A

SERIES OF

# GEOMETRICAL PROBLEMS,

SELECT OR ORIGINAL.

FOR THE USE OF SCHOOLS.

---

TORONTO.
PRINTED FOR THE AUTHOR.
1843.

# GEOMETRICAL PROBLEMS.

1. What are the derivations of the terms *geometry, point, line, rectilineal, angle, vertex, obtuse, acute, circle, circumference, periphery, centre, radius, diameter, semicircle, segment, triangle, trilateral, quadrilateral, multilateral, equilateral, polygon, isosceles, hypothenuse, scalene, rhombus, trapezium, diagonal, parallel, problem, theorem, proposition, hypothesis, corollary, deduction, postulate, axiom, coincide, lemma, area, perimeter?*

2. Define, *postulate, axiom, lemma, trapezium and parallel right lines.*

3. If two circles intersect each other having a common radius, the figure formed by joining the extremities of that radius with the points of intersection, is a rhombus.

4. If the common radius be produced to meet both circles, the figures formed by joining the points of intersection and the extremities of the diameters so formed, are equilateral triangles.

5. The equilateral triangle on the common radius is one-third of either of the former.

6. The figure formed by joining the points of intersection with the extremities of either diameter is four times as great as the equilateral triangle on the common radius.

7. The line joining the points of intersection divides that figure into parts, which are as three to one.

8. The rhombi, formed by joining the points of intersection with the near and remote extremities of the diameters, are as one to four.

9. The line joining the points of intersection cuts the common radius at right angles.

10. In drawing a right line equal to a given right line from a given point, show that the problem admits of eight different constructions.

11. The radius of the second circle, is equal to the *sum* or *difference* of the given line, and the side of the equilateral triangle.

12. If two sides of a triangle be equal, the opposite angles are also equal; prove this without producing the equal sides.

13. On what occasions is the latter part of the fifth proposition applied throughout the elements of Euclid?

14. In the construction for the sixth proposition of Euclid's elements, what objection to measuring off the required portion from A. instead of B.

15. Explain the nature of the *ex absurdo* or indirect process of proof.

16. To what purpose is the seventh proposition applied, and what is the form of argument made use of?

17. Prove the eighth proposition without an application of the seventh.

18. If an isosceles and an equilateral triangle be constructed on a common base, under what circumstances, will the vertex of the latter fall below, upon, or above that of the former?

19. In constructing for the bisection of a rectilineal angle, state the advantage of the particular position of the equilateral triangle.

20. Were the vertex situated above instead of below the base, in what case would that construction be inapplicable?

21. In the bisection of a given finite right line, would the latter be admissible.

22. The line bisecting the vertical angle of an isosceles triangle, bisects the base at right angles.

23 The line drawn from the vertex to the middle point of the base of an isosceles triangle, bisects the vertical angle and is perpendicular to the base.

24. If the perpendicular from the vertical angle of a triangle bisects the base, the triangle is isosceles.

25. If two isosceles triangles be described upon opposite sides of a common base, the line joining their vertices bisects the base at right angles.

26. Determine a point within a triangle equi-distant from its angular points.

27. Write down the complements and supplements of 45, 50, 55, 60, 65, 70, 75, 80, 85, degrees.

28. Why is an obtuse or a right angle the greatest angle of a triangle?

29. Prove the equal angles of an isosceles triangle necessarily acute.

30. If two sides of a triangle be unequal, the opposite angles are also unequal, and the greater angle is opposite the greater side, prove this by producing the shorter of the two sides.

31. To what propositions are the eighteenth and nineteenth analagous?

32. The difference between any two sides of a triangle is less than the third side.

33. To what propositions are the twenty-fourth and twenty-fifth analagous? Include them all under one general enunciation.

34. If any number of isosceles triangles be described on a common base, the perpendicular from the middle of the base passes through all their vertices.

35. The same construction being made, the line joining the vertices of any two, when produced, bisects the base at right angles.

36. Determine that point in a right line, equally distant from its extremity, and another point given in position.

37. Determine that point in a right line, at which a perpendicular of given length, subtends half the angle which it subtends at any other given point.

38. At what point of a right line does a given perpendicular subtend an angle of 7° 30 minutes?

39. Given the position at which a perpendicular to a right line at a given point, subtends an angle of 3°, 45 minutes, required the height of the perpendicular.

40. Distinguish between the fourth, eighth, and twenty-sixth propositions?

41. From a given point, draw the shortest line possible to a given right line.

42. If a perpendicular be drawn bisecting a given right line, any point in this perpendicular is at equal distances, and any point without the perpendicular is at unequal distances from the extremities of the line.

