, the contents of the exterior portions



s C B A, C D E, &c. must be added to the area of the quadralateral *s x* F E. But when the principal lines are taken outside the figure, the portions included between them and the boundaries of the field are to be deducted from the content of the quadralateral, and the remainder will give the true content of the field.



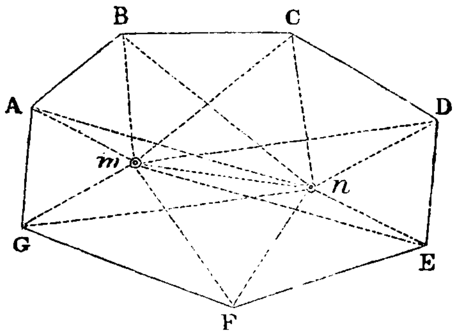
When there are obstructions within the figure, such as wood, water, hills, &c., measure the lengths and positions of the four-sided figure *a b c d*, taking care to measure the offsets from the different corners as you go along.

PROBLEM IX.

To survey any piece of land by two stations.

Choose two stations, from which all the corners of the ground can be seen, if possible; measure the distance between the stations; at each station take the angles formed by every object, from the station line, or distance. Then the station line, and these different angles being laid down from a regular scale, and the external points of intersection connected, the connecting lines will give the boundary.

The two stations may be taken within the bounds, in one of the sides, or without the bounds of the ground to be surveyed.



Let *m* and *n* be two stations, from which all the marks A, B, C, &c., can be seen, plant the instrument at *m* and by it, measure the angles A *m n*, B *m n*, C *m n*, &c. Next measure *m n*, and planting the instrument at *n*, measure the angles A *n m*, B *n m*, C *n m*, &c. These observations being planned the lines joining the points of external intersection, will give a true map of the ground. The method of finding the content will be shown further on.

The principal objects on the ground may be delineated on the map, by measuring the angles at each station, which every object makes with the station line $m\ n$. When all the objects to be surveyed cannot be seen from two stations, then three or four may be used, or as many as may be found necessary; taking care to measure the distance from one station to another; placing the instrument at every station, and observing the angles formed by all the visible objects with the respective station line; then the intersection of the lines forming these respective angles, will give the positions of all the remarkable objects thus observed.

In this manner may very extensive surveys be taken; and the positions of hills, rivers, coasts, &c., ascertained.

PROBLEM X.

To survey a large estate.

The following method of surveying a large estate was first given by Emerson, in his "Surveying," page 47. It has been followed by Hutton and Keith.

When the estate is very large, and contains a great number of fields, it cannot be accurately surveyed and planned by measuring each field separately, and then adding all the separate results together; nor by taking all the angles, and measuring the boundaries that enclose it. For in these cases the small errors will be so multipled as to render it very much distorted.

1. Walk over the estate two or three times, in order to get a perfect idea of its figure. And to help your memory, make a rough draft of it on paper, inserting the names of the different fields within it, and noting down the principal objects.

2. Choose two or more elevated places in the estate for your stations, from which you can see all the principal parts of it; and let these stations be as far distant from each other

as possible, as the fewer stations you have to command the whole, the more exact the work will be.

In selecting the stations, care should be taken that the lines which connect them may run along the boundaries of the estate, or some of the hedges, to which offsets may be taken when necessary.

3. Take such angles, between the stations, as you think necessary, and measure the distance from station to station, always in a right line; these things must be done till you get as many lines and angles as are sufficient for determining all the station points. In measuring any of these station distances, mark accurately where these lines meet with any hedges, ditches, roads, lanes, paths, rivulets, &c., and where any remarkable object is placed, by measuring its distance from the station line; and where a perpendicular from it cuts that line; and always mind, in any of these observations, that you be in a right line, which you may easily know by taking a back-sight and fore-sight, along the station line. In going along any main station line, take offsets to the ends of all hedges, and to any pond, house, mill, bridge, &c., omitting nothing that is remarkable. All these things must be noted down; for these are the data by which the places of such objects are to be determined on the plan.

Be careful to set up marks at the intersections of all hedges with the station line, that you may know where to measure from when you come to survey the particular fields that are crossed by this line.