43. If several right lines be drawn from a point to a given right line, then those equally inclined to the perpendicular are equal and, *vice versa*.

44. Those which meet the right line at equal distances from the perpendicular are equal and, *vice versa*.

45. Those which make greater angles with the perpendicular are greater, and *vice versa*.

46. Those which meet the line at greater distances from the perpendicular are greater, and *vice versa*.

47. It is required to erect a line of fence across the corner of a rectangular plot of ground, which shall bisect the mouth of a circular cistern situated near it, and make equal angles with the enclosures.

48. If the line drawn bisecting the vertical angle of a triangle, bisects the base also, the triangle is isosceles.

49. In general, a right line being drawn from the vertex of a triangle to the base; if any two of the five following equalities be established, the others may be deduced:—

1. The equality of the sides.
2. The equality of the angles at the base.
3. The equality of the angles under the line drawn and the base.
4. The equality of the angles under the line drawn and the sides.
5. The equality of the segments of the base.

50. If two triangles be constructed on a common base, the difference between their vertical angles is the sum of the angles contained by the coterminous sides.

51. It is required to construct a cistern on a concession line which shall be equally distant from two dwelling houses in the vicinity, situated on the same or opposite sides of the line.

52. The right line which bisects the vertical exterior angle of an isosceles triangle, is parallel to the base.

53. Given the position of a window, determine that of a mirror on the opposite wall, which shall reflect the images of external objects to the eye of a person, the position of which is also given.

54. The right line drawn from the vertical angle of an isosceles triangle, bisecting the base, is parallel to lines joining the extremities of the base and of the sides, produced through the vertex to their own length.

55. It is required to divide a quadrangular field into three portions, by lines drawn from the extremities of one to some point in the opposite side, so that the length of fence may be as short as possible.

56. The right line drawn from the vertex of an isosceles triangle parallel to the base, bisects the vertical exterior angle.

57. If the line drawn from the vertical angle of a triangle, parallel to the base, bisects the vertical exterior angle, the triangle is isosceles.

58. If two right lines which intersect each other be parallel respectively to two others, the angles included by these pairs of lines will be equal respectively.

59. If a line be perpendicular to one of two parallel lines, it will also be perpendicular to the other.

60. If one of two lines containing a given angle be parallel to one of two others containing an equal angle, the remaining lines are also parallel.

61. Through a given point to draw a straight line so that the parts of it intercepted between that point and perpendiculars drawn from two other given points may have a given ratio.

62. To trisect a right angle.

63. To trisect a given finite right line.

64. To divide a given right line into any number of equal parts.

65. A blank book of given magnitude is to be ruled, so that each opening shall contain thirteen lines, equi-distant from each other, with half an inch of margin above and below.

66. Define a *reentrant* angle and explain the nature of *concave* and *convex* figures.

67. All the exterior angles of any rectilineal figure equal four right angles, together with the excess of every reentrant angle above two right angles.

68. A triangle cannot have a reentrant angle, a quadrilateral may have one, and a figure of $n$ sides must have at least three ordinary angles.

69. The sum of the angles of every rectilineal figure is equal to an even number of right angles.

70. If one angle of a triangle contains $90^{\circ}$ it is equal to the sum of the other two, and *vice versa*.

71. If one angle of a triangle be greater than the sum of the other two, it is greater than a right angle, and *vice versa*.

72. If one angle of a triangle be less than a right angle, it is less than the sum of the other two angles.

73. In an isosceles triangle the vertical exterior angle is double of either of the angles at the base.

74. In an equilateral triangle any exterior angle is double of either of the interior.

75. Determine the magnitude of the angle of a regular pentagon, hexagon, octagon, and decagon.

76. What conditions are necessary, that any particular, *regular* figure may constitute the elements of a pavement?

77. Of what regular figures may a pavement be composed?
78. When the angle of the constituent figure is a *maximum*, what is the description of pavement?

79. If a right line be drawn from the vertical angle of a triangle to the middle point of the base, show that the vertical angle

is right, acute, or obtuse, according as that line is equal to, greater or less than half the base.

80. On the same supposition, the bisector of the base is equal to, greater or less than half the base, according as the vertical angle is right, acute or obtuse.