These fields must be measured as soon as you have completed your station line whilst they are fresh in your memory. In this manner all the station lines must be measured, and the situations of all adjacent objects determined. It will be proper to lay down the work on paper every night, that you may see how you go on.

4. With respect to the internal parts of the estate, they must be determined by new station lines; for, after the main stations are determined, and every thing adjoining to them, then the estate must be subdivided into two or three

parts by new station lines: taking the inner stations at proper places, where you can have the best view. Measure these station lines as you did the first, and all their intersections with hedges, ditches, roads, &c., also take offsets to the bends of hedges, and to such objects as appear near these lines. Then proceed to survey the adjoining fields by taking the angles which the sides make with the station line at the intersections, and measuring the distances to each corner from these intersections; for, every station line will be a basis to all future operations, the situation of every object being entirely dependent on them; and therefore they should be taken of as great length as possible: and it is best for them to run along some of the hedges or boundaries of one or more fields, or to pass through some of their angles.

All things being determined for these stations, you must take more inner stations, and continue to divide and subdivide, till at last you come to single fields; repeating the same work for the inner stations as for the outer ones, till the whole is finished. The oftener you close your work, and the fewer lines you make use of, the less you will be liable to error.

5. An estate may be so situated that the whole cannot be surveyed together, because one part of the estate may not be seen from another. In this case you may divide it into three or four parts, and survey these parts separately, as if they were lands belonging to different persons, and at last join them together.

6. As it is necessary to protract or lay down the work as you proceed in it, you must have a scale of due length to do it by. To get such a scale, measure the whole length of the estate in chains; then consider how many inches long the map is to be; and from these you will know how many chains you must have in an inch: then make your scale accordingly, or choose one already made.

7. The trees in every hedge-row may be placed in their proper situation, which is soon done by the plane table; but may be done by the eye without an instrument; and being

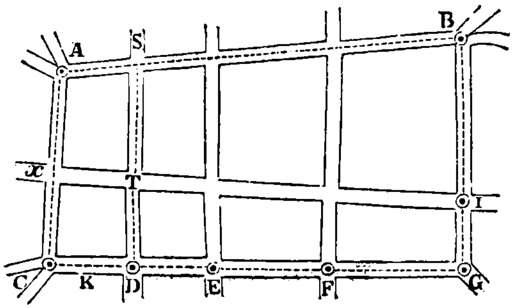
thus taken by guess in a rough draft, they will be exact enough, being only to look at; except it be such as are at any remarkable places, as at the ends of hedges, at stiles, gates, &c., and these must be measured or taken with the plane table, or some other instrument. But all this need not be done till the draft is finished. And observe, in all hedges, what side the gutter or ditch is on, and to whom the fence belongs.

PROBLEM XI.

To survey a town or city.

To survey a town or city, it will be proper to have an instrument for taking angles, such as a theodolite or plane table; the latter is a very convenient instrument, because the minute parts may be drawn upon it on the spot. A chain of 50 feet long, divided into 50 links, will be more convenient than the common surveying chain, and an offset staff of 10 feet long will be very useful. Begin at the meeting of two or more of the principal streets, through which you can have the longest prospects, to get the longest station lines. There having fixed the instruments, draw lines of direction along these streets, using two men as marks, or poles set in wooden pedestals, or perhaps some remarkable places in the houses at the farther ends, as windows, doors, corners, &c. Measure these lines with the chain, taking offsets with the staff, at all corners of streets, bendings, or windings, and to all remarkable objects, as churches, markets, halls, colleges, eminent buildings, &c. Then remove the instrument to another station along one of these lines, and there repeat the same process as before. And so continue until the whole is finished.

Thus, fix the instrument at A, and draw lines in the directions of all the streets meeting there; then measure A C, noting the street at x. At the second station C, draw the directions of all the streets meeting there; measure from C to D, noting the place of the street K, as you pass by it. At



the third station D, take the direction of all the streets meeting there, and measure D S, noting the cross street at T. Proceed in like manner through all the principal streets, after which proceed to the smaller and intermediate streets; and last of all to the lanes, alleys, courts, yards, and every other place which it may be thought proper to represent in the plan.

PROBLEM XII.

To compute the content of any survey.

1. In small and separate pieces, the method generally employed is, to compute their contents from the measures of the lines taken in surveying them, without drawing any correct map of them: rules for this purpose have been given in the preceding part of the work. But in large pieces, and whole estates, consisting of a great number of fields, the usual method is, to make an unfinished but correct plan of the whole, and from this plan, the boundaries of which include the whole estate, compute the contents quite independent of the measures of the lines and angles that were taken in surveying. Divide the plan of the survey into triangles and trapeziums, by drawing new lines through it:

measure all the bases and perpendiculars of all these new figures, by means of the scale from which the plan was drawn, and from these dimensions compute the contents, whether triangles, or trapeziums, by the proper rules for finding the areas of such figures.

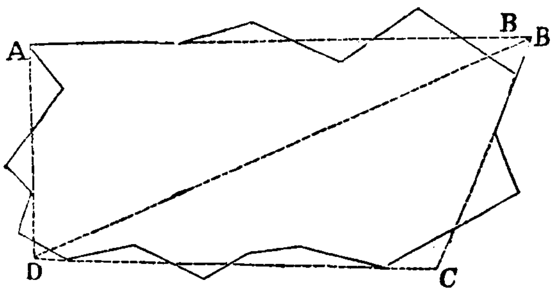
The chief difficulty in computing consists in finding the contents of land bounded by curved or very irregular lines, or in reducing such crooked sides or boundaries to straight lines, that shall enclose an equal area with those crooked sides, and so obtain the area of the curved figure by means of the right-lined one, which in general will be a trapezium.

The reduction of crooked sides to straight ones is easily performed thus:

Apply a horse-hair or silk thread across the crooked side in such a manner, that the small parts cut off from the crooked figure by it, may be equal to those taken in. A little practice will enable you to exclude exactly as much as you include; then, with a pencil, draw a line along the thread or horse-hair. Do the same by the other sides of the figure, and you will thus have the figure reduced to a straight-sided figure equal to the curved one: the content of which, being computed as before directed, will be the content of the curved figure proposed.

The best way of using the thread or horse-hair is, to string a small slender bow with it, either of whalebone or wire, which will keep it stretched.

If it were required to find the contents of the following crooked-sided figure; draw the four dotted straight lines A B, B C, C D, and D A, excluding as much from the survey as is taken in by the straight lines; by which the crooked figure is reduced to a right-lined one, both equal in area. Then draw the diagonal B D, which being measured by a proper scale, and multiplied by half the sum of the perpendiculars let fall from A and C upon B D (measured on the same scale), will give the area required.



Many other methods might have been given for computing the contents of a survey, but they are omitted, the above being, perhaps, the most expeditious.

MISCELLANEOUS PROBLEMS.

1. The three sides of a triangle are 12, 20, and 28; what is the area? *Ans.* $60\sqrt{3}$.

2. Find the difference between the area of a triangle whose sides are 3, 4, and 5 feet; and the area of an equilateral triangle having an equal perimeter?

Ans. ·928 of a square foot.

3. There is a segment of a sphere, the diameter of whose base is 24 inches, and its altitude 10 inches; required its solidity? *Ans.* 2785·552 inches.

4. There is a bushel in the form of a cylinder, whose depth is 8 inches, and breadth $18\frac{1}{2}$ inches; required to determine the breadth of another cylindrical vessel of the same capacity as the former, whose depth is only $7\frac{1}{2}$ inches?

Ans. 19.107 inches.

5. A ladder 40 feet long, may be so planted, that it shall reach a window 33 feet from the ground on one side of the street; and by only turning it over, without moving the foot out of its place, it will do the same by a window 21 feet high on the other side; what is the breadth of the street?

Ans. 56 feet $7\frac{3}{4}$ inches.

6. In turning a one-horse chaise within a ring of a certain diameter, it was observed that the outer wheel made two turns while the inner made but one; the wheels were both 4 feet high; and supposing them fixed at the statutable

distance of 5 feet asunder on the axle-tree, what was the circumference of the track described by the outer wheel?

Ans. 63 feet, nearly.