81. Erect a perpendicular to the extremity of a line without producing it.

82. The diagonals of a parallelogram bisect each other.

83. If the diagonals of a quadrilateral figure bisect each other, the figure is a parallelogram.

84. If the opposite sides of a quadrilateral figure are equal to each other, the figure is a parallelogram.

85. If the diagonals of a quadrilateral bisect the figure, it is a parallelogram.

86. If the opposite angles of a quadrilateral figure are equal to each other, the figure is a parallelogram.

87. In a right angled parallelogram, the diagonals are equal.

88. If the diagonals of a parallelogram are equal to each other, it is a rectangle.

89. In general, if in a quadrilateral figure, the following properties be established, the others may be deduced.

1. 2. The parallelism of either pair of opposite sides.
3. 4. The equality of either pair of opposite sides.
5. 6. The equality of either pair of opposite angles.
7. 8. The bisection of either diagonal by the other.
9. 10. The bisection of the area by either diagonal.

90. These *ten* data combined in pairs give *forty-five* different combinations; to establish, therefore, any of the eight remaining properties, there will arise *three hundred and sixty* different questions respecting such quadrilaterals.

91. The right line drawn from the vertex of a triangle bisecting the base, bisects the area.

92. If two triangles have two sides, of the one equal to two sides of the other and the included angles supplemental, they will have the same area.

93. What data are necessary to establish the equality of two triangles?

94. When two sides and an angle opposite one in each are equal; show that the case admits of two solutions.

95. If the base of a triangle be divided into any number of equal parts, lines drawn from the vertex to the several points of

division will divide the area of the triangle into as many equal parts.

96. The line joining the points of bisection of the sides of a triangle is parallel to the base.

97. A parallel to the base of a triangle through the point of bisection of one side, will bisect the other side.

98. The line joining the points of bisection of the sides of a triangle cuts off a triangle which is one fourth of the given triangle.

99. The lines which join the middle points of the three sides of a triangle, divide the triangle into four equal parts.

100. The line joining the points of bisection of each pair of sides, is equal to half the third side.

101. The area of a triangle is equal to one half of the base into the perpendicular altitude.

102. If the four sides of a quadrilateral figure be bisected, and the middle points of each pair of conterminous sides joined, the joining lines will form a parallelogram, whose area is equal to half that of the parallelogram.

103. The area of a trapezium having two parallel sides, is equal to half the base into the sum of those parallel sides.

104. A parallelogram is equal to a triangle on the same base and of twice the altitude.

105. If a parallelogram and a triangle have equal altitudes, and the base of the triangle be double that of the parallelogram, they will have equal areas.

106. Describe a square which shall have the same area as a given equilateral and equiangular octagon.

107. Determine the side of a square, which shall be equal to the sum of two others, whose sides are given.

108. Find a right line whose square is equal to the difference of two squares, whose sides are given.

109. The square of the base of a triangle is less than, equal to, or greater than the sum of the squares of the sides, according as the vertical angle is less than, equal to, or greater than a right angle.

110. Through a given point within or without a circle, todraw a chord of given length.

111. The vertical angle of a triangle is less than, equal to, or greater than a right angle according as the square of the base is

less than, equal to, or greater than the sum of the squares of the sides.

112. Determine a square which shall be equal to the sum of any number of squares, whose sides are given.

113. The rectangle under any two lines, equals twice the rectangle under either of them, and half the other, to three times the rectangle under either of them and a third of the other, to $n$ times the rectangle under either of them, and an *nth* part of the other.

114. The square of the sum of any two lines equals the rectangle under the sum and either of them.

115. The square of the greater of two lines equals the rectangle under those lines, together with the rectangle under the greater and difference.

116. The rectangle under the sum of two lines, and one of them equals the square of that one, together with the rectangle under the lines.

117. The rectangle under two lines equals the square of the less, together with the rectangle under the less and difference.

118. The difference of the squares of two lines equals the rectangle under their sum and difference.

119. The difference between the rectangle under two lines, and the square of one of them is the rectangle under that one and their difference.

120. The difference of the squares of two lines, exceeds the square of their difference by twice the rectangle under the less and differenee.

121. The square of a line is four times the square of its half.

122. Half the square of a line is double the square of half the line.

123. The square of the sum of any two lines equals the sum of their squares, together with twice the rectangle under them.

124. The square of a line equals the sum of the squares of all the parts, together with twice the rectangle under every distinct pair of them.

125. The arithmetic mean between any two lines, is half the sum of the extremes, and the common difference is half the difference of the extremes.

126. The square of the arithmetic mean equals the rectangle

under the extremes, together with the square of the common difference.

127. The rectangle under any two lines together with the square of half their difference, equals the square of half their sum.

128. The square of the sum of two lines equals the rectangle under them, together with the square of half their difference.

129. The rectangle under the sum and difference of two lines, together with the square of the less, equals the square of the greater.

130. The rectangle under the parts of a line is the greatest when the parts are equal.

131. The sum of the squares of the parts into which a line is divided is a minimum, when the line is bisected and equals the square of half the line.