7. A cable which is 3 feet long, and 9 inches in compass, weighs 22 lbs.; what will a fathom of that cable weigh, which measures a foot about? *Ans.* $78\frac{2}{9}$ lbs.

8. How many solid cubes, a side of which equals 4 inches, may be cut out of a large cube, whose side is 8 inches? *Ans.* 8.

9. Determine the areas of an equilateral triangle, a square, a hexagon, the perimeter of each being 40 feet?

Ans. 76·980035 — 100 — 115·47.

10. A person wants a cylindrical vessel 3 feet deep, that shall contain twice as much as another cylindrical vessel whose diameter is $3\frac{1}{2}$ feet, and altitude 5 feet; find the diameter of the required vessel? *Ans.* 6·39 feet.

11. Three persons having bought a conical sugar-loaf, wish to divide it into three equal parts by sections parallel to the base; it is required to find the altitude of each person's share, the altitude of the loaf being 20 inches?

Ans. Altitude of the upper part = 13·867, of the middle part = 3 604, of the lower part 2·528 inches.

12 There is a frustum of a pyramid, whose bases are regular octagons; each side of the greater base is 21 inches, and each side of the less base 9 inches, and its perpendicular length 15 feet, how many solid feet are contained in it?

Ans. 119·2 feet.

13. Requiring to find the height of a May-pole, I procured a staff 5 feet in length, and placing it in the sunshine, perpendicular to the horizon, I found its shadow to be 4·1 feet. Next I measured the shadow of the May-pole, which I found to be 65 feet; from this date the height of the pole is required? *Ans.* 79.26 feet.

14. Given two sides of an obtuse-angled triangle, which are 20 and 40 poles; required the third side, that the triangle may contain just an acre of land?

Ans. 58·876 or 23·099.

15. A circular fish-pond is to be made in a garden, that shall take up just half an acre; what must be the length of the chord that strikes the circle? *Ans.* $27\frac{3}{4}$ yards.

16. A gentleman has a garden 100 feet long, and 80 feet broad. Now a gravel walk is to be made of an equal width all round it; what must the breadth of the walk be, to take up just half the ground? *Ans.* 12·9846 feet.

17. A silver cup, in form of a frustum of a cone, whose top diameter is 3 inches, its bottom diameter 4, and its altitude 6 inches, being filled with liquor, a person drank out of it till he could see the middle of the bottom; it is required to find how much he drank? *Ans.* ·152127 ale gallons.

18. I have a right cone, which cost me £5 13*s*. 7*d*., at 10*s*. a cubic foot, the diameter of its base being to its altitude as 5 to 8; and would have its convex surface divided in the same ratio, by a plane parallel to the base; the upper part to be the greater; required the slant height of each part?

Ans. { 3·9506486, the slant height of the upper part.
1·0854612, the slant height of the under part.

19. How many acres of the earth's surface may be seen from the top of a steeple whose height is 400 feet, the earth being supposed to be a perfect sphere, whose circumference is 25000 miles. *Ans.* 12120981·338267112 acres.

20. Two boys meeting at a farm-house, had a tankard of milk set down to them; the one being very thirsty drank till he could see the centre of the bottom of the tankard; the other drank the rest. Now, if we suppose that the milk cost $4\frac{1}{2}d.$, and the tankard measured 4 inches diameter at the top and bottom, and 6 inches in depth; it is required to know what each boy had to pay, proportionable to the quantity of milk he drank?

Ans. { 14·1802815 farthings for the first.
3·8197185 farthings for the second.

21. If the linear side of a certain cube, be increased one inch, the surface of the cube will be increased 246 square inches: determine the side of the cube.

Ans. 20 inches.

22. If from a piece of tin, in the form of a sector of a circle, whose radius is 30 inches, and the length of its arc 36 inches, be cut another sector whose radius is 20 inches; and if then the remaining frustum be rolled up so as to form the frustum of a cone; it is required to find its content, supposing that one-eighth of an inch to be allowed off its slant height for the bottom, and the same allowance of the circumference, of both top and bottom, for what the sides fold over each other, in order to their being soldered together?

Ans. 685·3263 cubic inches.

23. Three men bought a grinding-stone of 40 inches diameter, which cost 20*s*., of which sum the first man paid 9*s*., the second 6*s*., and the third 5*s*., how much of the stone must each man grind down, proportionably to the money he paid?

Ans. The first man must grind down 5·167603 inches of the radius; the second 4·832397 inches, and the third 10 inches.

24. There is a frustum of a cone, whose solid content is 20 feet, and its length 12 feet; the greater diameter is to the less as 5 to 2; what are the diameters?

Ans. { 2·02012 feet. ·80804 feet.

25. A farmer borrowed of his neighbour part of a hayrick, which measured 6 feet in length, breadth, and thickness; at the next hay-time he paid back two equal cubical pieces, each side of which was 4 feet. Has the debt been discharged? *Ans.* No; 88 cubic feet are due.

26. There is a bowl in form of the segment of an oblong spheroid, whose axes are to each other in the proportion of 3 to 4, the depth of the bowl one-fourth of the whole transverse axis, and the diameter of its top 20 inches; it is required to determine what number of glasses a company of 10 persons would have in the contents of it, when filled, using a conical glass, whose depth is 2 inches, and the diameter of its top an inch and a half.

Ans. 114·0444976 glasses each.

27. If a cubical foot of brass were to be drawn into wire

of $\frac{1}{40}$ of an inch in diameter; it is required to determine the length of the said wire, allowing no loss in the metal?

Ans. $55\frac{5}{9}$ miles.

28. How many shot are there in an unfinished oblong pile, the length and breadth of whose base being 48 and 30, and the length and breadth of the highest course being 24 and 6? *Ans.* 17356.

29. How many shot are there in an unfinished oblong pile of 12 courses; length and breadth of the top contain 40 and 10 shot respectively? *Ans.* 8606 shot.

30. Of what diameter must the bore of a cannon be cast for a ball of 24 pounds weight, so that the diameter of the bore may be $\frac{1}{10}$ of an inch more than that of the ball?

Ans. 5·757098 inches.

31. What is the content of a tree, whose length is $17\frac{1}{2}$ feet, and which girts in five different places as follows, viz., in the first place 9·43 feet, in the second 7·92, in the third 6·15, in the fourth 4·74, and the fifth 3·16?

Ans. 42.5195.

32. What three numbers will express the proportions subsisting between the solidity of a sphere, that of the circumscribing cylinder, and circumscribing equilateral cone?

Ans. 4, 6, 9.

33. Given the side of an equilateral triangle 10, it is required to find the radii of its circumscribing circle?

Ans. 5·7736.

34. Given the perpendicular of a plane triangle 300, the sum of the two sides 1150, and the difference of the segment of the base 495; required the base and the sides?

Ans. 945, 375, and 780.

35. A side wall of a house is 30 feet high, and the opposite one 40, the roof forms a right angle, at the top, the lengths of the rafters are 10 feet and 12; the end of the shorter is placed on the higher wall, and *vice versâ;* required the length of the upright, which supports the ridge of the roof, and the breadth of the house?

Ans. 41·803, length of upright, and 12 feet the breadth of the house.