132. Of all rectangles having the same perimeter, the square contains the greatest area.

133. Of all rectangles equal in area. the square is contained by the least perimeter.

134. If a perpendicular be drawn from the vertex of a triangle to the base, the rectangle under the sum and difference of the sides equals the rectangle under the sum and difference of the segments.

135. The difference between the squares of the sides of a triangle equals twice the rectangle under the base and the distance of the perpendicular from the middle point.

136 If a line be drawn from the vertex of an isosceles triangle to the base or its production, the difference between the squares of this line and the side of the triangle is the rectangle under the segments of the base.

137. The square of the sum of two lines, the sum of their squares and the square of their difference are in arithmetic progression, the common difference being twice the rectangle under the lines.

138. The square of the sum of two lines equals four times the rectangle under them, together with the square of their difference.

139. The sum of the squares of any two lines equals twice the square of half their sum, and twice the square of half their difference,

140. Given the sum and difference of two magnitudes to find the magnitudes themselves.

141. In any rectangle, given two of the five following data:—

1. The sum of the sides.
2. The difference of the sides.
3. The area.
4. The sum of the squares of the side
5. The difference of the squares of the sides to determine the rectangle.

142. If a line be cut in extreme and mean ratio, the rectangle under its segments, equals the difference of their squares.

143. On the same supposition. If $l$, $P$, $p$, represent respectively the given line, the greater and lesser part,

Then (1.) $l^2 + p^2 = 3 P^2$ (2.) $(l + p)^2 = 5 P^2$
(3.) $l (P - p) = P p$ (4.) $p^2 = P. (P - p)$

144. To divide a given right line into two such parts, that the retangle contained by the whole line and one of the parts shall be *twice thrice....n* times the square of the other part.

145. If a line be drawn from the angular point of a triangle to the point of bisection of the opposite side, the sum of the squares of the other sides equals twice the sum of the squares of the bisector, and half the bisected side.

146. The sum of the squares of the sides of a quadrilateral figure equals the sum of the squares of the diagonals and four times the square of the line joining their points of bisection.

147. The sum of the squares of the sides of a parallelogram equals that of the diagonals.

148. If the sum of the squares of the sides of a quadrilateral figure equals the sum of the squares of the diagonals, the quadrilateral will be a parallelogram.

149. If from the three angles of a triangle lines be drawn to the points of bisection of the opposite sides; three times the sum of the squares of the sides, equals four times that of the bisectors.

150. On the same supposition the sum of the squares of the sides, equals three times the sum of the squares of the distances between the angles and the common intersection.

151. If with the middle point of a finite right line as centre a circle be described, the sums of the squares of the distances of all points in this circle, from the extremities of the right line, is a constant quantity.

152. If from any point within or without a rectangle, lines be drawn to the angular points, the sums of those which are drawn to the opposite angles are equal.

153. If two sides of a trapezium be parallel to each other: the squares of its diagonals are together equal to the squares of its two sides, which are not parallel, and twice the rectangle contained by its parallel sides.

154. The squares of the diagonals of a trapezium, are together double the squares of the two lines joining the bisections of the opposite sides.

155. Of all straight lines which can be drawn from two given points to meet on the convex circumference of a given circle, the sum of those two will be the least which make equal angles with the tangent at the point of concourse.

156. If a circle be described on the radius of another circle; any straight line drawn from the point where they meet to the outer circumference, is bisected by the interior one.

157. If two circles cut each other and from either point of intersection, diameters be drawn; the extremities of these diameters and the other points of intersectinn shall be in the same straight line.

158. Draw a straight line which shall touch two given circles.

159. If from a point in the circumference of a circle any number of chords be drawn, the locus of their points of bisection will be a circle.

160. Determine the arithmetic, geometric, and harmonic means between two given straight lines.

161. If from any two points in the circumference of a circle, there be drawn two straight lines, to a point in a tangent to that circle; they will make the greatest angle when drawn to the point of contact.

162. If from any point within an equilateral triangle perpendiculars be drawn to the sides, they are together equal to a perpendicular drawn from any of the angles to the opposite side.

163. If the points of bisection of the sides of a given triangle be joined; the triangle so formed will be one fourth of the given triangle.

164. Of all triangles upon the same base and between the same parallels, the isosceles triangle has the least perimeter.

165. To bisect a triangle by a line drawn from a given point in one of its sides.

166. To determine a point within a given triangle, from which lines drawn to the several angles will divide the triangle into three equal parts.

167. To trisect a triangle from a given point within it.

168. If from the three angles of a triangle lines be drawn to the points of bisection of the opposite sides, these lines will intersect each other in the same point.