A TABLE

OF THE AREAS OF THE SEGMENTS OF A CIRCLE,

Whose diameter is 1, *and supposed to be divided into* 1000 *equal parts.*

Height.	Area Seg.	Height.	Area Seg.	Height.	Area Seg.	Height.	Area Seg.
·001	·000042	·038	·009763	·075	·026761	·112	·048262
·002	·000119	·039	·010148	·076	·027289	·113	·048894
·003	·000219	·040	·010537	·077	·027821	·114	·049528
·004	·000337	·041	·010931	·078	·028356	·115	·050165
·005	·000470	·042	·011330	·079	·028894	·116	·050804
·006	·000618	·043	·011734	·080	·029435	·117	·051446
·007	·000779	·044	·012142	·081	·039979	·118	·052090
·008	·000951	·045	·012554	·082	·030523	·119	·052736
·009	·001135	·046	·012971	·083	·031076	·120	·053385
·010	·001329	·047	·013392	·084	·031629	·121	·054036
·011	·001533	·048	·013818	·085	·032180	·122	·054689
·012	·001746	·049	·014247	·086	·032745	·123	·055345
·013	·001968	·050	·014681	·087	·033307	·124	·056003
·014	·002199	·051	·015119	·088	·033872	·125	·056663
·015	·002438	·052	·015561	·089	·034441	·126	·057326
·016	·002685	·053	·016007	·090	·035011	·127	·057991
·017	·002940	·054	·016457	·091	·035585	·128	·058658
·018	·003202	·055	·016911	·092	·036162	·129	·059327
·019	·003471	·056	·017369	·093	·036741	·130	·059999
·020	·003748	·057	·017831	·094	·037323	·131	·060672
·021	·004031	·058	·018296	·095	·037909	·132	·061348
·022	·004322	·059	·018766	·096	·038496	·133	·062026
·023	·004618	·060	·019239	·097	·039087	·134	·062707
·024	·004921	·061	·019716	·098	·039680	·135	·063389
·025	·005230	·062	·020196	·099	·040276	·136	·064074
·026	·005546	·063	·020681	·100	·040875	·137	·064760
·027	·005867	·064	·021168	·101	·041476	·138	·065449
·028	·006194	·065	·021659	·102	·042080	·139	·066140
·029	·006527	·066	·022154	·103	·042687	·140	·066833
·030	·006865	·067	·022652	·104	·043296	·141	·067528
·031	·007209	·068	·023154	·105	·043908	·142	·068225
·032	·007558	·069	·023659	·106	·044522	·143	·068924
·033	·007913	·070	·024168	·107	·045139	·144	·069625
·034	·008273	·071	·024680	·108	·045759	·145	·070328
·035	·008638	·072	·025195	·109	·046381	·146	·071033
·036	·009008	·073	·025714	·110	·047005	·147	·071741
·037	·009383	·074	·026236	·111	·047632	·148	·072150

Height	Area seg.	Height	Area seg.	Height	Area seg.	Height	Area seg.
·149	·073161	·194	·107051	·239	·144091	·284	·183619
·150	·073874	·195	·107842	·240	·144944	·285	·184521
·151	·074589	·196	·108636	·241	·145799	·286	·185425
·152	·075306	·197	·109430	·242	·146655	·287	·186329
·153	·076026	·198	·110226	·243	·147512	·288	·187234
·154	·076747	·199	·111024	·244	·148371	·289	·188140
·155	·077469	·200	·111823	·245	·149230	·290	·189047
·156	·078194	·201	·112624	·246	·150091	·291	·189955
·157	·078921	·202	·113426	·247	·150953	·292	·190864
·158	·079649	·203	·114230	·248	·151816	·293	·191775
·159	·080380	·204	·115035	·249	·152680	·294	·192684
·160	·081112	·205	·115842	·250	·153546	·295	·193596
·161	·081845	·206	·116650	·251	·154412	·296	·194509
·162	·082582	·207	·117460	·252	·155280	·297	·195422
·163	·083320	·208	·118271	·253	·156149	·298	·196337
·164	·084059	·209	·119083	·254	·157019	·299	·197252
·165	·084801	·210	·119897	·255	·157890	·300	·198168
·166	·085544	·211	·120712	·256	·158762	·301	·199085
·167	·086289	·212	·121529	·257	·159636	·302	·200003
·168	·087036	·213	·122347	·258	·160510	·303	·200922
·169	·087785	·214	·123167	·259	·161386	·304	·201841
·170	·088535	·215	·123988	·260	·162263	·305	·202761
·171	·089287	·216	·124810	·261	·163140	·306	·203683
·172	·090041	·217	·125634	·262	·164019	·307	·204605
·173	·090797	·218	·126459	·263	·164899	·308	·205527
·174	·091554	·219	·127285	·264	·165780	·309	·206451
·175	·092313	·220	·128113	·265	·166663	·310	·207376
·176	·093074	·221	·128942	·266	·167546	·311	·208301
·177	·093836	·222	·129773	·267	·168430	·212	·209227
·178	·094601	·223	·130605	·268	·169315	·313	·210154
·179	·095366	·224	·131438	·269	·170202	·314	·211082
·180	·096134	·225	·132272	·270	·171089	·315	·212011
·181	·096903	·226	·133108	·271	·171978	·316	·212940
·182	·097674	·227	·133945	·272	·172867	·317	·213871
·183	·098447	·228	·134784	·273	·173758	·318	·214802
·184	·099221	·229	·135624	·274	·174649	·319	·215733
·185	·099997	·230	·136465	·275	·175542	·320	·216666
·186	·100774	·231	·137307	·276	·176435	·321	·217599
·187	·101553	·232	·138150	·277	·177330	·322	·218533
·188	·102334	·233	·138995	·278	·178225	·323	·219468
·189	·103116	·234	·139841	·279	·179122	·324	·220404
·190	·103900	·235	·140688	·280	·180019	·325	·221340
·191	·104685	·236	·141537	·281	·180918	·326	·222277
·192	·105472	·237	·142387	·282	·181817	·327	·223215
·193	·106261	·238	·143238	·283	·182718	·328	·224154