169. To bisect a parallelogram by a line drawn from a given point in one of its sides.

170. If the sides of an equilateral and equiangular pentagon be produced to meet, the angles formed by these lines are together equal to two right angles.

171. If the sides of an equilateral and equiangular hexagon be produced to meet, the angles formed by these lines are together equal to four right angles.

172 Generally, if every pair of alternate sides of a convex figure be produced to meet the sum of the angles so formed, is $2n—8$ right angles, $n$ being the number of the sides.

173. *The area of any two parallelograms described on the two sides of a triangle, is equal to that of a parallelogram on the base whose side is equal and parallel to the line drawn from the vertex of the triangle to the intersection of the two sides of the former parallelograms produced to meet.

174. The perimeter of an isosceles triangle is greater than the perimeter of a rectangular parallelogram, which is of the same altitude with, and equal to, the given triangle.

175. Determine a point in a line given in position, to which lines drawn from two given points, may have the greatest difference possible.

176. To divide a given right line into two such parts, that the square of the one shall be equal to the rectangle contained by the other and a given line.

177. To describe an isosceles triangle upon a given finite right line.

178. To describe an equilateral triangle equal to a given isosceles triangle.

179. Having given the difference between the diameter and side of a square, to describe the square.

180. To divide a circle into any number of concentric equal annuli.

181. In any quadrilateral figure circumscribing a circle, the opposite sides are equal to half the perimeter.

182. If an equilateral triangle be inscribed in a circle, the square described on a side thereof, is equal to three times the square described upon the radius.

*Pappus's Theorem.

183. To inscribe a square in a given right angled isosceles triangle.

184. To inscribe a square in a given quadrant of a circle.

185. To inscribe a square in a given semi circle.

186. To inscribe a square in a given segment of a circle.

187. To describe three circles of equal diameters, which shall touch each other.

188. To determine how many equal circles may be placed round another circle of the same diameter, touching each other and the interior circle.

189. Prove that the hexagonal form of the honey comb arises, necessarily, from the pressure of the fluid and the elasticity of the comb.

190. Of all triangles on the same base and between the same parallels, the isosceles has the greatest verticle angle.

191. If an equilateral triangle be inscribed in a circle, and through the angular points, another be circumscribed, to determine the ratio which they bear to each other.

192. In any triangle, if perpendiculars be drawn from the angles, to the opposite sides; they will all meet in a point.

193. Given one angle, a side adjacent to it and the difference of the other two sides, to construct the triangle.

194. Given one angle, a side opposite to it and the difference of the other two sides, to construct the triangle.

195. Given one angle a side opposite to it, and the sum of the other two sides, to construct the triangle.

196. Given the vertical angle the line bisecting the base and the angle which the bisecting line makes with the base, to construct the triangle.

197. Given the vertical angle, the perpendicular drawn from it to the base, and the ratio of the segments of the base made by it, to construct the triangle.

198. Given the vertical angle, the difference of the two sides, containing it and the difference of the segments of the base made by a perpendicular from the vertex, to construct the triangle.

199. Given the vertical angle and the radii of the inscribed and circumscribing circles, to construct the triangle.

200. What data are necessary to establish the similitude of two triangles?

201. In any trapezium, if two opposite sides be bisected, the sum of the squares of the two bisected sides, together with the

squares of the diagonals is equal to the sum of the squares of the bisected sides together with four times the square of the line joining those points of bisection.

202. If squares be described on the three sides of a right angled triangle and the extremities of the adjacent sides be joined; the triangles so formed are equal to the given triangle and to each other.

203. The two triangles formed by drawing straight lines from any point within a parallelogram to the extremities of the two opposite sides, are together half of the parallelogram.

204. If in the sides of a square at equal distances from the four angles, four other points be taken, one in each side; the figure contained by the straight lines which join them, shall also be a square.

205. The sum of the diagonals of a trapezium, is less than the sum of any four lines which can be drawn to the four angles from any point within the figure, except from the intersection of the diagonals.

206. To determine the figure formed by joining the points of bisection of the sides of a trapezium and its ratio, to the trapezium.

207. If on the two sides of a right angled triangle squares be described, the lines joining the acute angles of the triangle and the opposite angles of the squares, will cut off equal segments from the sides.

208. If squares be described on the hypothenuse and sides of a right angled triangle, and the extremities of the sides of the former and the adjacent sides of the others be joined; the sum of the squares of the lines joining them will be equal to five times the square of the hypothenuse.

209. Two bodies starting from the same point, move in circles whose diameters are in the same right line, and as two to one, so as always to appear in a line passing through the starting point, compare their distances from that point at any given time.