Height.	Area Seg.	Height.	Area Seg.	Height.	Area Seg.	Height.	Area Seg.
·329	·225093	·372	·266111	·415	·308110	·458	·350748
·330	·226033	·373	·267078	·416	·309095	·459	·351745
·331	·226974	·374	·268045	·417	·310081	·460	·352742
·332	·227915	·375	·269013	·418	·311068	·461	·353739
·333	·228858	·376	·269982	·419	·312054	·462	·354736
·334	·229801	·377	·270951	·420	·313041	·463	·355732
·335	·230745	·378	·271920	·421	·314029	·464	·356730
·336	·231689	·379	·272890	·422	·315016	·465	·357727
·337	·232634	·380	·273861	·423	·316004	·466	·358725
·338	·233580	·381	·274832	·424	·316992	·467	·359723
·339	·234526	·382	·275803	·425	·317981	·468	·360721
·340	·235473	·383	·276775	·426	·318970	·469	·361719
·341	·236421	·384	·277748	·427	·319959	·470	·362717
·342	·337369	·385	·278721	·428	·320948	·471	·363715
·343	·238318	·386	·279694	·429	·321938	·472	·364713
·344	·239268	·387	·280668	·430	·322928	·473	·365712
·345	·240218	·388	·281662	·431	·323918	·474	·366710
·346	·241169	·389	·282617	·432	·324909	·475	·367709
·347	·242121	·390	·283592	·433	·325900	·476	·368708
·348	·243074	·391	·284568	·434	·326892	·477	·369707
·349	·244026	·392	·285544	·435	·327882	·478	·370706
·350	·244980	·393	·286521	·436	·328874	·469	·371705
·351	·245934	·394	·287498	·437	·329866	·480	·372764
·352	·246889	·395	·288476	·438	·330858	·481	·373703
·353	·247845	·396	·289453	·439	·331850	·482	·374702
·354	·248801	·397	·290432	·440	·332843	·483	·375702
·355	·249757	·398	·291411	·441	·333836	·484	·376702
·356	·250715	·399	·292390	·442	·334829	·485	·377701
·357	·251673	·400	·293369	·443	·335822	·486	·378701
·358	·252631	·401	·294349	·444	·336816	·487	·379700
·359	·253590	·402	·295330	·445	·337810	·488	·380700
·360	·254550	·403	·296311	·446	·338804	·489	·381699
·361	·255510	·404	·297292	·447	·339798	·490	·382699
·362	·256471	·405	·298273	·448	·340793	·491	·383699
·363	·257433	·406	·299255	·449	·341787	·492	·384699
·364	·258395	·407	·300238	·450	·342782	·493	·385999
·365	·259357	·408	·301220	·451	·343777	·494	·386699
·366	·260320	·409	·302203	·452	·344772	·495	·387699
·367	·261284	·410	·303187	·453	·345768	·496	·388699
·368	·262248	·411	·304171	·454	·346763	·497	·389699
·369	·263213	·412	·305155	·455	·347759	·498	·390699
·370	·264178	·413	·306140	·456	·348755	·499	·391699
·371	·265144	·414	·307125	·457	·349752	·500	·392699